NOTES ON REFILTERING FOR PHASE CORRECTION   Rev A

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INTRODUCTION

Engineers post-process accelerometer data to better understand the characteristics of the shock and vibration environments. One of the most important post-processing tools is the filter.

Engineers can use filtering to clarify the behavior of a particular vibration mode. For example, the Pegasus drop transient is bandpass filtered so that the damping ratio of the fundamental body-bending mode can be calculated. A typical Pegasus XL vehicle has a fundamental frequency of 9.5 Hz with a damping ratio of 1.1%. The exact values depend on the payload and other variables.

This filtering is typically applied to digital accelerometer data. The most common filter type is the Butterworth filter. This filter is characterized by its -3 dB cutoff frequency.

The Butterworth filter is known to introduce a phase shift, however. The phase shift can be corrected by refiltering. *Linear phase response* is obtained through refiltering.

Again, refiltering is a technique which can be applied to digital data. It is impractical for analog data since it requires time reversals, as discussed in the text.

The purpose of this report is to discuss the characteristics of refiltering.

PROCEDURE

The refiltering method is taken from Reference 1. It is applied as follows:

1. Reverse the time history.
2. Apply the filter to the time history.
3. Reverse the time history again.
4. Apply the filter to the time history.

This process is shown in Figure 1 for an input X and an output Y.

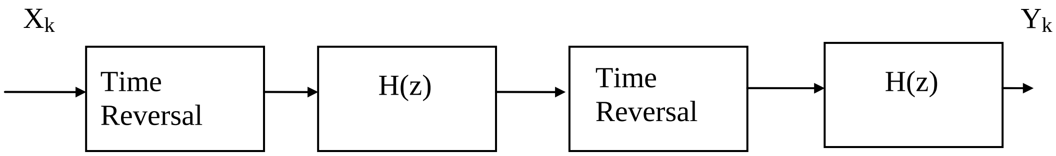

Figure 1. Refiltering Diagram

Note that H(z) represents the filter transfer function in terms of a Z-transform. The equivalent transfer function from X to Y is [$H(z)\,H(z^{-1})$]. Note that $z = \exp[j\omega\Delta t]$, where ω is the frequency and Δt is the time step.

TRANSIENT VIBRATION EXAMPLE

Direct Filtering

A sample raw time history is shown in Figure 2. The time history is composed of a 20 Hz sinusoid with 5% damping. The signal also contains random noise. The sample rate was 720 samples per second.

The raw time history was subjected to a 30 Hz lowpass filter. The filtered time history is also shown in Figure 2. Note that the filtered signal was given an offset of -20 G simply for plotting purposes.

The filter was a 6^{th} order Butterworth filter. The filter was applied in a direct manner which resulted in a phase shift.

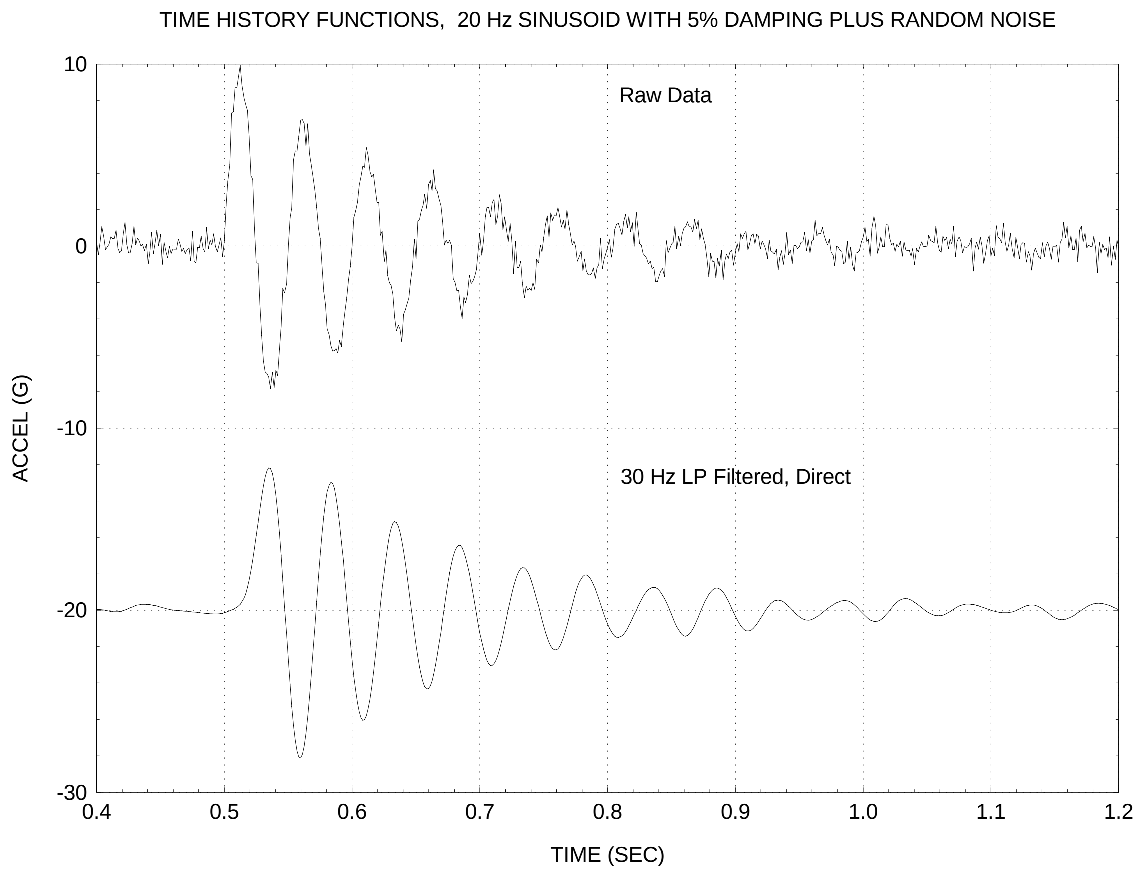

Figure 2. Transient Example

The filtered signal is delayed by about one-half of a period.

<u>Refiltering for Phase Correction</u>

The transient example was repeated except that refiltering was used for phase correction. The result is shown in Figure 3.

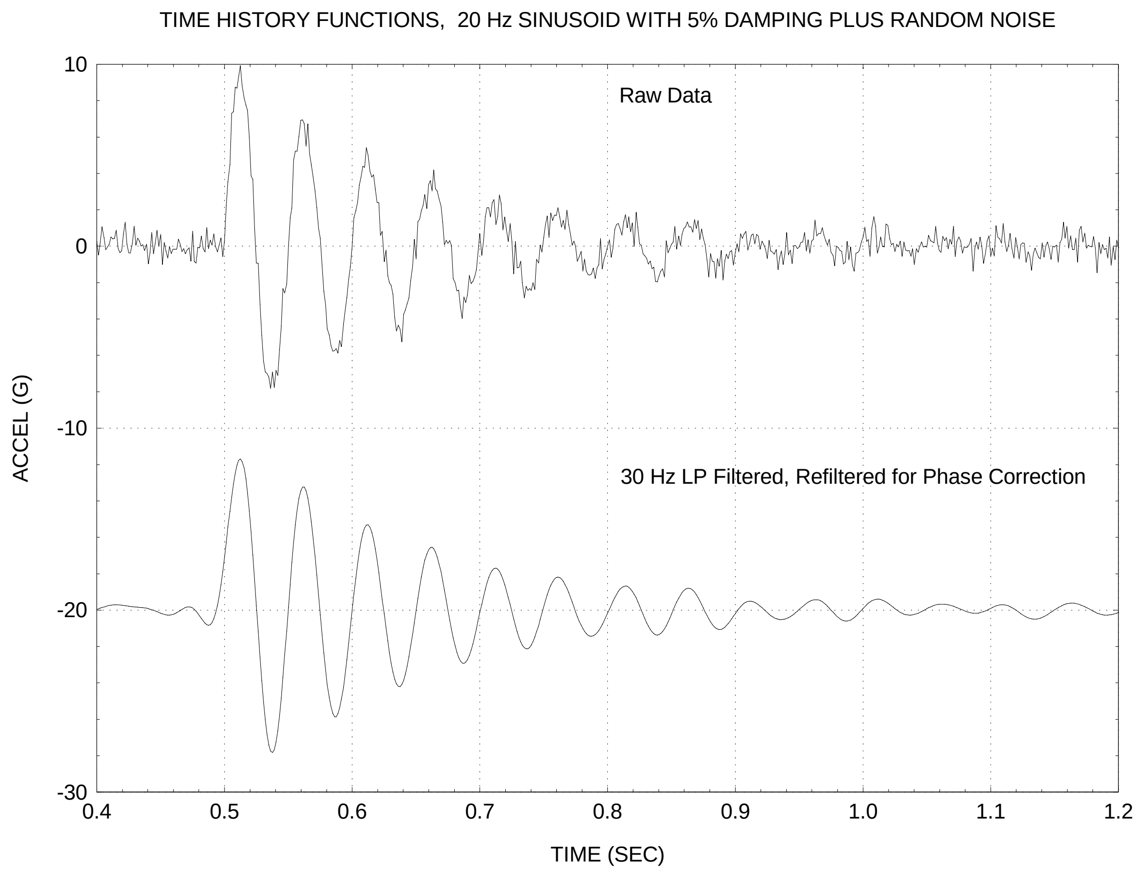

Figure 3. Transient Example with Refiltering

Refiltering removes the phase shift, although it appears to introduce a small ripple effect prior to the onset of the real event.

STEADY-STATE RANDOM VIBRATION EXAMPLE

A random vibration time history was synthesized, using the method in Reference 2. The duration was 60 seconds. The power spectral density amplitude was 1.0 G^2/Hz, as shown in Figure 4. The time history was lowpass filtered at 30 Hz using both the direct and refiltering techniques. The corresponding power spectral density functions are shown in Figure 4.

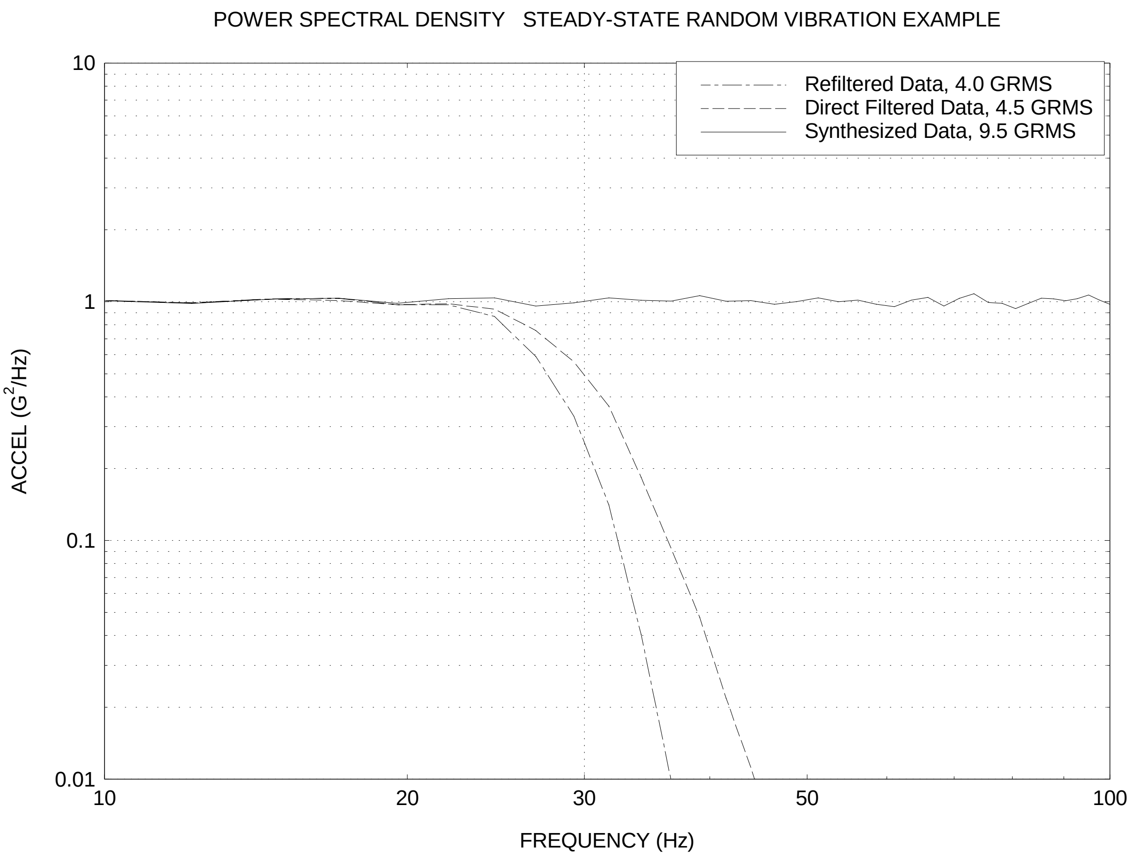

Figure 4. Steady-State Example

The power spectral density of the direct filtered data is -3 dB from the synthesized level at 30 Hz. The power spectral density of the refiltered data is -6 dB from the synthesized level at 30 Hz.

THEORY

The theoretical transfer function magnitude corresponding to the steady-state example is shown in Figure 5. Refiltering drives the cutoff frequency from -3 dB to -6 dB. In addition, the roll-off slope becomes steeper as shown in Figure 6. Computational details are given in Appendix A.

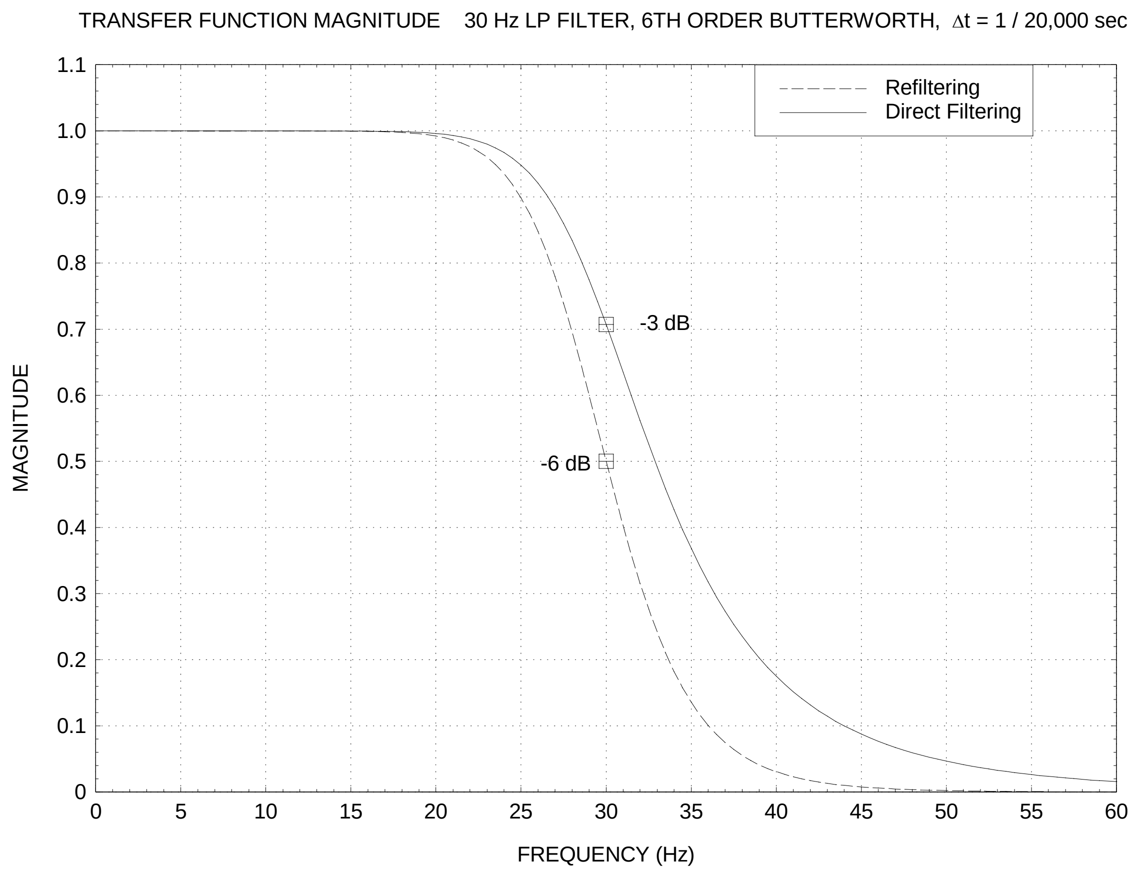

Figure 5. Transfer Function Magnitude

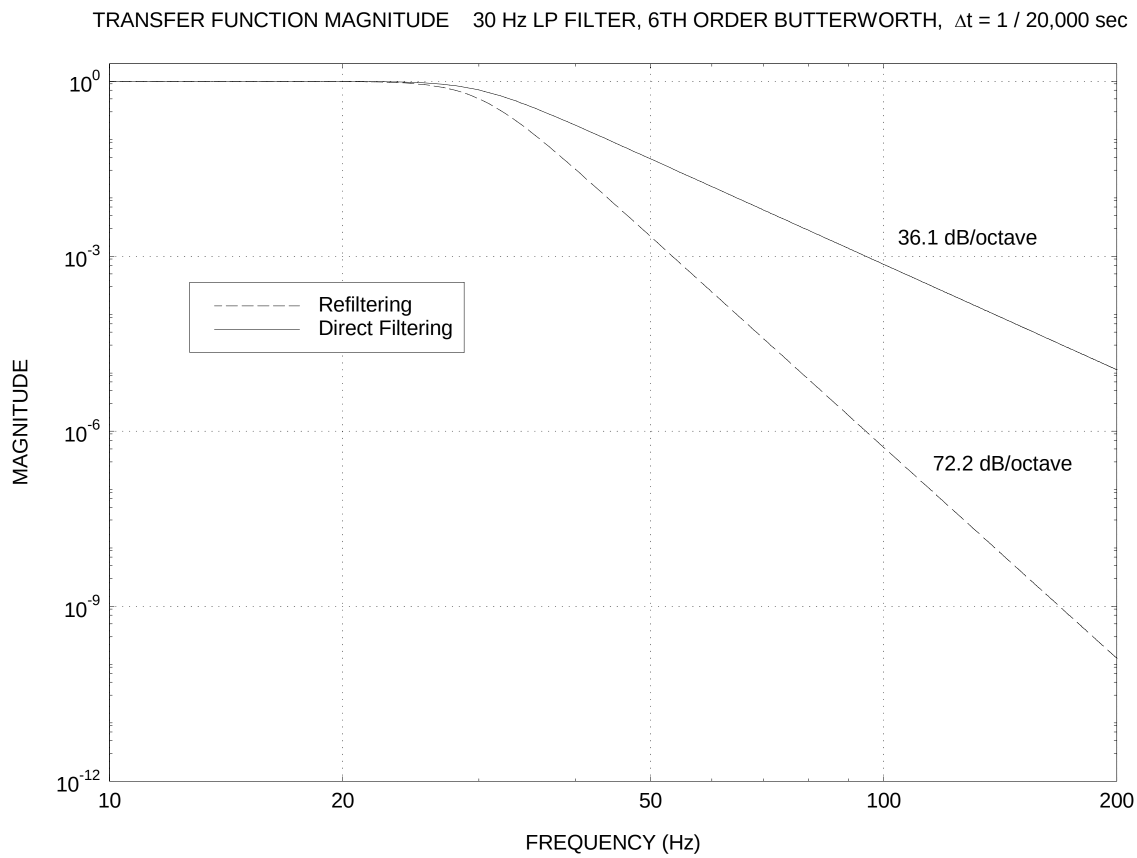

Figure 6. Roll-off Slopes

CONCLUSIONS

The refiltering method is an effective means for approximately achieving linear phase response.

A linear phase response is required for accurate time domain calculations. The shock response spectrum is an example of a time domain calculation which needs a linear phase input.

On the other hand, the refiltering method requires increased computational time.

Another characteristic of refiltering is additional amplitude attenuation near the cutoff frequency. This could either be an advantage or a drawback depending on the original motivation for filtering the data.

REFERENCES

1. Stearns and David, Signal Processing Algorithms in Fortran and C, Prentice Hall, Englewood Cliffs, New Jersey, 1993.

2. OSC ME File: MISC 030-114, An Improved Method for Power Spectral Density Synthesis, 1998.

APPENDIX A

The transfer function can be represented by a series of a_n and b_n coefficients as follows

$$H(z) = \frac{b_0 + b_1 z^{-1} + \cdots + b_L z^{-L}}{1 + a_1 z^{-1} + \cdots + a_L z^{-L}} \qquad \text{(A-1)}$$

Again, the transfer function is expressed as a Z-transform.

The filter coefficients for a 30 Hz lowpass filter are given in Tables A-1 and A-2. The filter design is 6 th order Butterworth with Δt = 1/20,000 seconds. The filter is implemented in a 3-stage cascade. Thus, L=3 in equation (A-1) for this example.

Table A-1. Numerator Coefficients

Stage	b0	b1	b2
1	0.22152413E-04	0.44304827E-04	0.22152413E-04
2	0.22059440E-04	0.44118880E-04	0.22059440E-04
3	0.22006117E-04	0.44012234E-04	0.22006117E-04

Table A-2. Denominator Coefficients

Stage	a1	a2
1	-0.19950447E+01	0.99513332E+00
2	-0.19866715E+01	0.98675978E+00
3	-0.19818693E+01	0.98195728E+00

The equivalent time-domain equation is

$$y_k = \left\{ \sum_{n=0}^{L} b_n x_{k-n} \right\} - \left\{ \sum_{n=1}^{L} a_n y_{k-n} \right\} \qquad \text{(A-2)}$$

Note that this equation is recursive.