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A SIMPLISTIC LOOK AT LIMIT STRESSES FROM RANDOM LOADING

By H.M. Lee

Structures and Dynamics Laboratory
Science and Engineering Directorate

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George C. Marshall Space Flight Center

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INTRODUCTION

Since random loads play such an important role in the design, analysis, and testing of most space shuttle payload components and experiments, the structures and dynamics community has long desired to more fully understand the relationship between the random load environment and the actual stresses resulting from that environment. The current philosophy at MSFC for calculation of random load factors embraces a statistical philosophy which utilizes Miles' equation:

$$G_{pk} = 3\sqrt{\frac{\Pi}{2} \cdot Q \cdot f_n \cdot PSD_i} \ ,$$

where

G_{pk} = peak random load factor (limit)

Q = resonant amplification factor

f_n = component natural frequency, Hz

PSD_i = input qualification criteria at f_n, G^2/Hz.

This equation involves calculation of the loads based on (1) analytical or tested values for significant resonant frequencies (f_n), (2) an historically based damping value of 5 percent (Q = 10) or component measured damping from testing, (3) the magnitude of the maximum expected flight environment at resonance (PSD_i), and (4) a statistically 3σ definition of peak load. If you remove the crest factor of 3 from the equation, the remaining expression, $\sqrt{\frac{\Pi}{2} \cdot Q \cdot f_n \cdot PSD_i}$, represents the root mean square response ($G_{\rm rms}$) of the component. This assumes that the component is a single degree-of-freedom harmonic oscillator driven at all frequencies by a white noise environment at constant PSD level and that the component does not affect the input. From a statistical point of view, the $G_{\rm rms}$ response can be set equal to the standard deviation (σ) by assuming that the realized ensemble of random input time histories are best represented by a Gaussian distribution with a mean of zero. Under these conditions the $G_{\rm rms}$ response is a 1σ response. Multiplying the $G_{\rm rms}$ by the crest factor 3 produces the well known 3σ response value which has a 99.73-percent probability of being greater than any instantaneous random load encountered. In Miles' equation, the other critical probabilistic term is the qualification input criteria value (PSD_i) at the component's natural frequency (f_n). From the historical data base, a 97.50-percent probability level is calculated (a 1.96σ value). This level then becomes the basis from which the actual component criteria is developed (fig. 1). Statistically, the criteria assures the analyst that the flight loads have only a 2.5-percent probability of an exceedance. Further confidence in the analytically derived criteria is gained from the fact that the criteria is created from straight-line enveloping of the data, and from the requirement to hard mount components during vibration testing.

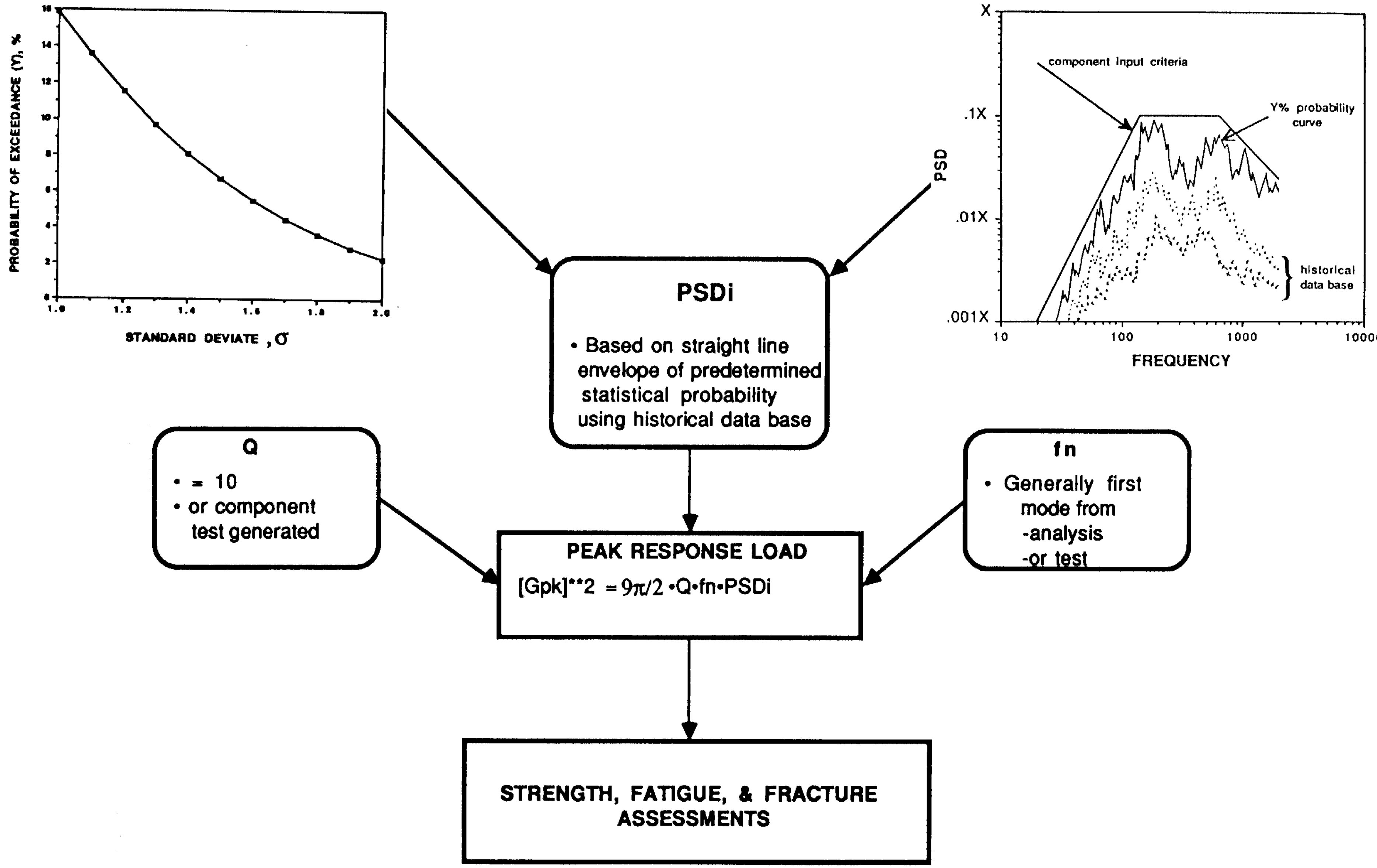


Figure 1. Statistical philosophy flow.

From the strength assessment and static test verification arena, the Miles' equation random load factors are applied as though they were truly a static loading. Thus, the limit static loads are, by definition, equivalent to these limit dynamic loads. The real questions to be answered are (1) is this a sound assumption or does the loads derivation process produce unjustifiable results? and (2) is there another approach that should be pursued both analytically and empirically?

Utilizing a simple continuous beam, the author will attempt to show some potential relationships between the dynamic and static limit loads and will try to answer the above questions.

THE CONTINUOUS BEAM

Analytical structural systems, which have their masses and elastic forces distributed rather than lumped together in concentrated masses and springs, belong to the class of vibrations of continuous media.[1] Since these systems have an infinite number of coordinates defining their configuration, they, of necessity, also possess an infinite number of natural frequencies and natural modes of vibration.

Vibration of the continuous beam is governed by the partial differential equation of motion:

$$\frac{\partial^2 Y}{\partial t^2} + a^2 \frac{\partial^4 Y}{\partial X^4} = 0 \ ,$$

where,

Y = deflection of beam, in

X = coordinate along length of beam, in

$a^2 = EIg/A\gamma$, 1/in^3

A = section area, in^2

γ = specific weight of beam, lb/in^3

ÉI = flexural rigidity of beam, lb-in^2 .

In order to predict the forced response of a structure, it is desirable to define the normal modes with this general solution equation. The normal modes obtained from this differential equation can be shown as one function:

$$Y_{i(x)} = y_i \left\{ \sin \frac{i\Pi X}{L} \right\} \{ \sin \omega_i t \} \ , \tag{1}$$

where y_i = maximum single amplitude displacement of i^{th} mode.

Figure 2 depicts the case of the pinned-pinned continuous beam with a mass located at the center. In order to calculate the natural frequencies for this potentially mass loaded beam, an equation evaluating the energy equilibrium condition must be developed. An examination of the beam as it undergoes vibration reveals that when the deflection is a maximum, all parts of the beam (including the concentrated

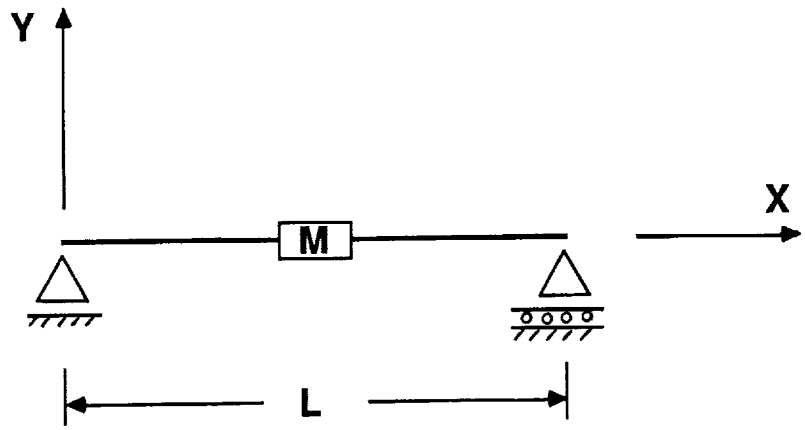


Figure 2. Continuous beam model.

mass) are motionless. When this occurs, the energy associated with the vibration has become converted into elastic strain energy (S.E.). As the beam then passes through its static equilibrium position, all strain energy is in the form of kinetic energy (K.E.). For the conservation of energy, the strain energy in the position of maximum deflection must equal the kinetic energy when passing through the static equilibrium position.

Hence: $$S.E._{max} = K.E._{max} = \text{constant}$$

$$S.E._{max} = \frac{1}{2}\,(\text{Stiffness})(\text{Displacement})^2 \ ,$$

$$= \frac{EI}{2}\int_0^L \left[\frac{\partial^2 Y_{i(x)}}{\partial X^2}\right]^2 dX \ ,$$

and

$$K.E. = \frac{1}{2}\,(\text{Mass})(\text{Velocity})^2 \ ,$$

$$= \frac{1}{2}\, m\omega_i^2 \int_0^L Y_{i(x)}^2 dX + \frac{1}{2}\, M\omega_i^2 y_i^2 \ ,$$

where:

m = mass per unit length of beam, lb-s^2/in^2

ω_i = natural frequency (rad/s) of i^{th} mode

M = concentrated mass, lb-s^2/in^2.

Substituting

$$Y_{i(x)} = y_i \left\{\sin \frac{i\Pi X}{L}\right\} \{\sin \omega_i t\} \ ,$$

into the equations for S.E. and K.E. and assuming that the term (sin $\omega_i t$) is equal to 1:

$$\text{S.E.} = \frac{i^4\Pi^4 EIy_i^2}{4L^3} , \tag{2}$$

$$\text{K.E.} = \frac{m\omega_i^2 L y_i^2}{4} + \frac{M\omega_i^2 y_i^2}{2} . \tag{3}$$

Equating these results in a solution for natural frequency,

$$\omega_i = i^2\Pi^2 \sqrt{\frac{EI}{2L^3\left(M + \frac{mL}{2}\right)}} ,$$

which can be rewritten as:

$$f_i = \frac{i^2\Pi}{2} \sqrt{\frac{EI}{2L^3\left(M + \frac{mL}{2}\right)}} , \tag{4}$$

where f_i = natural frequency (Hz) of i^{th} mode.

ASSUMPTIONS

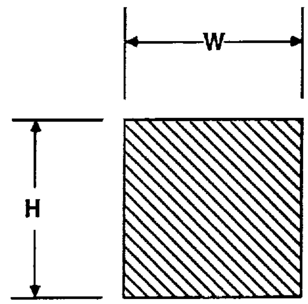
In the development of equations (1) through (4), and in the determination of the input environment and subsequent responses, the following basic assumptions were made:

1. The continuous beam is constructed of aluminum and has pinned end conditions.
2. The beam has a variable mass (M) at the midpoint of its length and the mass produces no change in the area moment of inertia of the beam.
3. Air damping and heat dissipation energy are negligible.
4. The beam is subjected to random excitation of constant magnitude (0.10 G^2/Hz) from 20 to 2,000 Hz.
5. A constant damping of 5 percent exists for all modes (Q = 10).
6. Random limit loads are calculated using Miles' equation with a crest factor of 3.0 .
7. Only the first four modes of the beam (from 20 to 2,000 Hz) will be considered.

Assumption 2 increases the response and raises the frequencies, while the actual effects of assumption 3 are to increase the response and only slightly lower the frequency. While number 4 is consistent with Miles' equation development, actual flight input criteria for shuttle hardware shows a tapering off of energy in the low and high frequency regimes. Thus, higher responses will be calculated using this assumption. Assumption 5 is historically a pretty reasonable value for the predominant mode (usually the first mode) of most payload hardware, but the premise that damping will remain constant across the frequency domain will produce an increase in responses. It should also be noted that an actual pinned-pinned beam would probably have a much smaller damping value, unless the boundaries had an inordinate amount of Coulomb damping.

CALCULATION OF BEAM FREQUENCIES

In order to look at an actual case, specific dimensions and associated properties have been selected for this study. These definite beam parameters were chosen such that the first four modes would fall in the range of 20 to 2,000 Hz. Those values are listed below:



where: $L = 38.0$ in $E = 1.0\times10^7$ lb/in^2
$H = 2.0$ in $\rho = 0.1$ lb/in^3
$W = 2.0$ in $I = 1.3333$ in^4
$m = 0.001036$ lb-s^2/in^2 $M = 0.0$ to 0.11813 lb-s^2/in.

Figure 3 depicts a plot of the change in frequency of the odd numbered modes as a function of the magnitude of the concentrated mass (M). This occurrence has been dubbed the mass loading effect.

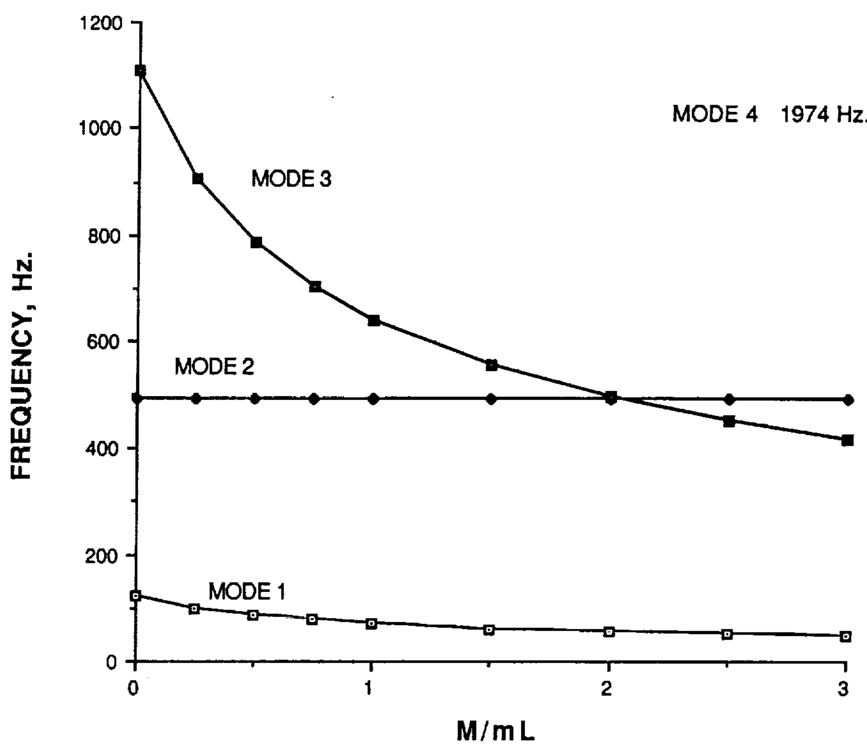


Figure 3. Pinned-pinned beam modes 1 to 4.

PEAK ACCELERATION FOR EACH MODE

Additionally, the effect of this concentrated mass can also be seen on the response of the beam to the assumed environment through use of the Miles' equation (G_{pk}). Figure 4 shows this mass loading effect for the odd numbered modes, which has been recognized and utilized in vibration environmental prediction techniques for a number of years. These correction factors not only show frequency changes but recognize the significant response level decrease that occurs by simply adding mass to a hardware system. This "mass damping" phenomena (fig. 5) has been required in the aerospace environmental prediction discipline since, in general, large variances in component mass to primary structure are common, and instrumentation programs to define such environments have been limited in the practical number of data acquisition systems that can be installed on flight vehicles. The most practical compromise has been to instrument the typical unloaded primary structure, define the environment for the unloaded structure, and apply correction factors to account for the mass loading of vehicle components.

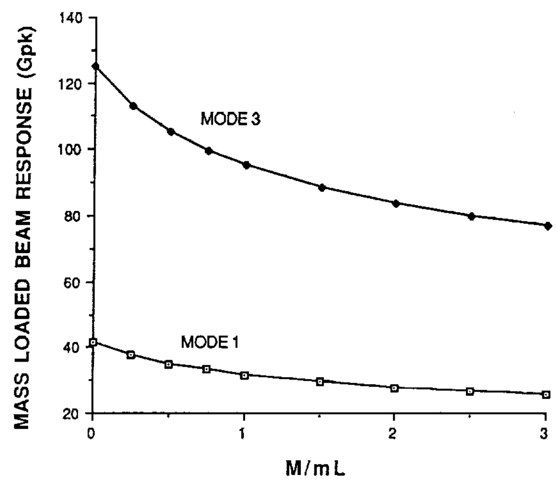


Figure 4. Peak acceleration for each mode.

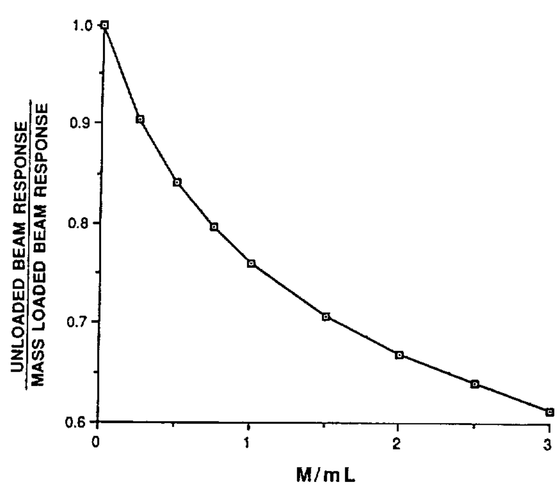


Figure 5. Mass loading effect on beam.

MAXIMUM DISPLACEMENT FOR EACH MODE

The next step, which is most critical for strength assessment, is the determination of the actual displacement equation for each mode. Using the dynamic relation:

$$\text{acceleration} = r\omega_i^2 \;\; ; \;\; (r = y_i)$$

$$G_{pk}(g) = y_i(2\Pi f_i)^2 \; ,$$

$$y_i = \frac{G_{pk}(g)}{4\Pi^2 f_i^2} \; . \tag{5}$$

Figure 6 shows the maximum displacement for each mode and how it too is affected by the mass loading relationship (appendix). In this example, it is clearly seen that mode 1 is dominant. What can also be seen is the fact that the so called "mass damping" effect produces a large increase in the maximum displacement of the odd numbered modes. The question now is what will be the resulting stresses on the beam, even though the natural frequencies and response accelerations have been decreased ?

THE STRESS EQUATION

Substituting the maximum displacement values from equation (5) into equation (1), the simplistic modal eigenvalues can be calculated for each of the four modes. Since in the random environment each mode from 20 to 2,000 Hz is excited, the true displacement must be a summation of each mode.

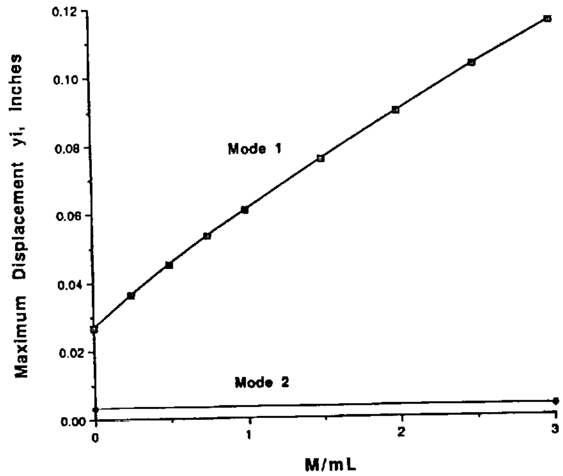
$$\begin{aligned} Y_{(x)} &= \sum_1^4 Y_{i(x)} \\ &= \sum_1^4 y_i \sin \frac{i\Pi X}{L} \; . \end{aligned} \tag{6}$$

Also, knowing that the stresses in a pinned-pinned continuous beam subjected to transverse loadings will be predominantly due to bending, we can use the elementary beam bending theory and resolve the stress state at any point. These two primary equations are:

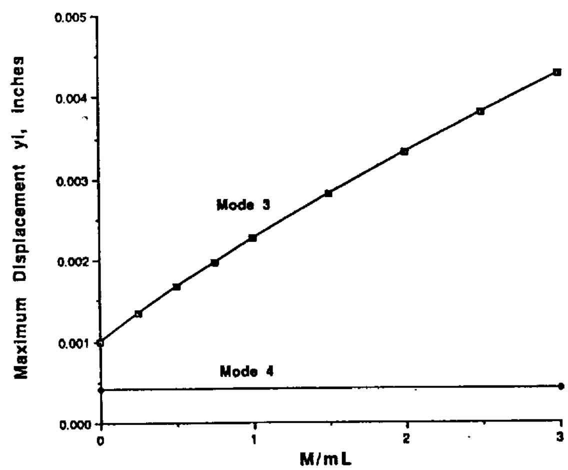
$$\frac{\partial^2 Y_{(x)}}{\partial X^2} = \frac{M_{(x)}}{EI} \quad \text{and} \quad \sigma_{(x)} = \frac{M_{(x)}\left(\frac{H}{2}\right)}{I} \; ,$$

which produce

$$\begin{aligned} \sigma_{(x)} &= \left(\frac{EH}{2}\right)\left(\frac{\partial^2 Y_{(x)}}{\partial X^2}\right) \\ &= \left(\frac{\Pi^2 EH}{2L^2}\right)\sum_1^4 i^2 y_i^2 \sin^2 \frac{i\Pi X}{L} \; . \end{aligned} \tag{7}$$



a. Modes 1 and 2



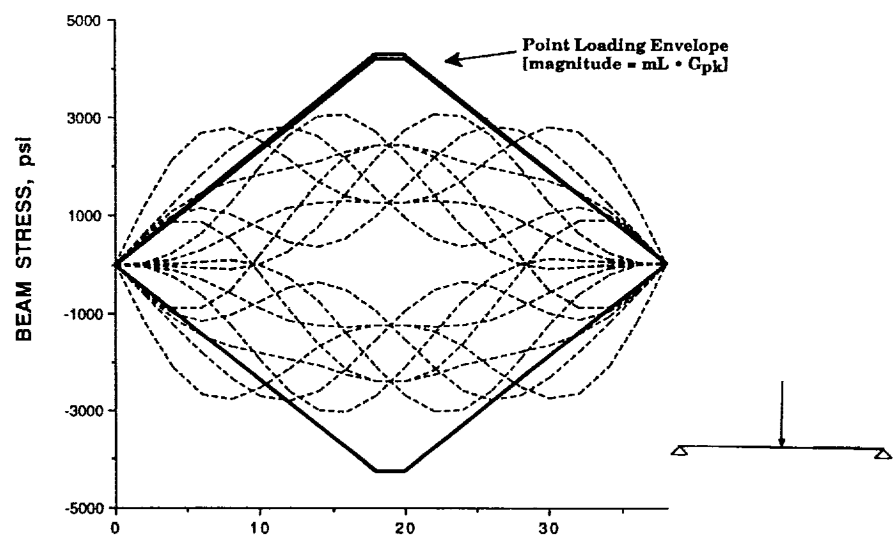
b. Modes 3 and 4

Figure 6. Maximum displacement for beam modes.

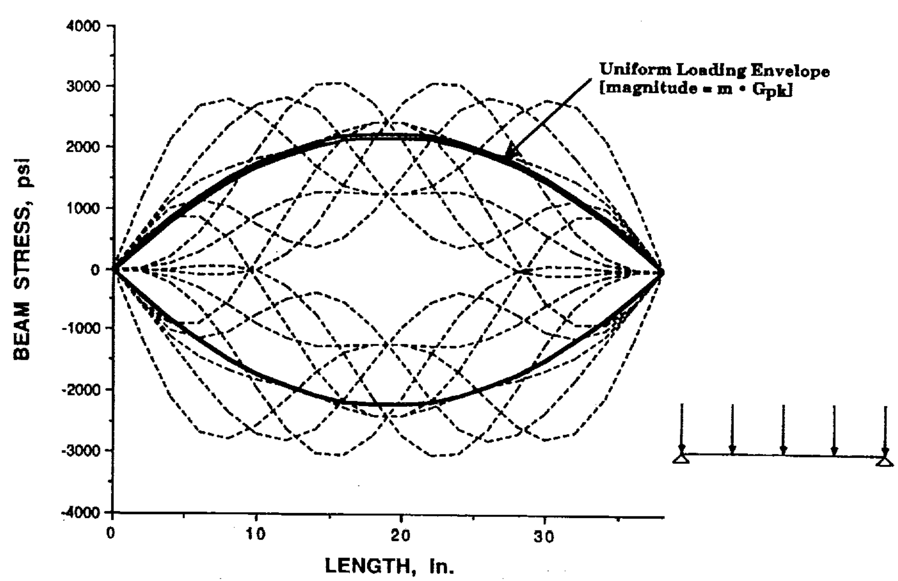
Placing the known constants into the equation, the stress solution for this pinned-pinned beam can be expressed as follows:

$$\sigma_{(x)} = \pm 68{,}348\, y_1 \sin\frac{\Pi X}{L} \pm 273{,}396\, y_2 \sin\frac{2\Pi X}{L}$$
$$\pm 615{,}140\, y_3 \sin\frac{3\Pi X}{L} \pm 1{,}093{,}584\, y_4 \sin\frac{4\Pi X}{L} \;. \tag{8}$$

The values of y_1, y_2, y_3, and y_4 are derived from equation (5) and were plotted in figure 6. Substituting these values into equation (8) and solving for all 16 possible combinations, for any given concentrated mass, results in some insight into the state of stress in the beam during the course of the random loads environment. Figure 7 relates the stresses from these potential combinations as a function of beam length, with a mass of zero at the center. It additionally shows how they compare to stresses produced by typical static tests, first, loaded through the center of gravity and, second, loaded in a uniform manner. From this figure, it is evident that most of the possible stress states in the beam are enveloped by the proposed point loading case. The uniform loading envelope, however, does not encompass the maximum stresses. When a concentrated center mass of three times that of the beam is added, the potential stress states are almost completely outside the static test envelopes (fig. 8). This fact certainly leads one to conclude that mass damping of some hardware, such as beams, panels, and floor structure, will reduce the response frequency and acceleration, but may not result in a corresponding reduction in stress.

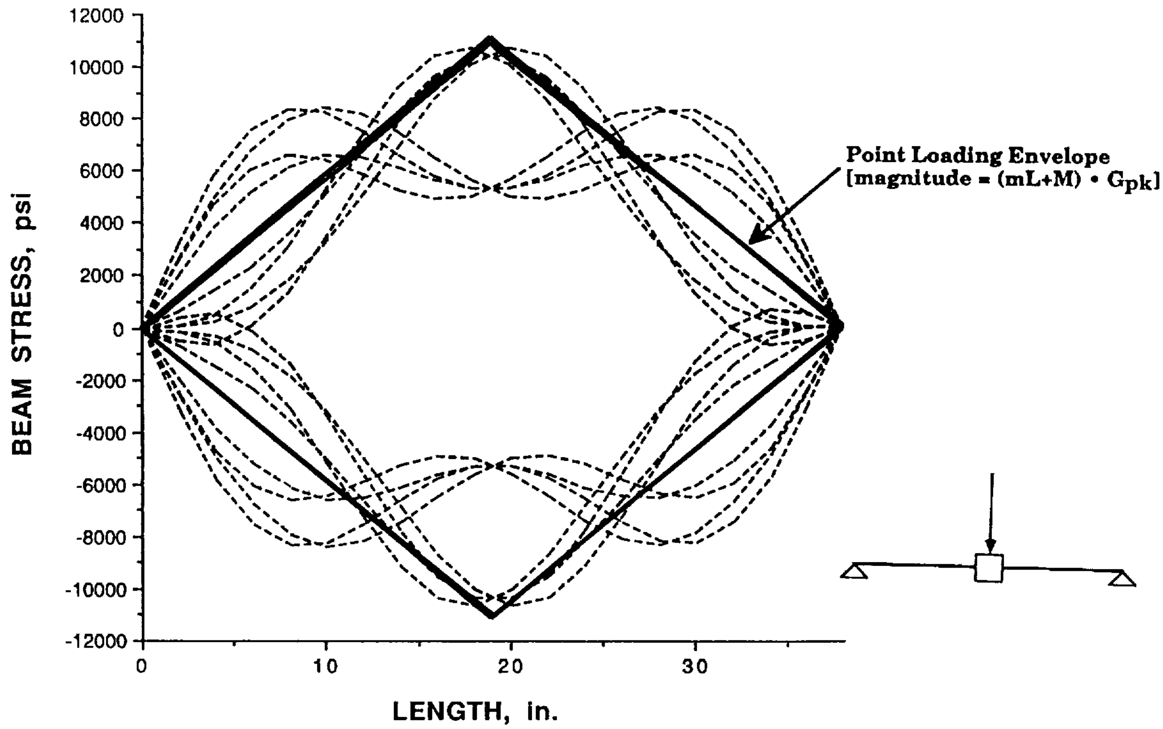


a. Point Loading Through Center of Gravity

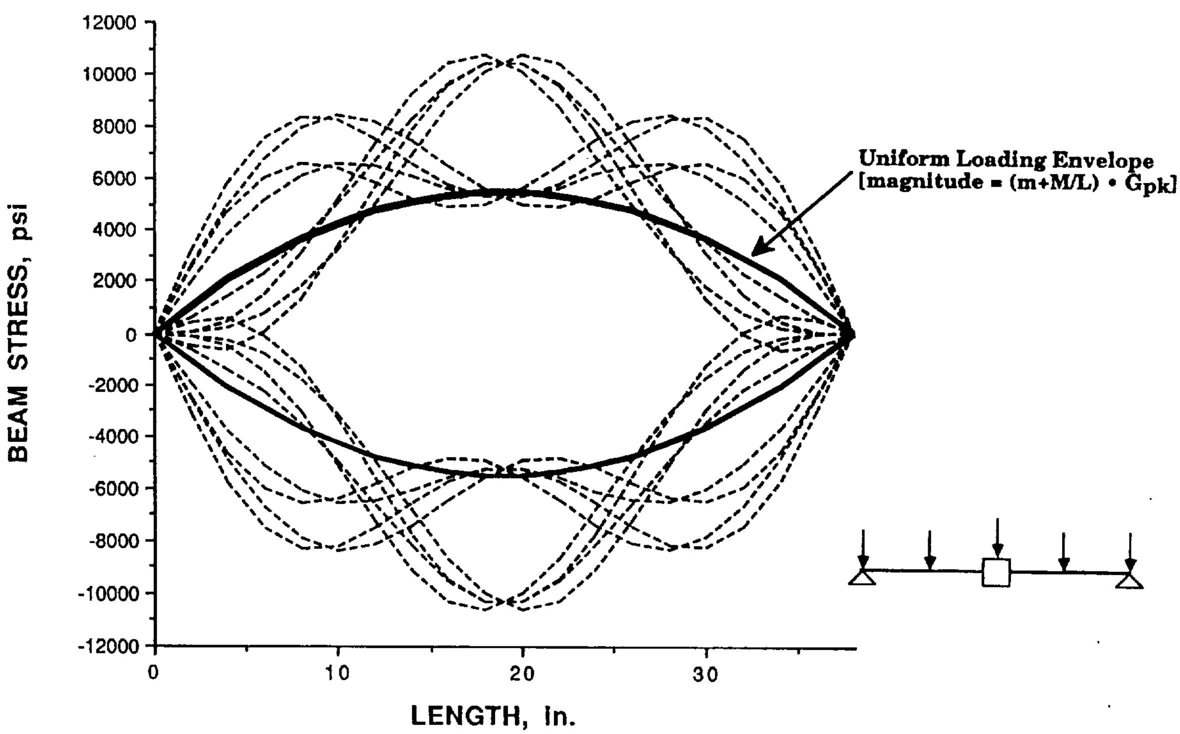


b. Uniform Loading Across Entire Beam

Figure 7. Beam stress with $M/mL = 0$.



a. Point Loading Through Center of Gravity



b. Uniform Loading Across Entire Beam

Figure 8. Beam stress with $M/mL = 3.0$.

COMPARISON WITH TEST SIMULATION

From figures 7 and 8, it is apparent that neither the uniform loading envelope nor the point loading envelope are truly representative of the calculated multiple mode stress distribution across the beam. It is clear, however, that the uniform loading envelope is the best shape fit for the response stress. This is to be expected, since the first mode of the beam generally produces the principle eigenvector set, and uniform loading is a good approximation of that mode. The bottom line is that the current static test loading procedure may not always envelope the response stress from random loading or, in some cases, the procedure could overly load some areas of the tested hardware.

Figure 9 shows the multiple mode stress along the beam, first for no mass at the center and next for *M/mL* = 3.0. These results are compared with the uniform loading envelopes multiplied by a factor (1.5, 2.0). In these cases, the proposed static tests would more accurately simulate the calculated random

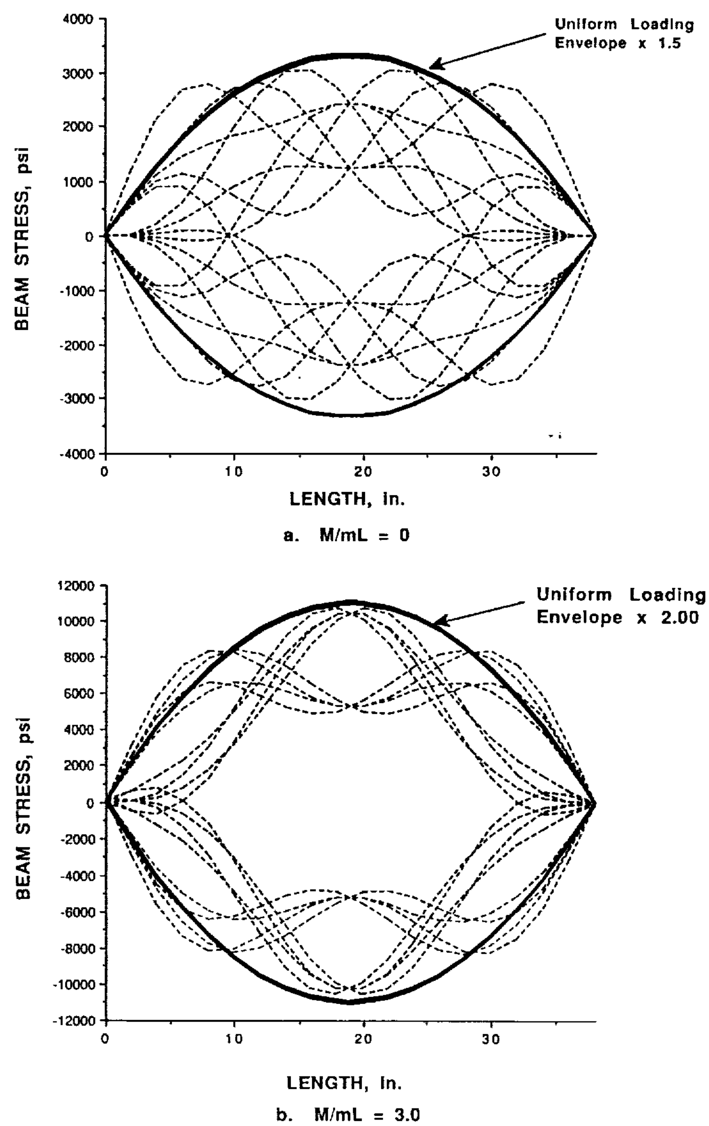


Figure 9. Multiple mode stress versus test simulation.

loading response stresses. If we assume that the first mode will actually be dominant and that all other modes would be negligible (either through possessing high damping and/or low input criteria at those frequencies), then the stress along the beam could be simulated quite nicely by a uniform loading envelope whose magnitude is controlled by deflection (fig. 10). This figure shows that when using a load controlled uniform envelope, stresses are over predicted for the beam by itself and under predicted for the mass loaded beam where $M/mL = 3.0$.

Correlation of the stress state for any static test (whether it be hydraulic load line, centrifugal, or below resonance sine burst) and the actual dynamic response is dependent upon how well the static test deflections simulate the predominant dynamic mode shape.

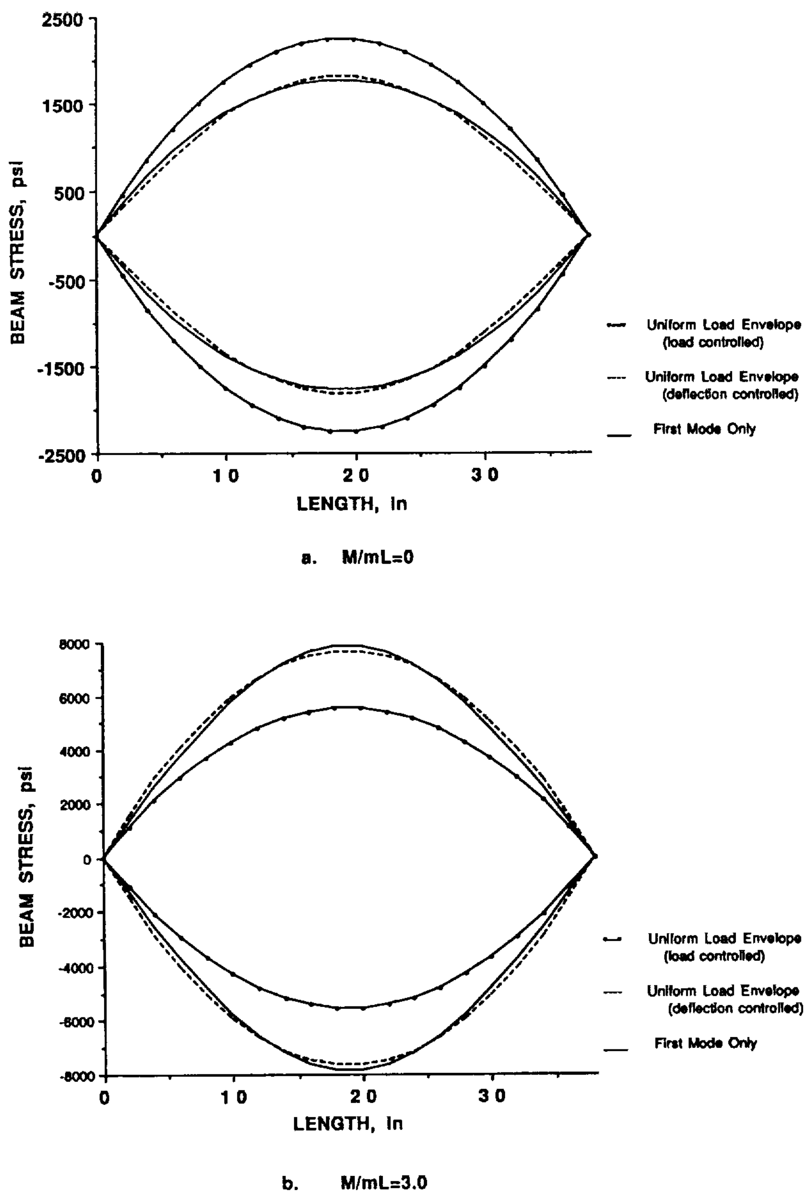


Figure 10. First mode stress versus test simulation.

CONCLUSIONS

Determining the stress distribution generated by random loading on space flight hardware is highly dependent upon the participation of each mode in the prescribed frequency domain. Analytically this includes:

- Knowledge of the environment as a function of frequency
- The magnitude of damping associated with each mode
- A reasonable definition of each mode shape (eigenvectors)
- Possessing an adequate strength model and/or having a sufficient set of instrumentation during ground testing.

The results of this study seem to indicate, at least for the simplistic beam case, that the stress state always has the potential to be of greater magnitude than a typically proposed static test. This fact is especially evident in structural support members not directly attached to a concentrated mass in the component. The study says, for instance, that we can probably predict the local loads where the mass is attached, but we may significantly miss them on structural members further away from the mass. An example of this may exist on hardware such as the atmospheric emission photometric imager (AEPI) shown in figure 11. In this case, the support structure weighed about 125 lb, while the actual experiment weight was some 244 lb. The random loads developed for this component were ±3.6 g's, ±2.7 g's, and ±5.1 g's in the x, y, and z axes, respectively.[2] Sixteen triaxial strain gauges were placed around the base of the AEPI during static loading (fig. 12). The unanswered question is, if the AEPI were subjected to random loading, would the static load testing prove to be too conservative, unconservative, or an exact prediction? The answer is, of course, dependent upon the similarity between the static deflections and the actual deflected shape which occurs during vibration. A test of this nature will be planned for the AEPI as soon as its reflight status is determined.

As stated previously, the premise under which most random loads (and subsequent static tests) are developed is that one (generally the first) mode will dominate. Historically, this is true when observing the acceleration of components, but may not always be so when examining the stress distribution of that component's support structure. As with the beam, we saw significantly higher stresses existing in areas away from the mass. The normal static test setup did not always envelope them until we multiplied the uniform load cases by some arbitrary factor (fig. 9). The point here is, that if we know enough about the predominant mode, we may be able to closely simulate it by altering the static test loading.

Another finding of this study was to show that the so called "mass damping" effect does indeed reduce the response acceleration of the hardware, but it additionally reduces the natural frequencies and subsequently may result in higher stresses than the original "undamped" condition. Again, this effect is simply due to an increase in the magnitude of the displacements. Expanding on the acceleration-displacement relationship (equation (5)) for a specific frequency, we find it reduces to:

$$y_i = \frac{36.76\sqrt{Q \cdot PSD_i}}{f_i^{3/2}} \;.$$

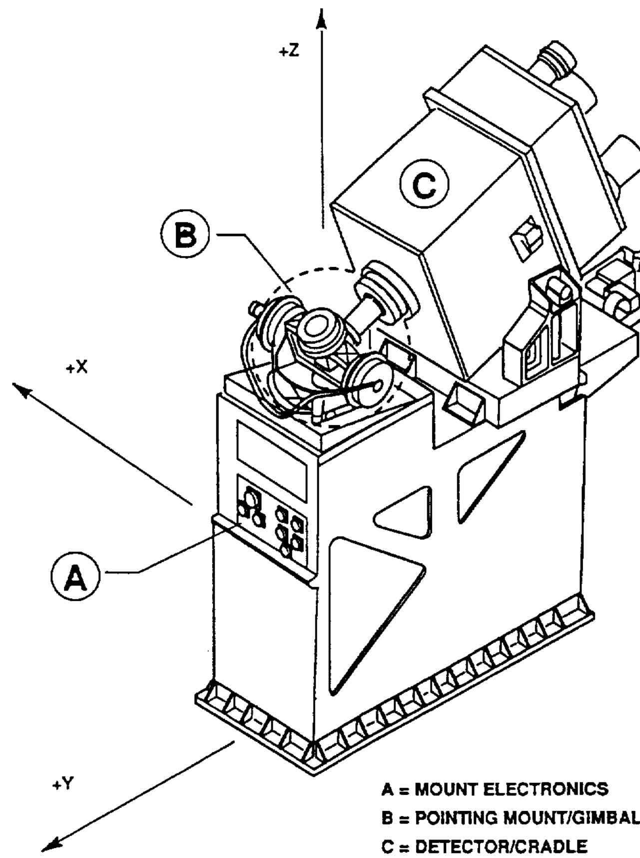


Figure 11. AEPI flight instrument.

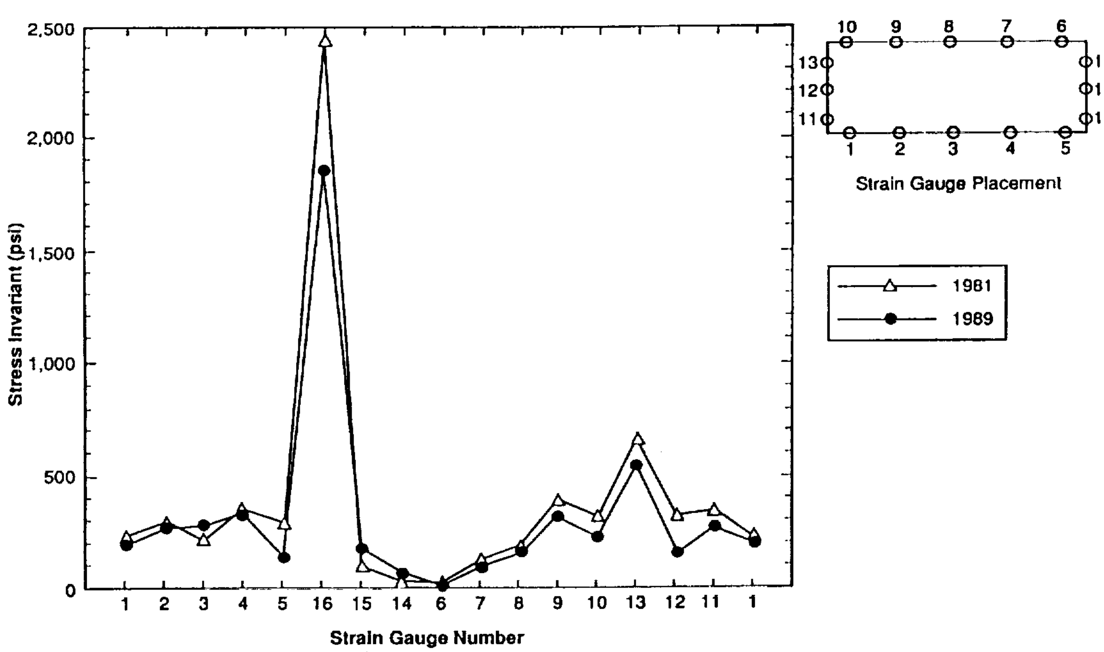


Figure 12. AEPI static tests.

Unless there are drastic changes in the damping and input criteria magnitudes, displacement will be greatly reduced with increase in the first resonant frequency. This fact is in contrast to traditional static test deflections which increase with higher frequency because the predicted response acceleration increases per the Miles' equation.

In summary, we have assumed that the Miles' equation is a reasonable technique to estimate response.[3] That response, however, is used primarily to develop the peak response acceleration for space flight hardware. This qualifies the technique as a force equation, since it is applied statically to the hardware center of gravity for strength analyses and verification testing. It is apparent from the study that this may not always be the best way to determine the resulting stress state. This is especially true for components that do indeed have some clearly predictable resonant frequencies, mode shapes, and masses (i.e.; not electronic packages). Those accomplishing a strength assessment need to know not only the predicted peak acceleration responses of major masses, but also there needs to be some understanding of the mode shape for the dominant hardware resonances. Utilizing the general shape of the predominant mode and the peak response accelerations at each major mass, a tailored enforced displacement loading may prove to more accurately replicate random environment stresses on flight hardware. For accomplishing strength analyses and associated verification testing, the process involved in determining the limit stresses from such an environment would be greatly improved.

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3. Ferebee, R.C., and Jones, J.H.: "Comparison of Miles' Relationship to the True Mean Square Value of Response for a Single Degree-of-Freedom System." NASA MSFC.

APPENDIX

CALCULATED DATA FOR MODAL PLOTS

i	M/mL *	f_i (Hz) †	G_{pk} ‡	y_i §
--	---------	----------	--------	-----------
1	0	123.39	41.7	0.026779
1	0.25	100.74	37.7	0.036321
1	0.50	87.25	35.1	0.045082
1	0.75	78.04	33.2	0.053300
1	1.00	71.24	31.7	0.061071
1	1.50	61.69	29.5	0.075791
1	2.00	55.18	27.9	0.089591
1	2.50	50.37	26.7	0.102890
1	3.00	46.63	25.6	0.115110
2	0-3.00	493.57	83.5	0.0033513
3	0	1,110.5	125.3	0.0009934
3	0.25	906.73	113.2	0.0013462
3	0.50	785.25	105.3	0.0016697
3	0.75	702.25	99.6	0.0019741
3	1.00	641.16	95.2	0.0022642
3	1.50	555.26	88.6	0.0028097
3	2.00	496.64	83.8	0.0033219
3	2.50	453.37	80.0	0.0038055
3	3.00	419.73	77.0	0.0042734
4	0-3.00	1,974.27	167.0	0.0004189

* $mL = 0.39378$ lb-s²/in

† $f_i = \frac{i^2\Pi^2}{2}\sqrt{\frac{EI}{2L^3\left(M + \frac{mL}{2}\right)}}$

‡ $G_{pk} = 3\sqrt{\frac{\Pi}{2}\cdot Q \cdot f_i \cdot PSD_i}$

§ $y_i = \frac{G_{pk}(g)}{4\Pi^2 f_i^2}$

APPROVAL

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By H.M. Lee

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

J.C. Blair

for J.C. BLAIR
Director, Structures and Dynamics Laboratory

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Utilizing a continuous beam model, this report compares the potential stresses imposed on the beam from a random environment with those resulting from a typical static load analysis or test simulation. The Miles' equation used to develop peak response accelerations is shown to become a force equation in the hands of strength assessment personnel. This may prove to be unrealistic since hardware dynamic stresses are related to deflection rather than load. Correlation of the stress state for any static analysis or test with the actual dynamic response stress is strictly dependent upon how well the static deflections simulate the predominant dynamic mode shape. The report proposes that the general shape of this predominant mode, along with the peak response accelerations of major masses be used in strength assessments. From these data, a tailored enforced displacement loading may prove to be more effective in reproducing random induced stresses on flight hardware.

14. SUBJECT TERMS
random environment, limit stress, Miles' equation

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