VIBRATION RESPONSE OF A THIN CYLINDRICAL SHELL TO EXTERNAL ACOUSTIC PRESSURE VIA STATISTICAL ENERGY ANALYSIS Revision D

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Variables

c_{air}	=	speed of sound in the air
$(\rho c)_{air}$	=	specific acoustic impedance of the air
C_L	=	longitudinal wave speed
σ_{rad}	=	radiation efficiency of the cylindrical shell
ρ_S	=	mass/area of the cylinder
ρ_v	=	mass/volume of the cylinder
E	=	elastic modulus
μ	=	Poisson's ratio
h	=	Thickness
d	=	Diameter
S	=	surface area
$n(f)$	=	modal density
M	=	mass of the cylinder
ω	=	excitation frequency (rad/sec)
f	=	excitation frequency (Hz)
f_c	=	critical frequency
f_r	=	ring frequency
v_o	=	nondimensional frequency parameter

Variables Continued

η	=	loss factor
λ	=	wavelength
ξ	=	fraction of critical damping
Q	=	quality factor
$\left\langle p^2 \right\rangle$	=	the space-time average mean-square acoustic pressure
$\left\langle v^2 \right\rangle$	=	normal component of the space-time average mean-square vibration velocity of the radiating surface
$\left\langle a^2 \right\rangle$	=	normal component of the space-time average mean-square vibration acceleration of the radiating surface
$S_a(f)$	=	acceleration power spectral density
$S_p(f)$	=	pressure power spectral density
T(f)	=	power transmissibility function
W_{rad}	=	radiated sound power
m	=	mode number or number of half-wavelengths in the longitudinal axis
n	=	mode number or number of half-wavelengths in the circumferential direction
k_m	=	axial wave number
k_n	=	circumferential wave number
k	=	acoustic wave number

Vibroacoustic Formulas

Consider a homogeneous, cylindrical shell subjected to a diffuse, broadband sound field where the surrounding medium is air. The shell could represent an aircraft fuselage segment or a rocket vehicle avionics module.

The primary goal is to calculate the response of the shell. Note that the shell is also a radiator or transponder because it transmits some of its vibration energy back into the sound field.

A secondary goal is to derive a transmissibility function that reasonably matches the Franken method, particularly near the ring frequency.

The following equation is a modified version of an equation in Reference 1, section 9.8. The original equation in this reference was derived for a flat panel in a reverberant field using statistical energy analysis.

The mean square, space-time average velocity $\left\langle v^2 \right\rangle$ of the cylindrical shell is

$$\left\langle v^2 \right\rangle = \left\langle p^2 \right\rangle \frac{\pi \, c_{air}{}^2 n(f)}{(\rho c)_{air} \; M\omega^2} \left\{ \frac{(\rho c)_{air} \sigma_{rad}}{(\rho c)_{air} \sigma_{rad} + \rho_s \omega \eta} \right\} \tag{1}$$

The corresponding acceleration is

$$\left\langle a^2 \right\rangle = \left\langle p^2 \right\rangle \frac{\pi \, c_{air}{}^2 n(f)}{(\rho c)_{air} \; M} \left\{ \frac{(\rho c)_{air} \sigma_{rad}}{(\rho c)_{air} \sigma_{rad} + \rho_s \omega \eta} \right\} \tag{2}$$

Equation (3) can be expressed in terms of the corresponding power spectral density functions.

$$S_a(f) = S_p(f) \frac{\pi \, c_{air}{}^2 n(f)}{(\rho c)_{air} \; M} \left\{ \frac{(\rho c)_{air} \sigma_{rad}}{(\rho c)_{air} \sigma_{rad} + \rho_s \omega \eta} \right\} \tag{3}$$

Rewrite equation (3) in terms of a power transmissibility function T(f).

$$S_a(f) = S_p(f)\, T(f) \tag{4}$$

$$T(f) = \frac{\pi \, c_{air}{}^2 n(f)}{(\rho c)_{air} \; M} \left\{ \frac{(\rho c)_{air} \sigma_{rad}}{(\rho c)_{air} \sigma_{rad} + \rho_s \omega \eta} \right\} \tag{5}$$

The modal density $n(f)$ is the parameter which usually has the greatest effect on shaping the power transmissibility function.

Ring Frequency

The ring frequency f_r is the frequency at which the longitudinal wavelength in the skin material is equal to the vehicle circumference. The ring frequency is not an explicit variable in equation (3), but it affects the modal density n(f). It also appears in certain formulas for the radiation efficiency σ_{rad}, although a wave number approach is used in this tutorial.

The ring frequency is

$$f_r = \frac{C_L}{\pi d} \tag{6}$$

The shell moves radially outward and the radially inward at the ring frequency if the cylinder has infinite length. On the other hand, the shell has pure tangential motion for the case of a finite shell with fixed-fixed boundary conditions, as show in Reference 5.

Note that the wave speed can be calculated from

$$C_L = \sqrt{\frac{E}{\rho_v}} \tag{7}$$

Critical Frequency

The critical frequency f_c is another parameter which has an indirect effect on equation (3). It is the frequency at which the airborne acoustic wavelength matches the panel bending wavelength.

$$f_c = \frac{{c_{air}}^2}{2\pi h}\sqrt{\frac{12(1-\mu^2)\rho}{E}} \tag{8}$$

Equation (8) is taken from Reference 1, equation (9.84).

The critical frequency may also be expressed as

$$f_C = \frac{{c_{air}}^2}{2\pi\, h\, C_L}\sqrt{12(1-\mu^2)} \tag{9}$$

A cylinder shell is considered as "acoustically thin" if $f_r < f_C$, as is the case with the example in Appendix A. The radiation efficiency has a peak at the ring frequency in acoustically thin shell. [1]

Furthermore, the cylindrical shell tends to vibrate as a flat plate above the ring frequency because the shell's curvature is less important.

See Reference 7 for further details.

Loss Factor

The loss factor η is related to other damping parameters by

$$\eta = 2\xi = \frac{1}{Q} \tag{10}$$

The loss factor varies with frequency. For a vibroacoustic analysis, the highest loss factor may occur at the critical frequency where the cylindrical shell has the greatest amount of coupling with the acoustic field.

Wavenumbers

The wavenumber k has a dimension of [radians/length]. It is the inverse of the wavelength.

$$k = \frac{2\pi}{\lambda} \tag{11a}$$

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[1] Acoustical thick shells occur when $f_r > f_C$. The radiation efficiency is less clear for thick shells.

The acoustic wave number is

$$k = \omega / c_{air} \tag{11b}$$

The axial wave number is

$$k_m = m\pi / L \tag{12}$$

The circumferential wave number is

$$k_n = \frac{n\pi}{\pi d} = \frac{n}{d} \tag{13}$$

<u>Acoustically Fast Modes</u>

Cylinder modes must be categorized as either acoustically fast (AF) or acoustically slow (AS) in order to determine their ability to interact with sound waves.

The distinction between these two classes is that an AF mode has a structural wavenumber smaller than the acoustic trace, according to Reference 6. In other words, the AF mode has a longer wavelength than the acoustic wave at the corresponding frequency. The converse is true for AS modes.

$$k^2 \geq k_m{}^2 + k_n{}^2 \quad \text{for AF modes} \tag{14}$$

A cylindrical shell can have AF modes in all frequencies domains in which modes occur, unlike plates.

The importance of the classification is that AF modes are superior in terms of their ability both to be excited by sound and to radiate sound. AS modes can only generate sound at structural boundaries or discontinuities.

<u>Radiation Efficiency</u>

The radiation efficiency of a vibrating body generating sound energy in air is

$$\sigma_{rad} = \frac{W_{rad}}{\left\langle v^2 \right\rangle (\rho c)_{air} S} \tag{15}$$

Equation (15) is taken from Reference 1, section 9.6.

A larger radiation efficiency means that a vibrating body will be generate a greater amount of sound energy.

Only a brief discussion of radiation efficiency is within the scope of this tutorial. A thorough treatment of this subject is given in References 5 and 6.

The radiation efficiency for AF modes is

$$\sigma_{rad} = \sqrt{1 - \frac{(k_m{}^2 + k_n{}^2)}{k^2}} \tag{16}$$

$$\sigma_{rad} \approx 1 \qquad \text{for} \qquad k^2 >> k_m{}^2 + k_n{}^2 \tag{17}$$

The original source for equation (17) is Reference 6, equation (2.16).

The radiation efficiency of AS modes is less than that of AF modes, but there is no readily available reliable formula because AS modes can only generate sound at structural boundaries or discontinuities. AS modes are assumed to have zero radiation efficiency in this tutorial for simplicity.

The radiation efficiency can be expressed as the average value of all modes in a given one-third octave band.

Modal Density

The modal density is the number of modes in a particular frequency band. Typically, the bandwidth is one-third octave.

The highest modal density may occur near the ring frequency, as shown in the example in Appendix A.

Formulas for estimating the modal density of a cylinder shell are given in References 2 and 5. See Appendix B. The preferred method in this tutorial, however, is a direct count method described in Reference 4.

Mass Law

As the excitation frequency increases beyond the ring frequency, the power transmissibility is assumed to converge to that of the mass law.

$$T(f) \approx \frac{S^2}{M^2} \quad \text{for} \quad f >> f_r \tag{18}$$

This condition is not directly derivable from equation (5) but is rather supplementary.

References

1. L. Beranek & I. Ver, Noise and Vibration Control Engineering, Principles and Applications, Wiley, New York, 1992.
2. L. Cremer and M. Heckl, Structure-Borne Sound, Springer-Verlag, New York, 1988.
3. T. Irvine, Vibration Response of a Cylindrical Skin to Acoustic Pressure via the Franken Method, Revision E, Vibrationdata, 2004.
4. T. Irvine, Natural Frequencies of a Finite, Thin-Walled Cylindrical Shell Revision B, Vibrationdata, 2006.
5. E. Szechenyi, Modal Densities and Radiation Efficiencies of Unstiffened Cylinders using Statistical Methods, Journal of Sound and Vibration, 1971.
6. G. Maidanik, Response of Ribbed Panels to Reverberant Fields, Journal of the Acoustical Society of America, Volume 34, Number 6, June 1962.
7. T. Irvine, Vibroacoustic Critical and Coincidence Frequencies of Structures, Vibrationdata, Rev B, 2008.

APPENDIX A

<u>Cylindrical Shell Parameters</u>

A cylindrical shell with the following values is subjected to a diffuse, broadband sound field. The sound pressure level is shown in Figure 5. The surrounding medium is ambient air.

The shell's boundary conditions are fixed-fixed.

d	=	48 inch
h	=	0.125 inch
L	=	48 in
E	=	1.0e+07 psi
ρ_v	=	0.1 lbm/in^3 = 0.000259 lbf sec^2/in^4
μ	=	0.3

The resulting ring frequency is f_r = 1309 Hz.
The critical frequency is f_c = 3843 Hz.

The shell is "acoustically thin" because the critical frequency is about three times greater than the ring frequency.

Furthermore, assume 5% damping for all modes.

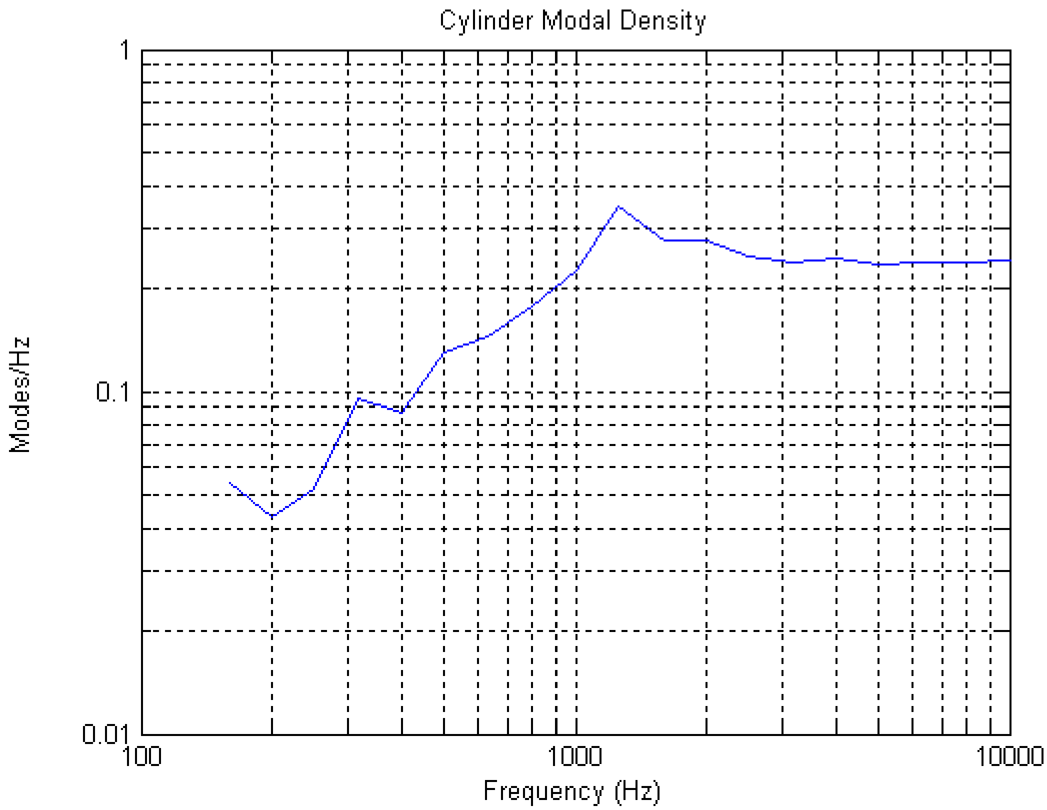

Figure A-1.

The modal density is calculated by directly counting the modes in each one-third octave band. The peak occurs near the ring frequency.

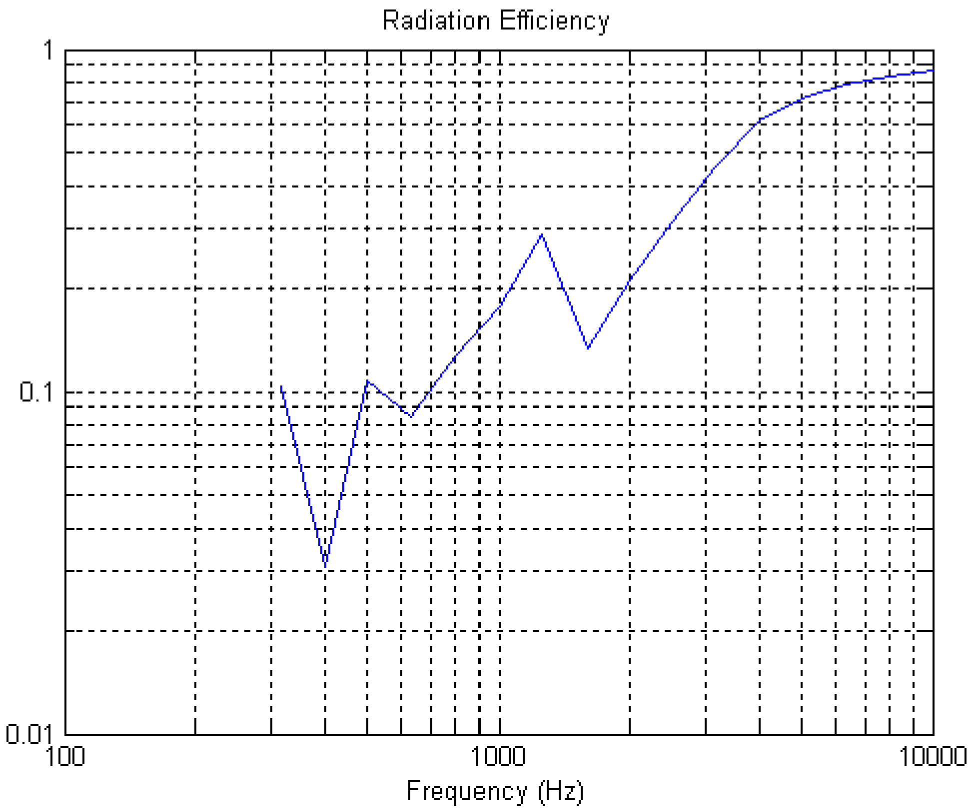

Figure A-2.

The cylindrical shell's radiation efficiency function is shown in Figure A-2.

The radiation efficiency is the average for all modes in each one-third octave band.

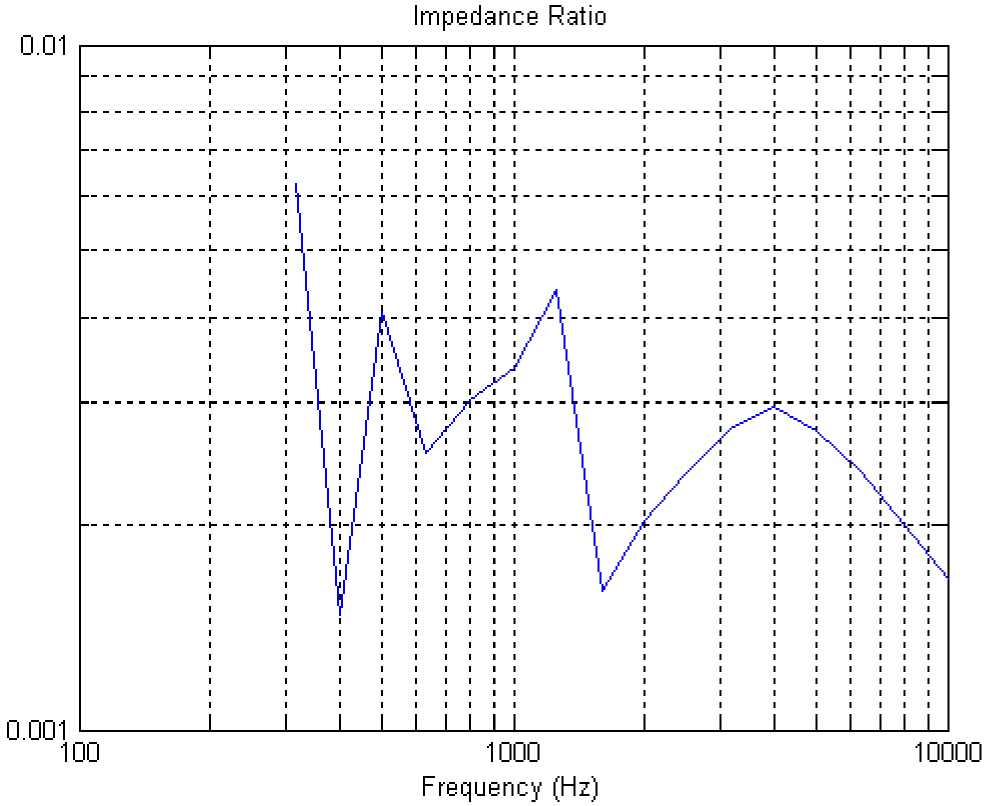

Figure A-3.

The impedance ratio is the term

$$\left\{\frac{(\rho c)_{air}\,\sigma_{rad}}{(\rho c)_{air}\,\sigma_{rad}+\rho_s\omega\eta}\right\}$$

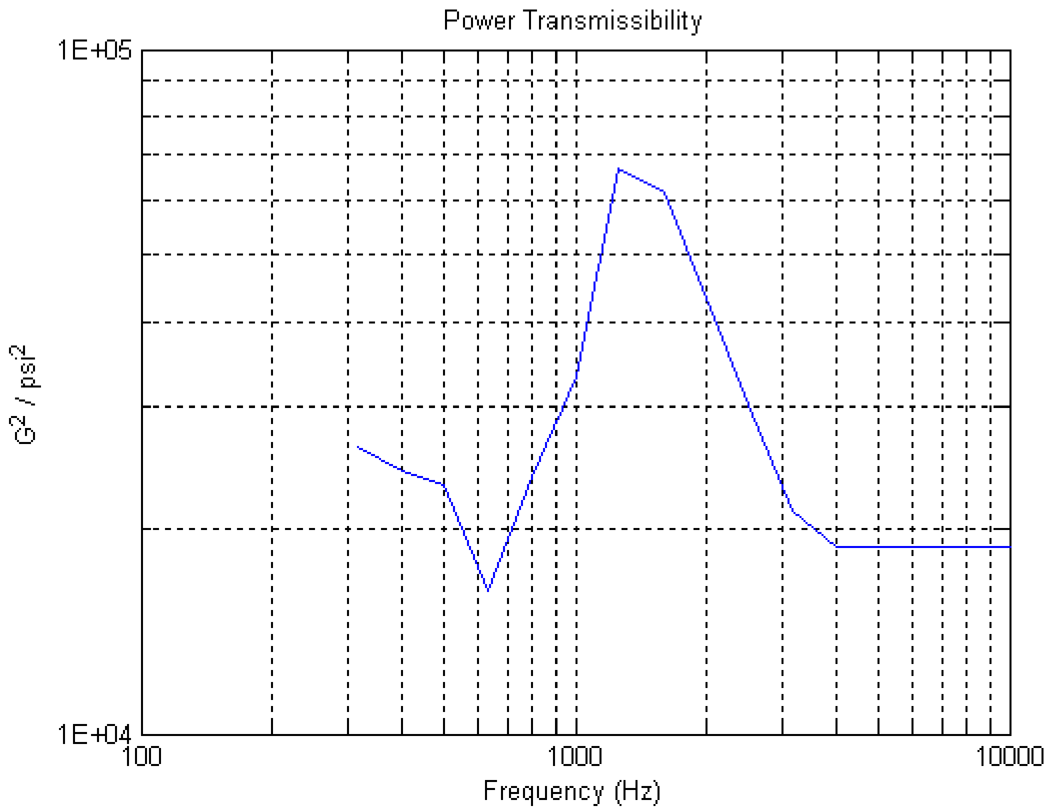

Figure A-4.

The cylindrical shell's transmissibility function is shown in Figure A-4.

The transmissibility function's lower frequency limit is determined by the first AF mode, which is 331 Hz.

The power transmissibility is constant at frequencies above 4000 Hz due to the supplementary mass law with a 4.5 dB margin.

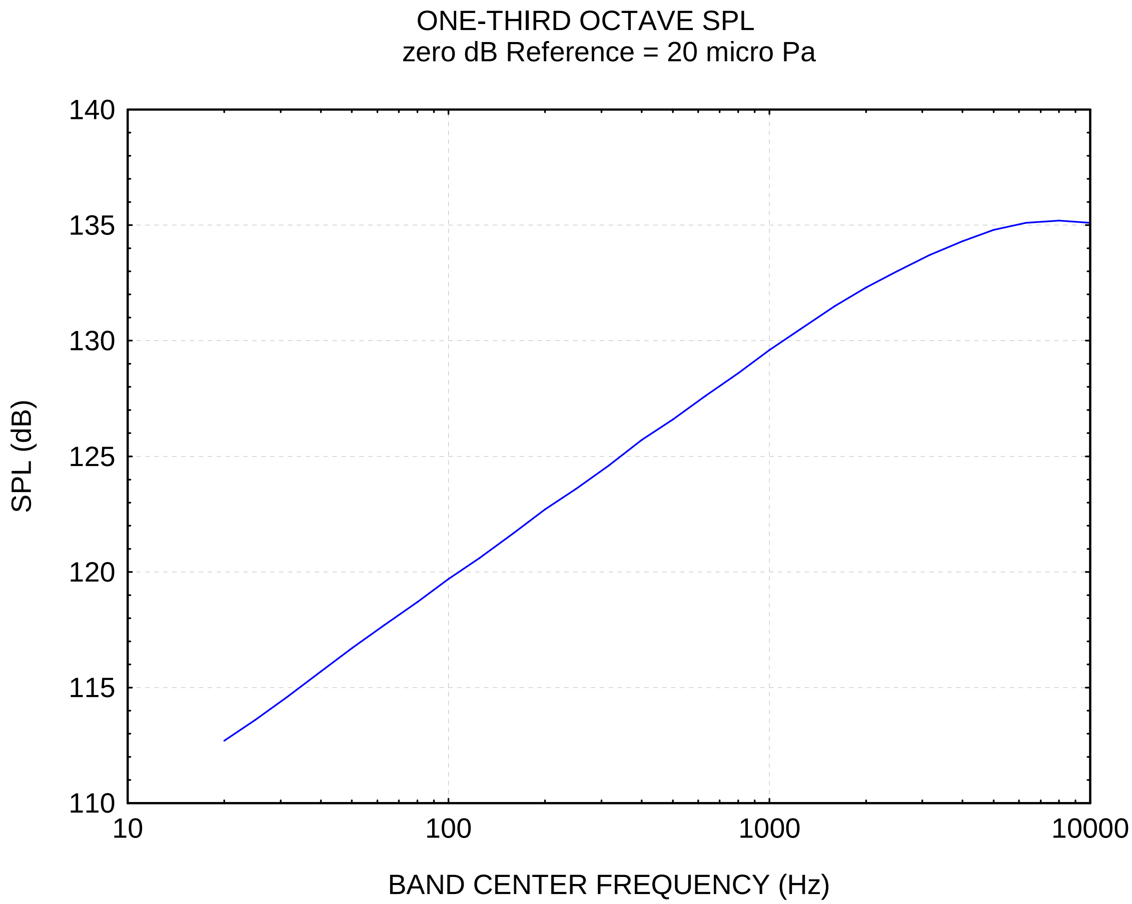

Figure A-5.

The overall level is 144.5 dB.

This is the external pressure field that is applied against the cylindrical shell.

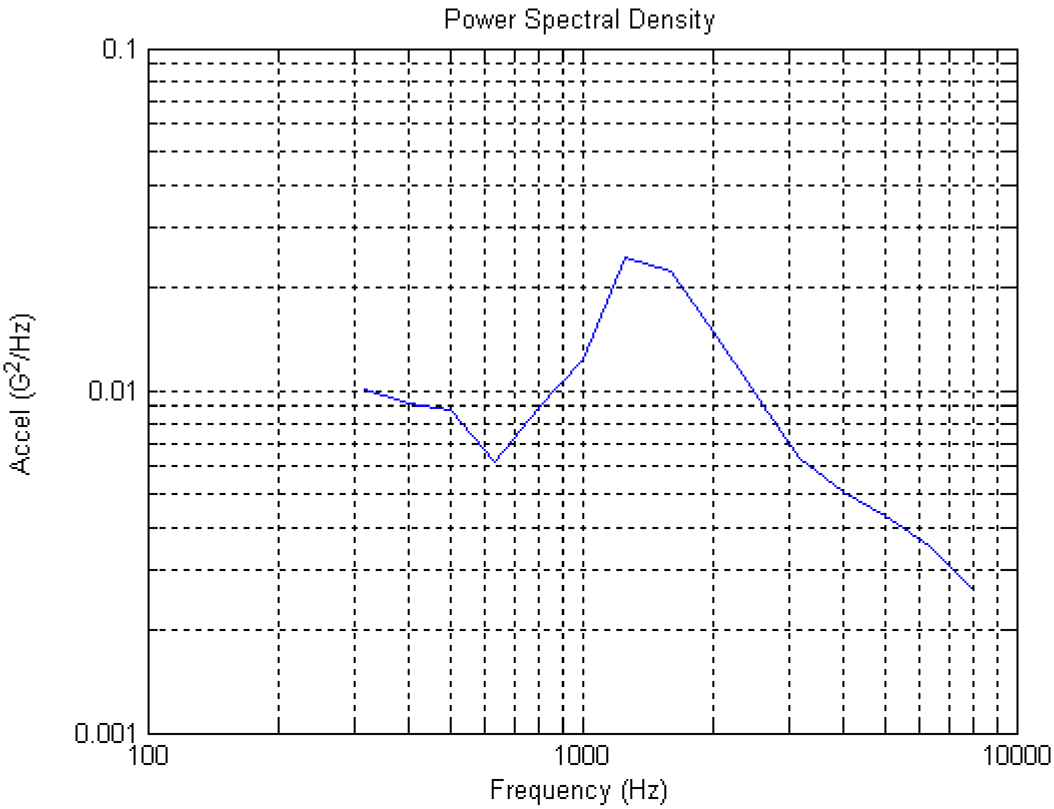

Figure A-6.

The cylindrical shell’s response PSD is shown in Figure A-6.

The response PSD is calculated by multiplying the power transmissibility function in Figure A-5 by the SPL in Figure A-6, with the appropriate unit conversion.

The overall acceleration is 5.1 GRMS over the domain up to 2000 Hz.

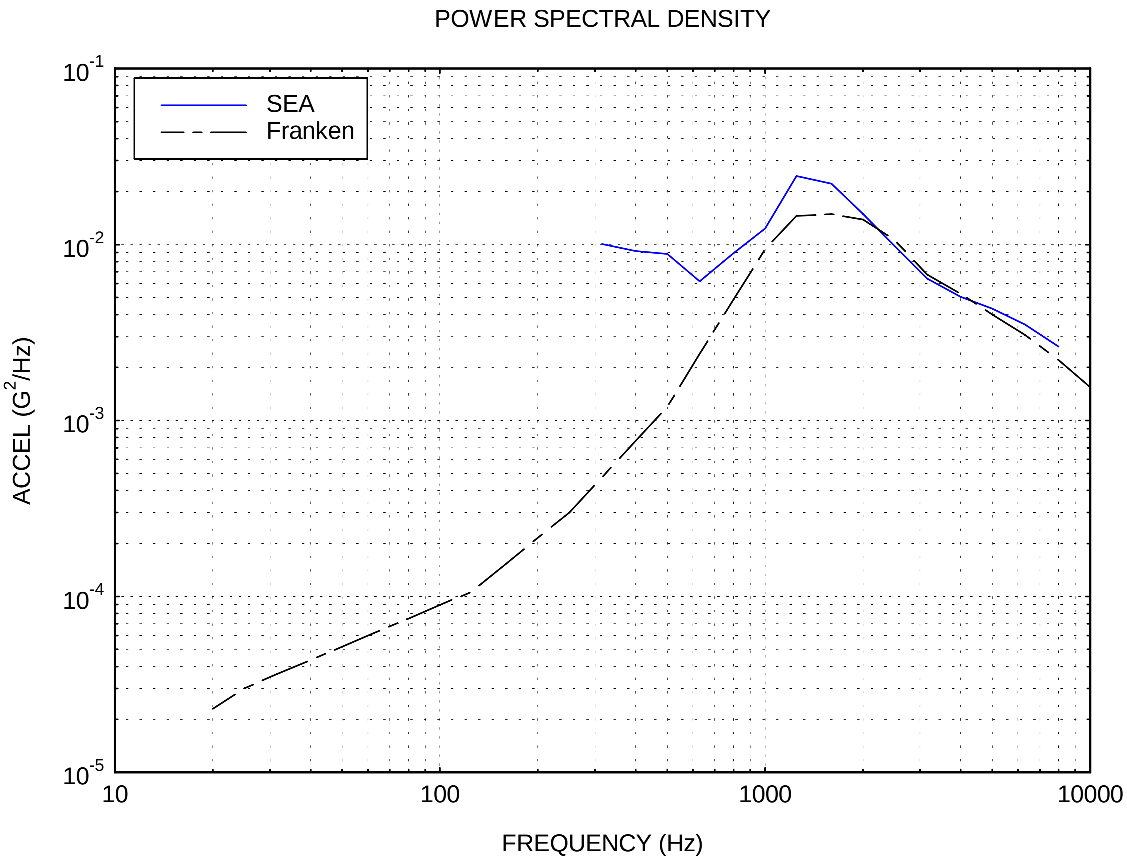

Figure A-7.

The SEA curve is the cylindrical shell's response PSD that was previously shown in Figure A-6.

The accuracy of the SEA curve below 600 Hz is questionable due to the scarcity of AF modes in this below this frequency.

The response via the Franken method is also shown in Figure A-7.

The following statements apply to the overall level over the domain up to 2000 Hz.

The SEA level is 5.1 GRMS.

The Franken level is 4.0 GRMS.

The SEA level is 1.7 dB higher.

APPENDIX B

Modal Density of a Cylindrical Shell

The following is taken from Reference 5.

Let

$$\nu_o = f_o / f_r \tag{B-1}$$

The normalized modal density B is

$$B = 2.5\sqrt{\nu_o} \quad \text{for} \quad \nu_o \le 0.48 \tag{B-2}$$

$$B = 3.6\,\nu_o \quad \text{for} \quad 0.48 < \nu_o \le 0.83 \tag{B-3}$$

$$B = 2 + \frac{0.23}{(F - 1/F)}\left[F\cos\left(\frac{1.745}{F^2\,\nu_o^2}\right) - \frac{1}{F}\cos\left(\frac{1.745\,F^2}{\nu_o^2}\right)\right] \quad \text{for} \quad \nu_o > 0.83 \tag{B-4}$$

The upper frequency band limit is $\nu_o F$. The lower limit is ν_o / F.

F=1.122 for one-third octave bands and F=1.414 for octave bands.

The modal density $n(\omega)$ is

$$n(\omega) = \frac{L}{\pi h \omega_R} B(\omega) \tag{B-5}$$

where

h = shell thickness

L = cylinder length

ω_R = ring circular frequency