LOAD TRANSFORMATION MATRIX
Revision A

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Introduction

The tutorial builds upon the component mode synthesis method in Reference 1.

The dynamic response of a system is modeled as

$$M\ddot{x} + K x = F \tag{1}$$

where

M	is the mass matrix
K	is the stiffness matrix
F	is the force vector
x	is the displacement vector

The mass and stiffness coefficients are determined by a finite element model.

The physical coordinates are separated into a set of interface coordinates x_I and a set of noninterface (interior) coordinates x_N.

The matrices and vectors in equation (1) are then partitioned as

$$\begin{bmatrix} M_{NN} & M_{NI} \\ M_{NI}{}^{T} & M_{II} \end{bmatrix} \begin{bmatrix} \ddot{x}_N \\ \ddot{x}_I \end{bmatrix} + \begin{bmatrix} K_{NN} & K_{NI} \\ K_{NI}{}^{T} & K_{II} \end{bmatrix} \begin{bmatrix} x_N \\ x_I \end{bmatrix} = \begin{bmatrix} F_N \\ F_I \end{bmatrix} \tag{2}$$

Solve for x_N using the upper half of equation (2).

$$M_{NN}\ddot{x}_N + M_{NI}\ddot{x}_I + K_{NN}x_N + K_{NI}x_I = F_N \tag{3}$$

$$K_{NN}x_N = F_N - M_{NN}\ddot{x}_N - M_{NI}\ddot{x}_I - K_{NI}x_I \tag{4}$$

$$x_N = K_{NN}^{-1}F_N - K_{NN}^{-1}M_{NN}\ddot{x}_N - K_{NN}^{-1}M_{NI}\ddot{x}_I - K_{NN}^{-1}K_{NI}x_I \tag{5}$$

The internal dynamic loads L of a payload or spacecraft are derived from the displacements x using a load transformation matrix LTM where

$$L = (LTM)_x = [LTM_N \quad LTM_I]\begin{bmatrix} x_N \\ x_I \end{bmatrix} \tag{6}$$

The LTM is derived from the FEM stiffness model. Each row of the LTM yields an internal load resulting from displacement s of the non-interface displacements x_N and interface displacements x_I.

The load is a general term that could represent a bending moment, shear force, stress, or relative displacement where there is a concern about loss of clearance.

The accuracy of the dynamic loads is increased by reformulating the transformation to involve inertial forces rather than displacements. This is due to the truncation of system modes.

Substitute equation (5) into (6).

$$L = [LTM_N \quad LTM_I]\begin{bmatrix} K_{NN}^{-1}F_N - K_{NN}^{-1}M_{NN}\ddot{x}_N - K_{NN}^{-1}M_{NI}\ddot{x}_I - K_{NN}^{-1}K_{NI}x_I \\ x_I \end{bmatrix} \tag{7}$$

$$\begin{aligned} L = \quad & LTM_N K_{NN}^{-1}F_N - LTM_N K_{NN}^{-1}M_{NN}\ddot{x}_N - LTM_N K_{NN}^{-1}M_{NI}\ddot{x}_I \\ & - LTM_N K_{NN}^{-1}K_{NI}x_I - LTM_I\, x_I \end{aligned} \tag{8}$$

$$L = LTM_N K_{NN}^{-1}\left[-M_{NN}\ddot{x}_N - M_{NI}\ddot{x}_I + F_N\right]$$

$$+\left[-LTM_N K_{NN}^{-1} K_{NI} - LTM_I\right]x_I \tag{9}$$

Recall from Reference 1 that the constraint mode matrix ϕ_{CN} is

$$\phi_{CN} = -K_{NN}^{-1} K_{NI} \tag{10}$$

$$L = LTM_N K_{NN}^{-1}\left[-M_{NN}\ddot{x}_N - M_{NI}\ddot{x}_I + F_N\right]$$

$$+\left[-LTM_N \phi_{CN} - LTM_I\right]x_I \tag{11}$$

References

1. T. Irvine, Component Mode Synthesis, Fixed-Interface Model, Vibrationdata, 2004.
2. NASA-HDBK-7005, Dynamic Environmental Criteria, 2001.