STATISTICAL ENERGY ANALYSIS MODEL AND CONNECTORS FOR AUTOMOTIVE VIBRATION ISOLATION MOUNTS

Brooks P. Byam

Department of Mechanical Engineering
Michigan State University
East Lansing, MI 48824-1229

Clark J. Radcliffe

Department of Mechanical Engineering
Michigan State University
East Lansing, MI 48824-1229

ABSTRACT

Consumer comfort is a top priority in today's vehicle design. Reduction of noise and vibration enhances comfort and improves the overall quality of the vehicle. Linear elastic vibration isolation mounts are traditionally applied within complex structures to combat noise and vibration. Statistical Energy Analysis (SEA) is becoming an established and tested noise and vibration modeling methodology that is frequently used to model the flow of noise and vibration in complex structures. A SEA model of a linear elastic vibration isolation mount is developed here which enables SEA analysis of elastically isolated systems for the first time.

A vibration isolation mount model has been absent from SEA models because of the lack of an SEA formulation. Linear elastic vibration isolation mount SEA equations are developed here for the first time. The basis of SEA theory is energy sharing among vibrational mode groups. Traditionally, vibration isolation mounts were thought of as compliant elements with no modes suitable to SEA analysis. An example mount is defined and the number of energy storage modes computed. The results show that a linear elastic vibration isolation mount should be modeled in SEA analysis and provide the equations for a SEA isolation mount model. Also included is the analysis for three unique SEA connectors, a mount to acoustic volume connector, a beam to mount connector, and a mount to flat plate connector.

INTRODUCTION

The acoustic response of automobiles is a key property to the perception of product quality. Automotive vehicles are complex structures where the consumer is configured in close proximity to the power plant or drive train. For this reason acoustic design has become an integral part of the overall product design process. The implementation of acoustic modeling in the acoustic design process is the most efficient and cost-effective method to assure product quality. Statistical Energy Analysis (SEA) is accepted as an accurate and reliable noise and vibration transmission modeling methodology for automotive vehicles (Elliot and Friberg, 1988 & Fredo, 1988). SEA is a useful tool for acoustic design predicting the response of and power flow between model elements.

Linear elastic vibration isolation mount systems are common components of complex structures that involve a source of noise and vibration. Typically configured between the noise and vibration source and the rest of complex structure, the task of the vibration isolation system is to reduce the flow of noise and vibration (Pinnington and White, 1981). For example, a linear elastic engine mount in an automotive vehicle separates the engine from the cradle of the car. Structural vibration through and sound radiation from linear elastic mounts are significant vibro-acoustic energy transmissions in complex structures. Clearly, vibration isolation systems are mechanisms for transferring noise and vibration power. This leads to the question of a linear elastic mount vibration isolation system in SEA modeling.

SEA vibration isolation models are not included in SEA models because a SEA model formulation is unavailable. The impact that a linear elastic vibration isolation mount will have on a SEA model will be investigated by defining the system as a SEA element. This will be accomplished by a modal development of a mount. In the past, compliant vibration mounts and other linear elastic structures were considered to have a zero mode count (Ricks, 1987). Analysis of actual non-zero mode count allows an SEA linear elastic vibration isolation mount that enhances the overall effectiveness of SEA for analytical vibro-acoustic modeling.

The key component of SEA elements is energy storage (Lyon and DeJong, 1995). The foundation of SEA theory is energy sharing among resonators (elements), therefore, the elements in an SEA model should have comparable energy storage. SEA equations are power balances of the coupled elemental energies. Vibrational modes are the mechanisms that characterize energy storage in a resonator. The number of modes or the modal density is the most descriptive parameter of total energy storage in a SEA element. The modal density of the vibration isolation system SEA element relative magnitude to the modal density of other important SEA elements determines the validity of the system as a SEA subsystem. The modal density of a typical linear elastic mount vibration isolation system will be compared with the modal density of a typical SEA element to evaluate equivalent energy storage capability and hence their ability to share energy. Other SEA parameters are the coupling loss factor and the internal loss factor. Previously unavailable model equations for these relationships as well as element energy dissipation are derived below.

MODEL DEVELOPMENT

Development of a Statistical Energy Analysis (SEA) noise transmission model is dependent on the energy storage in the structure and the type of coupling among the substructures.

Vibration models contain substructures that are small and stiff connected by point couplings with near zero energy storage. SEA models involve substructures that are large and compliant with modal couplings and high energy storage. A three element vibration model and a three element SEA model are shown in Figures 1 a) & 1 b), respectively. M1 and M3 in Figure 1 a) are rigid masses and K2 is the spring constant. The motion of the vibration model involves the movement of each rigid mass as a whole. In the case of the SEA model, the motion is internal via waves within the physical substructures. The groups of modes created by this wave motion are considered the elements of the SEA model. These groups of modes (SEA elements) create the energy that is shared throughout the model through modal couplings to other groups of modes (SEA elements). Figure 1 b) illustrates the SEA power balance where E1, E2, and E3 are the element internal energies, P12 and P23 are the power flow losses due to the modal couplings, and P11, P22, and P33 are the power flow losses due to internal damping.

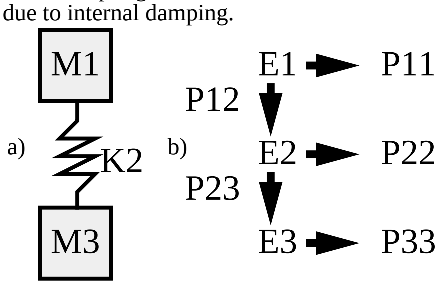

FIGURE 1: VIBRATION & SEA MODEL COMPARISON

Internal vibrational modes within the physical substructures are the mechanisms of energy storage. The number of modes within a substructure characterizes the capacity for energy storage in the corresponding SEA element. Energy storage is the essence of the SEA method and modal density for each SEA element is a critical component in SEA analysis. The modal density, $n_i(f)$, is used with element i modal energy, $e_i(f)$, to derive the element i total energy, $E_i(f)$, at a frequency band center frequency band f

$$E_i(f) = n_i(f)e_i(f) \tag{1}$$

The total element energy is the used within a SEA power balance relation between an element i and an element j.

$$\frac{1}{2\pi f} P_i = \eta_{ij} E_i - \eta_{ji} E_j + \eta_i E_i \tag{2}$$

P_i is the input power to element i, η_{ij} & η_{ji} are the coupling loss factors corresponding to modal couplings between elements, and η_i is the internal loss factor corresponding to internal damping of element i.

The derivation of a linear elastic vibration isolation mount system modal density requires a modal development of its dynamic behavior. Figure 2 a) shows a linear elastic vibration isolation mount system with N mounts configured between two flexible substructures. The SEA representation of the 3-element structure is shown in Figure 2 b). In the SEA model, the modes in all N mounts are considered as one energy storing element with energy E_{vis}. E_1 and E_2 are the total energies of the modes in flexible substructures, 1 and 2 respectively. Vibrational energy will flow from the energy storing modes in the first substructure to the energy storing modes in each mount of the vibration isolation system to the modes in the second substructure.

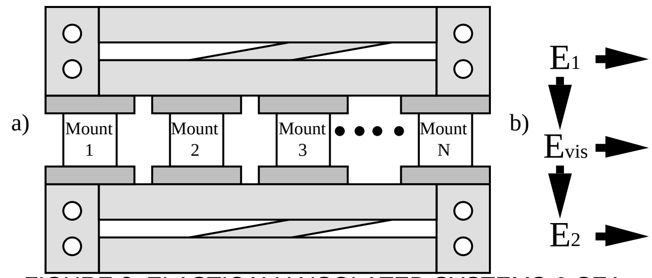

FIGURE 2: ELASTICALLY ISOLATED SYSTEMS & SEA REPRESENTATION

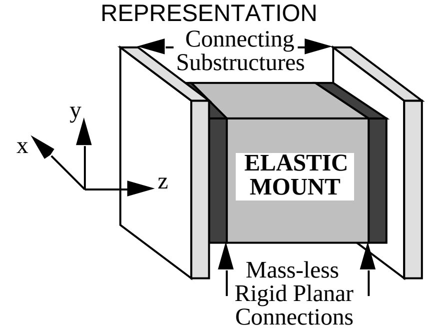

FIGURE 3: SINGLE MOUNT SCHEMATIC

Six degree of freedom (DOF) energy storing modes in each mount are excited by the modes of the flexible substructures. The connecting substructures perturb a single three-dimensional mount through mass-less rigid planar connections shown in the Figure 3. Shear and bending waves will be present in both the x and y directions (4 DOF) with longitudinal and torsion waves in the z direction (2 DOF). The waves are assumed to be uncoupled in this formulation. The modes present in each mount in the vibration isolation system will be governed by the six wave equations (Cremer, Heckl, and Ungar, 1973).

$$E\frac{\partial^2 u_{long_z}}{\partial z^2} = \rho\frac{\partial^2 u_{long_z}}{\partial t^2} \qquad EJ\frac{\partial^2 u_{torsion_z}}{\partial z^2} = \rho(1+\upsilon)\frac{\partial^2 u_{torsion_z}}{\partial t^2}$$
$$E\frac{\partial^2 u_{shear_x}}{\partial x^2} = \rho(1+\upsilon)\frac{\partial^2 u_{shear_x}}{\partial t^2} \qquad E\frac{\partial^2 u_{shear_y}}{\partial y^2} = \rho(1+\upsilon)\frac{\partial^2 u_{shear_y}}{\partial t^2}$$
$$EI_x\frac{\partial^4 u_{bending_x}}{\partial x^4} = \rho A\frac{\partial^2 u_{bending_x}}{\partial t^2} \qquad EI_y\frac{\partial^4 u_{bending_y}}{\partial y^4} = \rho A\frac{\partial^2 u_{bending_y}}{\partial t^2} \tag{3}$$

u is a displacement, E is Young's modulus, υ is Poisson's ratio, A is a mount cross-sectional area, I_x and I_y are the second moments of area, J is the second polar moment of area, and ρ is the density. Figure 4 a) - f) illustrates the six energy storing modes defined by the six equations in (3).

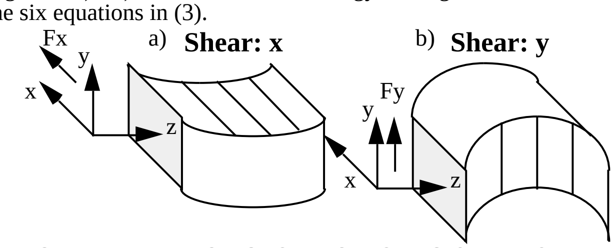

FIGURE 4: ENERGY STORING MODES OF A MOUNT

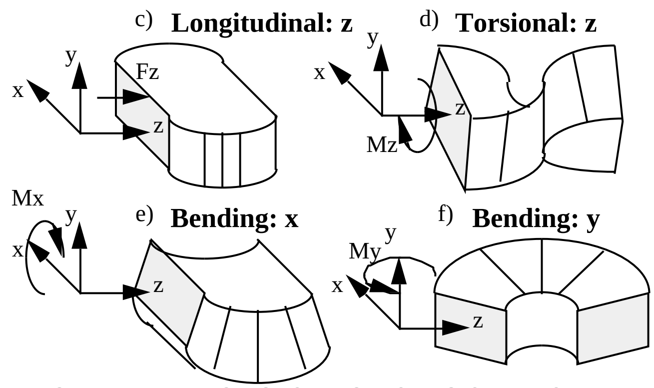

FIGURE 4: ENERGY STORING MODES OF A MOUNT

Six wave velocities results from solving (3)

$$c_{long_z} = \sqrt{\frac{E}{\rho}} \qquad c_{torsion_z} = \sqrt{\frac{EJ}{\rho(1+\upsilon)}}$$
$$c_{shear_x} = \sqrt{\frac{E}{\rho(1+\upsilon)}} \qquad c_{shear_y} = \sqrt{\frac{E}{\rho(1+\upsilon)}}$$
$$c_{bending_x} = \sqrt[4]{\frac{EI_x}{\rho A}}\sqrt{2\pi f} \qquad c_{bending_y} = \sqrt[4]{\frac{EI_y}{\rho A}}\sqrt{2\pi f} \tag{4}$$

These velocities are found independent of boundary conditions and type of structure. The definition of mount wave velocities leads to the derivation of a modal density for each mount which characterizes its energy storage capability.

MODAL DENSITY, *n*

Defining a mount modal density results in a term that characterizes its energy storage capacity. Considering of the mount modal density as an energy storage term, N mount modal densities may be summed together to obtain an overall modal density of an N-mount vibration isolation system. The modal density of a mount is the sum of longitudinal, torsion, two bending, and two shear modal density components.

$$n_{mount} = n_{long_z} + n_{torsion_z} + n_{shear_x} + n_{shear_y} + n_{bending_x} + n_{bending_y} \tag{5}$$

Each modal density component is twice the length over the wave velocity (Lyon and DeJong, 1995). Using this definition, the results of (4), and choosing the length between connecting faces of a rectangular mount as L, yields the longitudinal, shear, and bending modal densities for a mount in the vibration isolation system

$$n_{long_z} = 2L\sqrt{\frac{\rho}{E}} \qquad n_{torsion_z} = 2L\sqrt{\frac{\rho(1+\upsilon)}{EJ}}$$
$$n_{shear_x} = 2L\sqrt{\frac{\rho(1+\upsilon)}{E}} \qquad n_{bending_x} = \frac{2L}{\sqrt{2\pi f}}\sqrt[4]{\frac{\rho A}{EI_x}}$$
$$n_{shear_y} = 2L\sqrt{\frac{\rho(1+\upsilon)}{E}} \qquad n_{bending_y} = \frac{2L}{\sqrt{2\pi f}}\sqrt[4]{\frac{\rho A}{EI_y}} \tag{6}$$

Substituting the results of the above equation in (5), the total modal density of a mount is

$$n_{mount} = 2L\sqrt{\frac{\rho}{E}} + 4L\sqrt{\frac{\rho(1+\upsilon)}{E}} + 2L\sqrt{\frac{\rho(1+\upsilon)}{EJ}} + \frac{4L}{\sqrt{2\pi f}}\sqrt[4]{\frac{\rho A}{EI_x} + \frac{\rho A}{EI_y}} \tag{7}$$

n_{mount} is the modal density of a mount in the vibration isolation system. The modal density of the system will be defined by the mount modal density multiplied by the number of mounts, N, present in the system.

$$n_{system} = \mathrm{N} \times n_{mount} \tag{8}$$

Modes are the mechanisms of energy storage. Modal density is a characterization of the energy storage in a structure. n_{system} is the energy storage characterization of a N-mount vibration isolation system.

TYPICAL AUTOMOTIVE MOUNT MODAL DENSITY

The modal density of a typical linear elastic vibration isolation mount system will be compared with the modal density of an automotive structural component. If the modal densities are similar, it will indicate that there is an equivalent capability for energy storage in the vibration isolation system as compared to other automotive SEA elements, and the vibration isolation system can be considered an important SEA element. The number of mounts will be taken as 4 (N = 4). Figure 5 outlines the dimensions of the example mount to be used in this study.

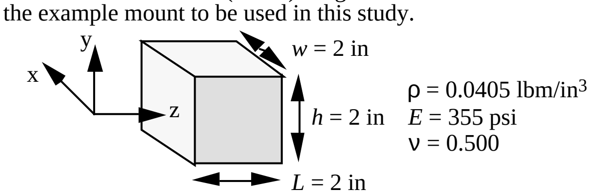

FIGURE 5: EXAMPLE MOUNT DIMENSIONS

The vibration isolation system modal density (8) is compared with the modal density of a 48" X 16" steel flat plate with a 0.04" thickness. The dimensions of the steel flat plate are characteristic of those for a typical body panel on a door or engine compartment of an automotive vehicle. A comparison of the number of modes (bandwidth multiplied by the modal density) in the 1/3 Octave Bandwidths at the eleven center frequencies from 500 Hz to 5000 Hz of these two elements is shown in Table 1. This shows that a four-mount vibration isolation system has capability for energy storage comparable to other automotive structural components.

Center Frequency (Hz) 1/3 Octave Bandwidth	Number of Modes Vibration Isolation System 4 Mounts Rubber 2"X2"X 2"	Number of Modes Small Body Panel Steel 48" X 16" 0.1" thick
500	6.593	4.025
630	8.039	5.110
800	9.694	6.405
1000	11.83	8.085
1250	14.45	10.18
1600	17.57	12.77
2000	21.63	16.13
2500	26.53	20.26
3150	32.71	25.55
4000	40.32	32.16
5000	49.81	40.46

TABLE 1: NUMBER OF MODES IN 1/3 OCTAVE BANDWIDTH

MOUNT INTERNAL LOSS FACTOR, η_i

The internal loss factor is the damping dissipation of the SEA element. It appears directly in the SEA power balance equation (2). SEA models consist of weakly coupled groups of vibrational and acoustic modes. The weak coupling implies that the energy storage mechanisms of substructures are not changed when they are connected. SEA theory says that a coupling energy flow exists between a set of ideal modes. Weak couplings imply that the dissipation energy flows in SEA elements is dominate. Damping domination places a great deal of importance on the identification of the internal loss factors of the mode groups in the SEA models. The internal loss factor is defined as twice of the damping ratio. The simple relationship is presented in (9) (Lyon, 1987). However, the damping ratio, ζ, must be determined experimentally.

$$\eta_{mount} = 2\zeta_{mount} \tag{9}$$

Typical values of internal loss factor for rubber are 0.1 - 0.5.

MOUNT COUPLING LOSS FACTORS, η_{ij}

The coupling loss factor is the SEA mechanism that characterizes the dissipation of the modal energy through the modal energy transfer between SEA elements. The coupling loss factor appears in the SEA power balance equation (2). In this study, the systems have the following characteristics: only the sources being considered exist, the response and excitation are proportional and at the same frequency, and the response changes in the same manner as the excitation. So, η_{ij} has a reciprocal relationship with η_{ji}, specifically (Lyon, 1987)

$$\eta_{ij} = \eta_{ji}\left(\frac{n_j}{n_i}\right) \tag{10}$$

Two coupling losses are important to vibration isolation mount modeling: a radiation loss of the mount structure to an acoustic volume and a structural loss between the mount and other elements.

Radiation coupling loss of a structural subsystem as a result of a modal coupling to an acoustic volume is given by

$$\eta_{struct-act} = \frac{P_{rad}}{\omega M_{struct}\left\langle v^2\right\rangle_{struct}} \tag{11}$$

where P_{rad} is the power radiated to the volume and $\omega M_{struct}\left\langle v^2\right\rangle_{struct}$ is power in the structure (Lyon, 1987). The power radiated by a structure is defined by

$$P_{rad} = \left(\left\langle v^2\right\rangle A\right)_{struct}(\rho c)_{act}\sigma_{rad} \tag{12}$$

where ρ is the density of air, c is the speed of sound in air, A is the structural surface area, and σ_{rad} is the radiation efficiency (Lyon, 1987). Radiation efficiency is dependent on a parameter called the critical frequency, given by

$$f_c = \frac{c^2}{2\pi}\left(\frac{A}{I}\sqrt{\frac{\rho}{E}}\right)_{struct} \tag{13}$$

where I is the second moment of area (Lyon, 1987). f_c has the units of Hertz per unit width. Critical frequency is the frequency where the bending wave speed in a structure equals the speed of sound. Free sound waves are radiated from a structure when the dominate source of sound radiation, bending waves, travel through a structure at the speed of sound. The supersonic bending waves create a radiation angle or a mach angle which can be related to the critical frequency

$$\sin\theta = \sqrt{\frac{f_c}{f}} \tag{14}$$

The structure velocity is equal to the wave front particle velocity, $\left(p/\rho c\right)_{act}$, times the cosine of the Mach angle. Using this relation, a radiation impedance of structure velocity to the generated pressure in the acoustic volume is given by

$$Z_{rad} = \frac{A_{struct}(\rho c)_{act}}{\sqrt{1-f_c/f}} \tag{15}$$

So the radiation efficiency for supersonic bending waves in structures is (Lyon, 1987)

$$\sigma_{rad} = \frac{1}{\sqrt{1-f_c/f}} \tag{16}$$

Radiation loss in the subsonic case (below the critical frequency) bending waves do not radiate until the wave incidents an obstacle or a boundary. The reaction force from this interaction can be related to the sound pressure in the local sound field similarly to the above analysis for supersonic waves. This development is lengthy and will not be presented here, however, the full analysis is given in Lyon (1987, Chap. 5). The radiation efficiency for the subsonic bending waves is

$$\sigma_{rad} = \frac{2\pi}{c_{act}}\left(\frac{CI}{A^2}\sqrt{\frac{E}{\rho}}\right)_{act}\sin^{-1}\left(\frac{f}{f_c}\right)^{\frac{1}{2}} \tag{17}$$

where C is the perimeter of the structure.

The coupling loss factor for the mount to acoustic volume connector is

$$\eta_{mnt-act} = \begin{cases} \dfrac{(\rho c)_{act}}{2\pi f}\left(\dfrac{A}{M}\right)_{mnt}\dfrac{1}{\sqrt{1-f_c/f}} & f > f_c \\[2ex] \dfrac{\rho_{act}}{f}\left(\dfrac{CI}{MA}\sqrt{\dfrac{E}{\rho}}\right)_{mnt}\sin^{-1}\left(\dfrac{f}{f_c}\right)^{\frac{1}{2}} & f < f_c \end{cases} \tag{18}$$

Structural coupling loss due to modal coupling of two structural subsystems requires the definition of some preliminary parameters. The applied load forces and induced velocities within a structural subsystem are related by an input and a transfer mobility (Lyon, 1987). A mobility, Υ, is the ratio of complex velocity, v, over a complex load force, l.

$$v = \Upsilon l \tag{19}$$

The mean square response over a frequency bandwidth Δf of a single subsystem is defined as

$$\frac{\langle v^2\rangle_{\Delta f}}{\langle l^2\rangle_{\Delta f}} = |\Upsilon|^2 = \frac{G}{\omega\eta M} \tag{20}$$

where G is the conductance, η is the internal loss factor, and M is the mass. Conductance is the real part of the complex mobility and is a ratio of modal density to mass (Lyon, 1987).

$$G = \frac{n}{4M} \tag{21}$$

G represents the ability of the structure to absorb power.

Connecting subsystems requires the use of transfer and input mobilities. The transfer mobility, Υ_{ij}, relates velocity at i to load force at j on the subsystem. The input mobility, Υ_{ii}, relates velocity at i to load force at i on the subsystem. The mobilities of a subsystem are collected into a mobility transfer matrix (Lyon, 1987)

$$\begin{pmatrix} v_i \\ v_j \end{pmatrix} = \begin{bmatrix} \Upsilon_{ii} & \Upsilon_{ij} \\ \Upsilon_{ij} & \Upsilon_{jj} \end{bmatrix} \begin{pmatrix} l_i \\ l_j \end{pmatrix} \tag{22}$$

It is helpful to write connected subsystems in a two-port description to easily obtain relations between the velocities and load forces of the different subsystems. Subsystems i and j are combined by a two port bond graph method where there is compatibility at the junctions (Karnopp and Rosenberg, 1975). Each port of the bond graph has a flow variable and an effort variable. The connection of two 2-port subsystems, i and j, at a junction **M** is shown in Figure 6.

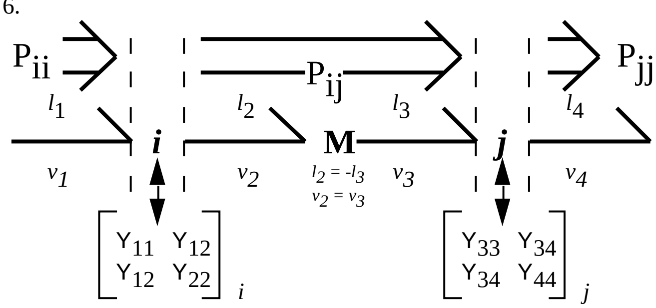

FIGURE 6: 2-PORT BOND GRAPH SUBSYSTEMS CONNECTED AT A JUNCTION

It can be shown, using the 2-port bond graph method and the transfer mobility matrices of subsystems i and j that the velocity v_4 is related to the load force l_1

$$v_4 = \frac{\Upsilon_{12}\Upsilon_{34}}{\Upsilon_{22}+\Upsilon_{33}} l_1 \tag{23}$$

Defining the structural coupling loss requires rewriting the velocity at port four, v_4, in terms of the mean square response of structure j when structure i is driven by a band of noise of some frequency interval Δf.

$$\frac{\langle v_4^2\rangle_{\Delta f}}{\langle l_1^2\rangle_{\Delta f}} = \frac{|\Upsilon_{12}|^2|\Upsilon_{34}|^2}{|\Upsilon_{22}+\Upsilon_{33}|^2} \tag{24}$$

In the above equation, $\langle l_1\rangle_{\Delta f}^2$ is the band of noise. $\langle v_2^2\rangle_{\Delta f}$ and $\langle v_4^2\rangle_{\Delta f}$ are the free motions of the subsystems i and j and will be renamed $\langle v_i^2\rangle_{\Delta f} = \langle v_2^2\rangle_{\Delta f}$ and $\langle v_j^2\rangle_{\Delta f} = \langle v_4^2\rangle_{\Delta f}$. This is done to make the equations functions of the entire subsystems not the internal ports. $|\Upsilon_{12}|^2$ may now be represented by $\langle v_i^2\rangle_{\Delta f} / \langle l_1^2\rangle_{\Delta f}$ and $|\Upsilon_{34}|^2$ may be written as the mean square response of system j, $G_j/\omega\eta_j M_j$. Substituting, (24) may be rewritten as (the Δf as been dropped for convenience).

$$\frac{\langle v_j^2\rangle}{\langle l_1^2\rangle} = \frac{\langle v_i^2\rangle}{\langle l_1^2\rangle} \frac{1}{|\Upsilon_i+\Upsilon_j|^2} \frac{G_j}{\omega\eta_j M_j} \tag{25}$$

Υ_i and Υ_j are the structural mobilities at the coupling. The above equation will now be rewritten in terms of the modal energy per frequency bandwidth. The modal energies of systems i and j are $M_i\langle v_i^2\rangle / n_i\Delta f$ and $M_j\langle v_j^2\rangle / n_j\Delta f$ respectively. Writing (25) in terms of the system's modal energies requires both sides of the equation to be multiplied by $M_jG_i/n_j\Delta f$.

$$G_i \frac{M_j\langle v_j^2\rangle}{n_j\Delta f} = \frac{\langle v_i^2\rangle}{\Delta f} \frac{G_iG_j}{|\Upsilon_i+\Upsilon_j|^2} \frac{1}{n_j\omega\eta_j} \tag{26}$$

Set G_i on the left hand side of the above equation equal to $n_i/4M_i$ and move it to the right hand side resulting in

$$\frac{M_j\langle v_j^2\rangle}{n_j\Delta f} = \left(\frac{4}{\omega n_j} \frac{G_iG_j}{|\Upsilon_i+\Upsilon_j|^2}\right) \frac{1}{\eta_j} \frac{M_i\langle v_i^2\rangle}{n_i\Delta f} \tag{27}$$

The above equation shows that the modal energy of system j is equal to the source system i modal energy times a ratio of the quantity in parentheses and the internal loss factor η_j. The quantity in parentheses is the coupling loss factor which is dependent on the modal density of system j and the properties at the junction of system i and j.

$$\eta_{ji} = \frac{4}{\omega n_j} \frac{G_iG_j}{|\Upsilon_i+\Upsilon_j|^2} \tag{28}$$

TYPICAL AUTOMOTIVE MOUNT COUPLING

A typical automotive vibration isolation system is configured between a beam and a flat plate is shown in Figure 7. Any load force perpendicular to the beam's axis will result in bending energy storage in the beam. The vibration isolation mount will have longitudinal, shear, and bending energy storage and the flat plate, bending energy storage.

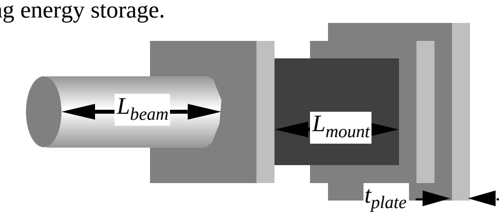

FIGURE 7: TYPICAL AUTOMOTIVE MOUNT COUPLING

The coupling locations, assuming that the substructures are connected rigidly at their faces, for the beam-mount connection with respect to the beam are (0, 0, L_{beam}) and with respect to the mount (0, 0, 0). The mount-plate coupling locations with respect to the mount are (0, 0, L_{mount}) and with respect to the plate (0, 0, 0).

The derivation of the mobilities will follow Goyder and White (1980) for the three elements in the beam-mount-plate model and requires the wave equations presented in (3). As an example, the longitudinal wave motion mobility of the mount in the beam-mount connection will be derived. The longitudinal wave equation in equation (3) is rewritten with a harmonic complex load force.

$$\rho\frac{\partial^2 u}{\partial t^2} = E\frac{\partial^2 u}{\partial z^2} + Le^{-i\omega t} \tag{29}$$

For a harmonic time dependence $u = e^{-i\omega t}$

$$\frac{\partial^2 u}{\partial z^2} + \omega^2\frac{\rho}{E}u = -\frac{L}{AE} \tag{30}$$

Spatial Fourier transforms and contour integration yields the mobility (Goyder and White, 1980)

$$\Upsilon = \frac{v}{L} = \frac{i\omega u}{L} = \frac{1}{2A\sqrt{\rho E}}e^{i\omega\sqrt{\frac{\rho}{E}}z} \tag{31}$$

The beam-mount with respect to the mount coupling location, z, can be substituted into (32) to obtain the longitudinal wave motion mount mobility for the beam-mount connection.

$$\Upsilon_{mount_{long}}\Big|_{beam-mount} = \frac{1}{2A\sqrt{\rho E}} \tag{32}$$

Similar equation manipulations may be applied to all wave motions in the two connections (beam-to-mount, mount-to-plate). The resulting four mobilities required for the two couplings beam-to-mount and mount-to-plate are shown in equation (34).

$$\begin{aligned}
\Upsilon_{beam}\Big|_{beam-mount} &= \frac{1}{4A\rho\sqrt{\omega}}\sqrt[4]{\frac{A\rho}{EI}}\left[e^{i\sqrt{\omega}\sqrt[4]{\frac{\rho A}{EI}}L_{beam}} - ie^{-\sqrt{\omega}\sqrt[4]{\frac{\rho A}{EI}}L_{beam}}\right]\\
\Upsilon_{mount}\Big|_{beam-mount} &= \frac{1}{2A\sqrt{\rho E}} + \frac{1}{2A\sqrt{\rho E(1+\upsilon)}} + \frac{(1+i)\sqrt{\omega}}{4EI}\sqrt[4]{\frac{EI}{A\rho}}\\
\Upsilon_{mount}\Big|_{mount-plate} &= \frac{1}{2A\sqrt{\rho E}}e^{i\omega\sqrt{\frac{\rho}{E}}L_{mount}} + \frac{1}{2A\sqrt{\rho E(1+\upsilon)}}e^{i\omega\sqrt{\frac{\rho(1+\upsilon)}{E}}L_{mount}}\\
&\quad + \frac{\sqrt{\omega}}{4EI}\sqrt[4]{\frac{EI}{A\rho}}\left[e^{i\sqrt{\omega}\sqrt[4]{\frac{\rho A}{EI}}L_{mount}} + ie^{-\sqrt{\omega}\sqrt[4]{\frac{\rho A}{EI}}L_{mount}}\right]\\
\Upsilon_{plate}\Big|_{mount-plate} &= \frac{1}{16t^2}\sqrt{\frac{3(1-\upsilon)}{\rho E}}
\end{aligned}$$

(33)

The coupling loss factors for a beam-to-mount connector and a mount-to-plate connector may now be obtained from (33), (21), (8), (28), and with the modal densities of a plate and a beam.

$$\eta_{beam-mnt} = \frac{\dfrac{n_{mnt}}{2\pi f\left(4M_{mnt}M_{beam}\right)}}{\left|\left(\dfrac{1}{2A\sqrt{\rho E}} + \dfrac{1}{2A\sqrt{\rho E(1+\upsilon)}} + \dfrac{(1+i)\sqrt{\omega}}{4EI}\sqrt[4]{\dfrac{EI}{A\rho}}\right)_{mnt} + \left(\dfrac{1}{4A\rho\sqrt{\omega}}\sqrt[4]{\dfrac{A\rho}{EI}}\left[e^{i\sqrt{\omega}\sqrt[4]{\frac{\rho A}{EI}}L_{beam}} - ie^{-\sqrt{\omega}\sqrt[4]{\frac{\rho A}{EI}}L_{beam}}\right]\right)_{beam}\right|^2}$$

$$\eta_{mnt-plate} = \frac{\dfrac{n_{plate}}{2\pi f\left(4M_{mnt}M_{plate}\right)}}{\left|\left(\dfrac{1}{2A\sqrt{\rho E}}e^{i\omega\sqrt{\frac{\rho}{E}}L_{mount}} + \dfrac{1}{2A\sqrt{\rho E(1+\upsilon)}}e^{i\omega\sqrt{\frac{\rho(1+\upsilon)}{E}}L_{mount}} + + \dfrac{\sqrt{\omega}}{4EI}\sqrt[4]{\dfrac{EI}{A\rho}}\left[e^{i\sqrt{\omega}\sqrt[4]{\frac{\rho A}{EI}}L_{mount}} + ie^{-\sqrt{\omega}\sqrt[4]{\frac{\rho A}{EI}}L_{mount}}\right]\right)_{mnt} + \left(\dfrac{1}{16t^2}\sqrt{\dfrac{3(1-\upsilon)}{\rho E}}\right)_{plate}\right|}$$

(34)

CONCLUSIONS

A linear elastic vibration isolation mount system is an important SEA element. Linear elastic mounts have the capability to store a comparable amount of energy as other automotive vehicle SEA elements. The modal density of a linear elastic engine mount is of the same magnitude of common automotive vehicle SEA elements. SEA equations have been developed for the linear elastic vibration isolation mount system.

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