NYQUIST AND NICHOLS PLOTS

By Tom Irvine
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Email: tomirvine@aol.com

Introduction

The purpose of this tutorial is to show how some concept from control theory are applied to structural dynamics problems.

Transfer Functions

Consider a transfer function $\mathrm{H(s)}$.

$$\mathrm{H(s)} = \frac{\mathrm{N(s)}}{\mathrm{D(s)}} \tag{1}$$

where $\mathrm{N(s)}$ and $\mathrm{D(s)}$ are simple polynomials.

The poles are roots of the denominator. The zeros are the roots of the numerator. The poles and zeros may be complex.

Physically realizable control systems must have a number of poles greater than or equal to the number of zeros. Systems that satisfy this relationship are called *proper.*

Bode Plot

Bode plots in control's analysis represent transfer functions which give the response to an input where the excitation frequency is an independent variable. Both the magnitude and phase are plotted with respect to the excitation frequency. The magnitude is expressed in terms of decibels.

Similar frequency response functions are used in vibration analysis, although the magnitude is typically given in straight logarithmic form.

Nyquist Plot

A Nyquist plot is a polar plot of the frequency response function of a linear system. It is used to predict the stability and performance of a control system. It is also used to extract modal parameters in vibration analysis.

The Nyquist plot shows the imaginary versus the real component of the function.

Nichols Plot

A Nichols plot gives the magnitude versus the phase of a frequency response on orthogonal axes.

Example

Consider the single-degree-of-freedom system in Figure 1.

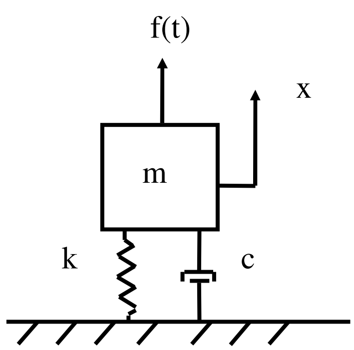

Figure 1.

The mass m is 1 lbm = 0.00259 lbf sec^2/in.

The stiffness k is 1000 lbf/in.

The viscous damping ratio ξ is 0.05.

The natural frequency is

$$\omega_n = \sqrt{\frac{k}{m}} \tag{2}$$

$$\omega_n = \sqrt{\frac{1000\,\text{lbf}\,/\,\text{in}}{0.00259\,\text{lbf sec^2/in}}} = 621.3\ \text{rad}\,/\,\text{sec} \tag{3}$$

$$f_n = 98.9\ \text{Hz} \tag{4}$$

The viscous damping coefficient is

$$C = 2\xi\omega_n m \tag{5}$$

$$C = 2\left(0.05\right)(620.5\ \text{rad}\,/\,\text{sec})\left(0.00259\,\text{lbf sec^2/in}\right) \tag{6}$$

$$C=0.161\ \text{lbf sec/in} \tag{7}$$

The transfer function H(s) from the Laplace transform is

$$H(s) = \left\{\frac{1}{m}\right\}\left\{\frac{1}{s^2 + 2\xi\omega_n s + \omega_n{}^2}\right\} \tag{8}$$

The substitution $s=j\omega$ is made in the transfer function H(s).

$$H(j\omega) = \left\{\frac{1}{m}\right\}\left\{\frac{1}{\omega_n{}^2 - \omega^2 + j2\xi\omega_n\omega}\right\} \quad (9)$$

$$H(j\omega) = \left\{\frac{1}{m}\right\}\left\{\frac{\omega_n{}^2 - \omega^2 - j2\xi\omega_n\omega}{\sqrt{\left(\omega_n{}^2 - \omega^2\right)^2 + (2\xi\omega_n\omega)^2}}\right\} \quad (10)$$

The poles p are

$$p_{1,2} = -\xi\omega_n \pm j\,\omega_n\sqrt{1-\xi^2} \quad (11)$$

The poles for the sample problem are

$$p_{1,2} = -31.06 \pm j\,620.5 \text{ rad/sec} \quad (12)$$

The system is stable because the real portion of each pole is negative.

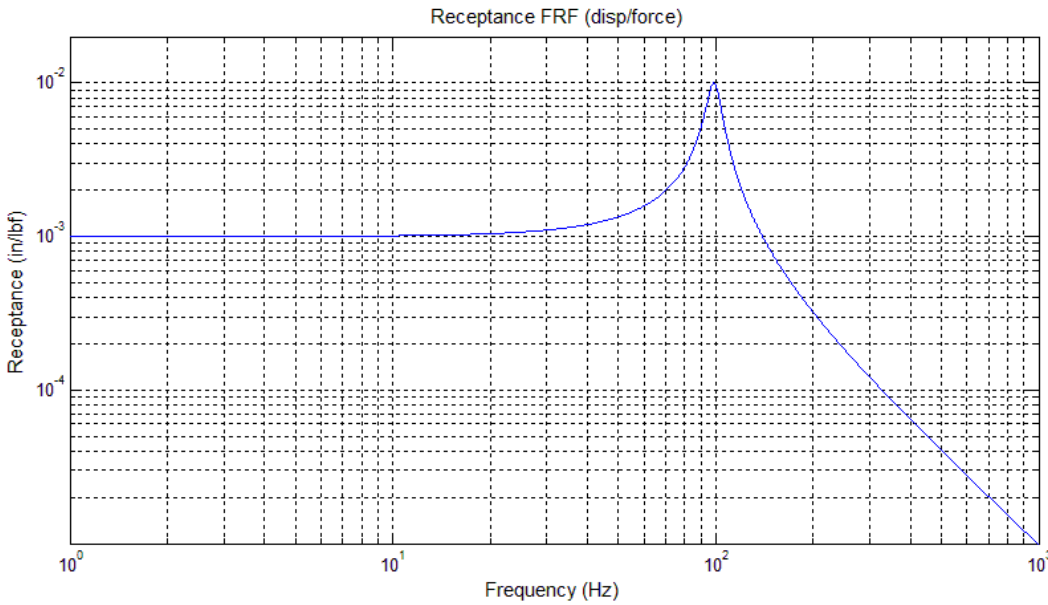

Figure 1.

As the frequency approaches zero, the receptance converges to (1/k).

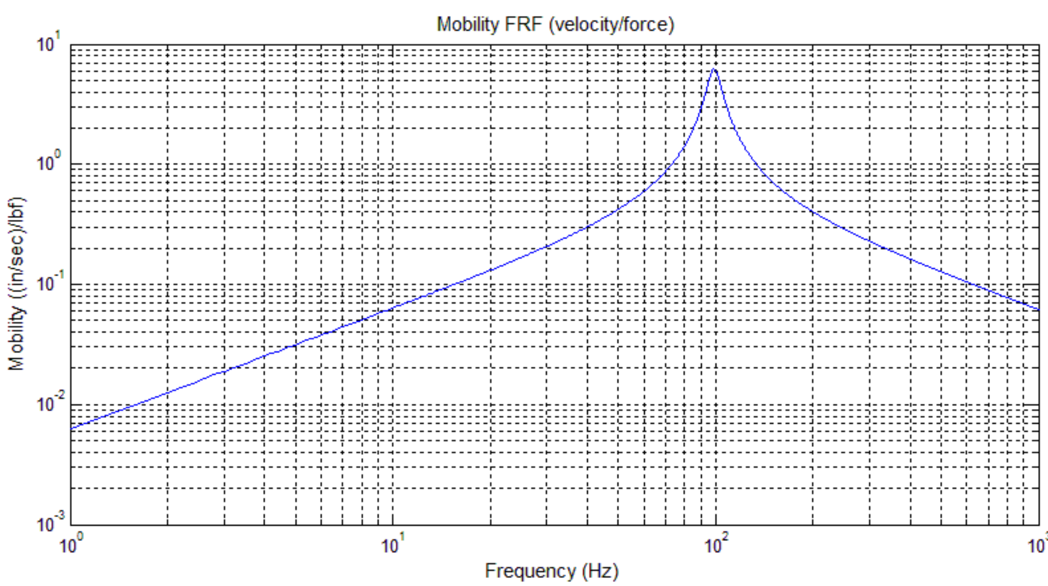

Figure 2.

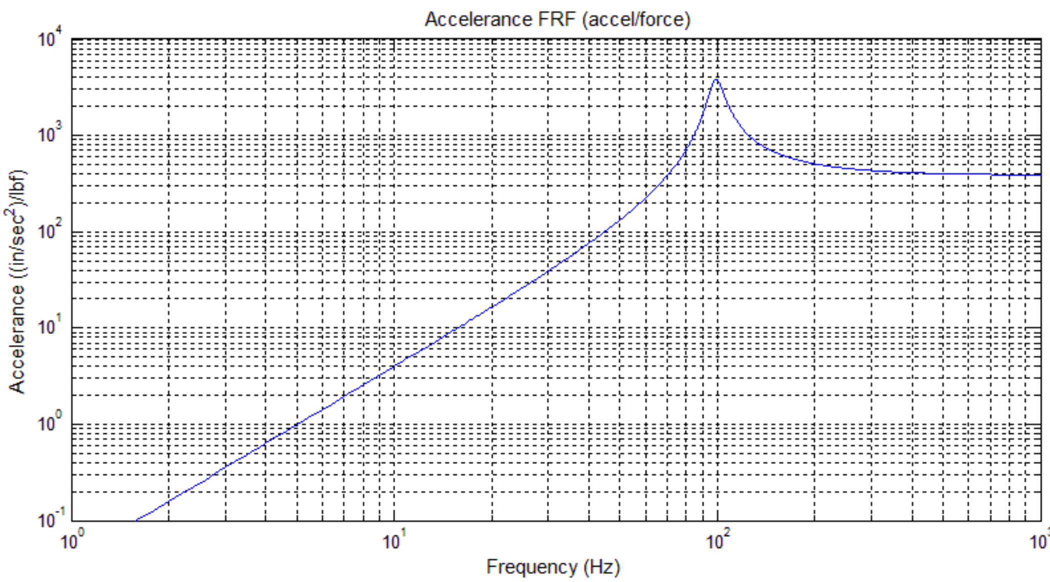

Figure 3.

As the excitation frequency increases beyond the natural frequency, the accelerance converges to (1/m).

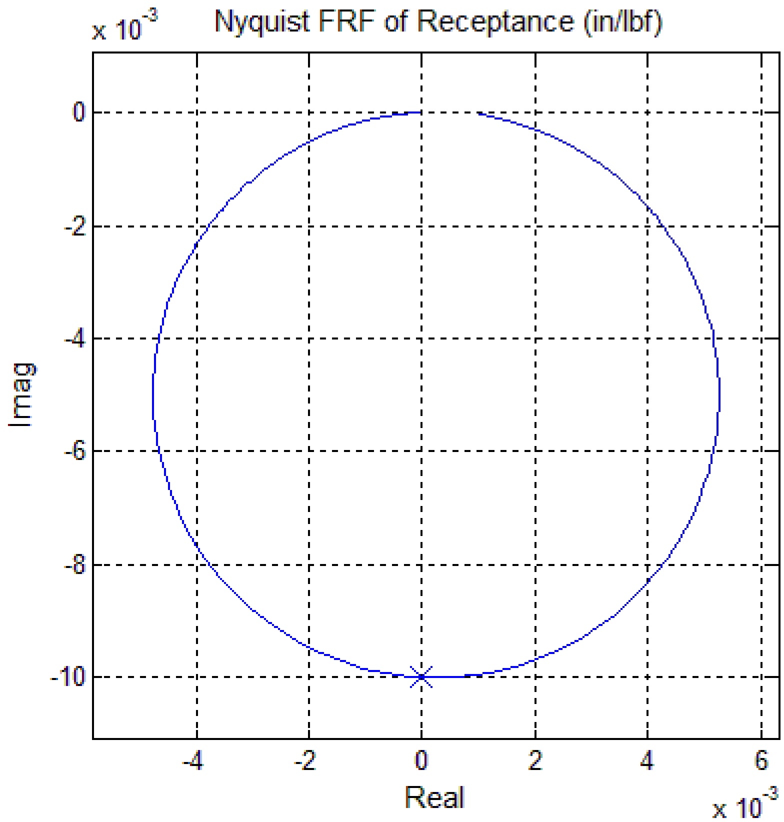
Nyquist FRF of Receptance (in/lbf)
x 10^-3
0
-2
-4
-6
-8
-10
Imag
-4
-2
0
2
4
6
Real
x 10^-3

Figure 4.

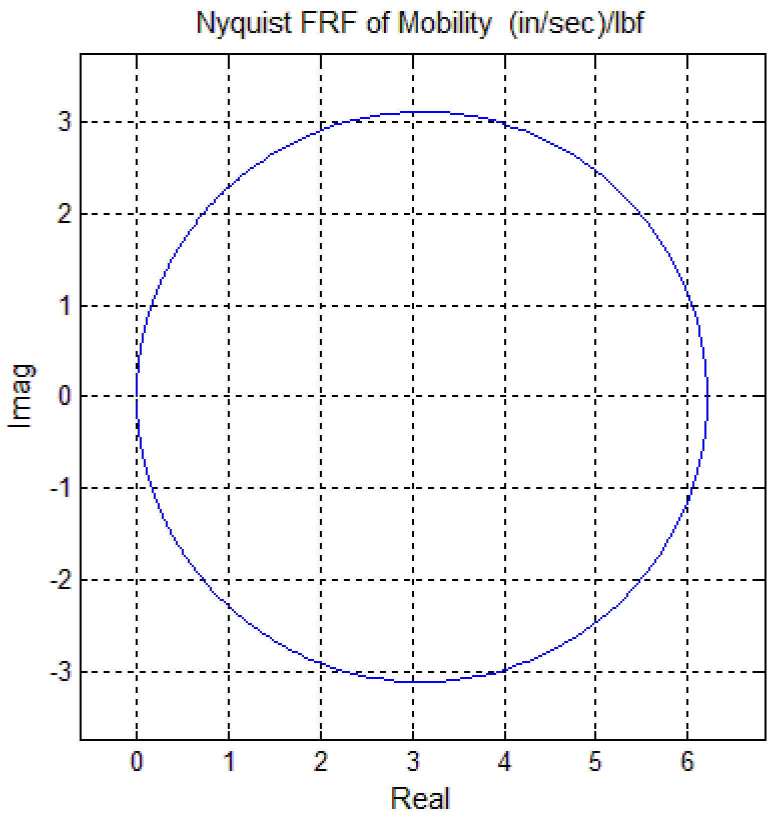

Figure 5.

Recall that $C = 0.161$ lbf sec/in.

The radius of the mobility circle is

$$\frac{1}{2C} = \frac{1}{2(0.161)} = 3.11$$

The Mobility Nyquist curve is a perfect circle.

The Receptance and Accelerance Nyqust curves are nearly circles.

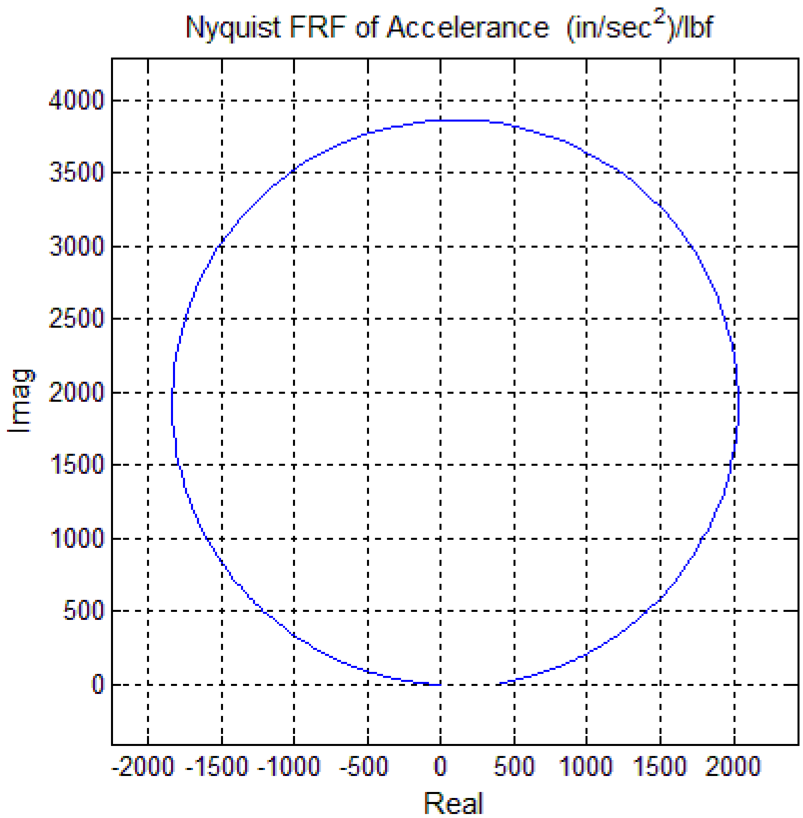

Figure 6.

The peak imaginary value is equal to

$$\frac{1}{2\xi m} = \frac{1}{2(0.05)(0.00259\,\text{lbf sec\^{}2/in})} = 3860\ (\text{in}/\text{sec\^{}2})/\text{lbf}$$

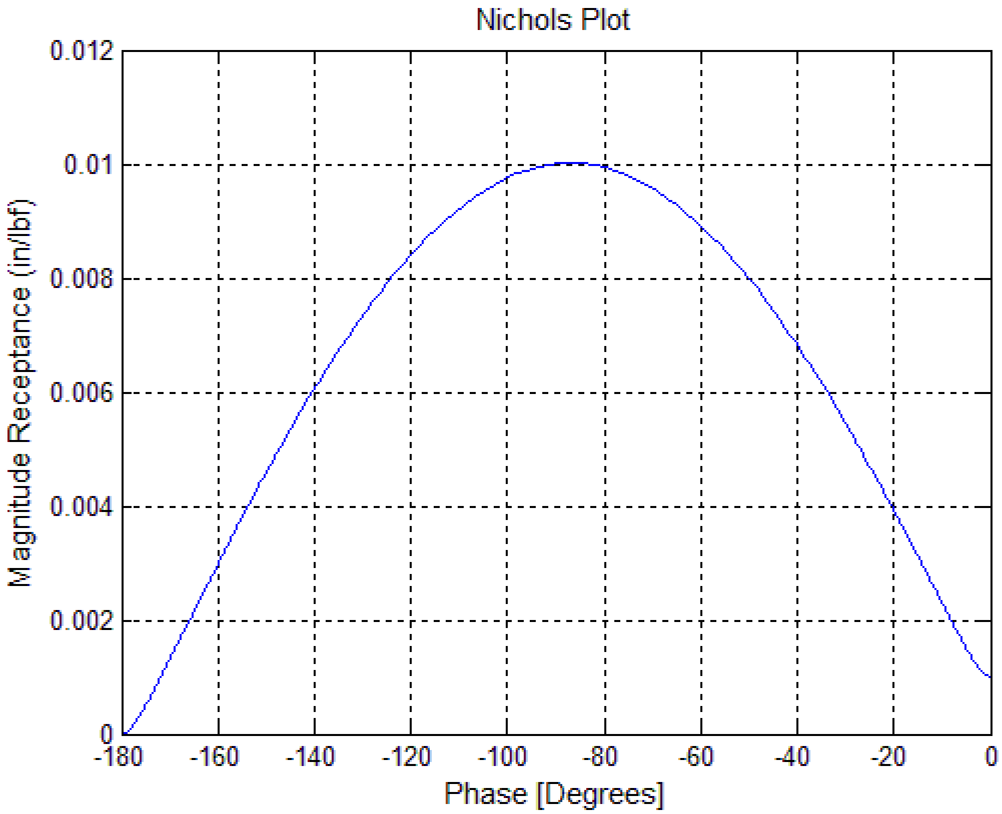

Figure 7.

Reference

1. T. Irvine, An Introduction to Frequency Response Functions, Vibrationdata, 2000.