Sine on Random Analysis: Alternatives and Challenges

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ABSTRACT

All frequency domain representations of environments include assumptions about the nature of the environment. Analysis methods which include these frequency domain loads also apply these assumptions. In combining loads, it is important that the analysis does not violate the assumptions used in defining the loads. This is the basic difficulty in combining sine with random or shock loads in any analysis.

This paper addresses the analysis challenges and pitfalls of creating a combined loading environment. An equivalent transient approach is demonstrated on a generic payload designed for captive carry by a helicopter using loads from Mil-Std-810.

INTRODUCTION AND BACKGROUND

Requirements for a product being designed or delivered often include shock and vibration environments to which the product will be subjected. These environments are usually supplied as frequency domain definitions that are applicable to standardized tests, such as sine dwell/sweep, random vibration, or shock response. These frequency domain definitions may be the only available definition of the actual environment.

Analysis methods have been developed and are in common use for each of these excitation types. All frequency domain representations of environments include assumptions about the nature of the environment; the real environments seldom exactly match those assumptions. For example, a helicopter environment includes sinusoidal excitations due to the rotor rotational speed and blade pass frequency which are accurately represented by discrete sine tones. Helicopter environments also include random vibration caused by engine noise and turbulence as well as shock loads caused by lifting or dropping payloads or firing weapons. The available descriptions of these environments have been prepared by separating accelerometer measurements from flight into components that match each of these types of loads in order to allow suppliers to design or test their equipment using standardized tests and analysis methods that are consistent with each of these types of frequency domain data representations. However, the real environments have all these types of loads often occurring simultaneously. The structural design or evaluation needs to consider some method of load combination. In the frequency domain, the load combination challenges include combining stresses or forces resulting from different sine tones for steady state frequency analysis or different modes if a shock response analysis is performed. In addition, the random results, which are only available as statistical averages, also need to be added in some way.

In combining loads, it is important that the analysis does not violate the assumptions used in defining the loads or processing the flight data to get the frequency domain loads. This is the basic difficulty in combining sine with random or shock loads in any analysis. This paper suggests that in some cases, it is valuable to reconstruct a transient from the frequency domain definitions in order to assess some aspects of the structural performance under the combined loading, as opposed to relying solely on frequency domain analysis for each load component and arbitrary load combinations.

Realistic loads are obtained from Mil-Std-810 F for a helicopter since this data is representative or the direct source of many shock and vibration requirements. Mil-Std-810 F provides the distinct sinusoidal tones from the rotor and blade pass frequencies and random vibration levels as well as shock from dropping and lifting payloads and gun fire.

The methods that are described here have been applied in the design of actual payloads that included these shock and vibration requirements. The methods described are applicable to many different structures, but the results shown here are from a generic model constructed to avoid using any proprietary data but realistic enough to illustrate the methods.

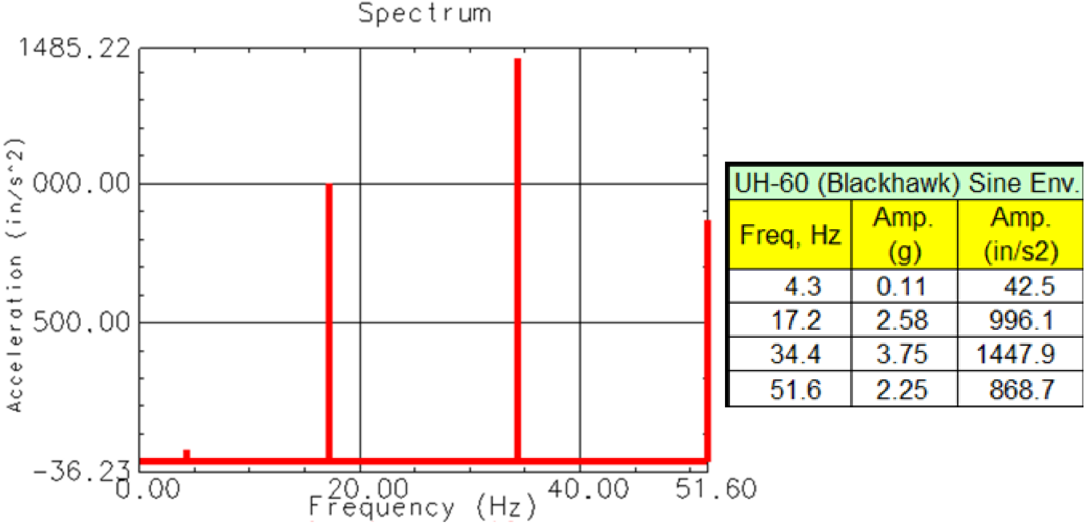
STANDARD METHODS

In this section, a review and comments about each of the frequency domain methods are provided, followed by some load combination approaches.

Sine Loads

Sinusoidal loads, defined in Mil-Std-810, for many different helicopters, can be a big problem for the payload designers. Representative amplitude versus frequency is shown in Figure 1. Note that the specification provides only a single discrete frequency for each sine tone. In the discussion, the specification states that frequencies vary less than one percent.

The analysis for a sine vibration is just the steady state or particular solution to the dynamic differential equations of motion for the structure. The response of each structural mode for sine tones below that frequency is at the same amplitude as the input and is greatly attenuated for sine tones above that mode frequency. At the mode frequency, the input sine tone causes a response which is greatly amplified with peak amplitude inversely proportional to the damping at that mode frequency. The response to a series of sine tones is a response at each input frequency with no response at other frequencies. Even structural mode frequencies will not show up in the sine response because these structural modes will contribute only at each input frequency.



UH-60 (Blackhawk) Sine Env.		
Freq, Hz	Amp. (g)	Amp. (in/s2)
4.3	0.11	42.5
17.2	2.58	996.1
34.4	3.75	1447.9
51.6	2.25	868.7

Figure 1. Blackhawk sine environment from Mil-Std_810 F.

Thus designing for the sine environment requires understanding damping for the payload and knowing its modes to a high degree of accuracy. If the modes are separated a few percentage points from the sine tone, the response will be greatly reduced over that which would occur if the input were aligned in frequency with a mode. However, if a sine tone matches a natural frequency, the response could be higher than the vibration input amplitude by a factor of 50 for a typical structural damping of 1%. Thus, sinusoidal response analysis requires determination of the appropriate damping and the uncertainty in mode frequencies of the structure, but, in the case of the helicopter, there is little uncertainty in the frequency of the excitation.

The best way to handle the structural frequency and damping uncertainties is to perform a modal test if hardware is available. Realistic damping and frequencies can be measured to allow a sine domain analysis to be completed with high accuracy. If the payload item is just being designed, testing a similar structure may be effective in identifying appropriate damping. One approach to handle the uncertainty in the structural mode frequencies is to move the sine tone closer to the closest mode frequency in order to identify the worst case condition. Another approach might be to broaden the input sine tone to a band width equal to the mode uncertainty, as shown in Figure 2, even though in actuality it is the structural mode not the input that carries the uncertainty.

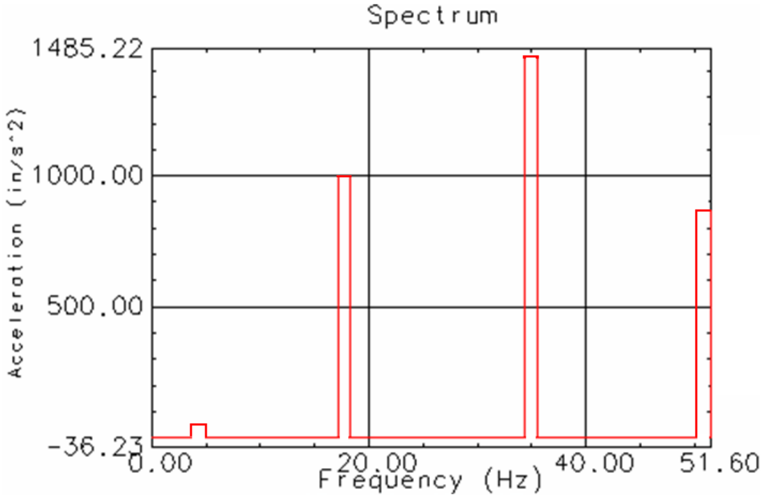


Figure 2. Broadened sine tones to account for uncertainties in the input.

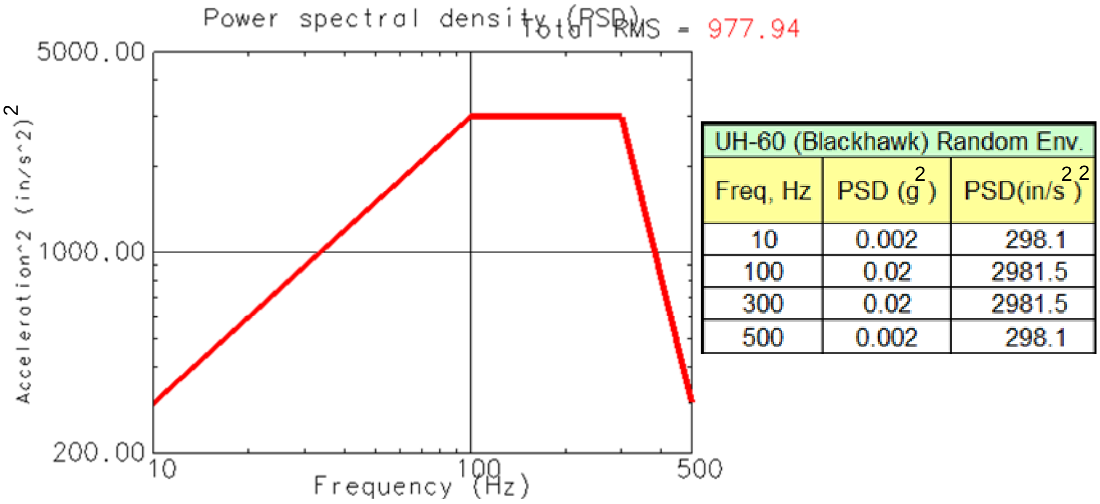
Random

Random loads provide the statistics of the environment after the sine tones have been removed from the flight data. A representative random environment from Mil-Std-810F is shown in Figure. 3. The challenge with random is the interpretation of the results, since the response predictions provide only the statistics of the response – not a deterministic actual response. In performing a test to qualify equipment for this environment, a shake table is driven to some physical acceleration with random amplitude and phase which has the specified power spectral density (PSD). This motion is not unique and will be different every time the test is performed. For direct use of the specified PSD in a random analysis, it is assumed that the environment is a stationary random process with zero mean. The response computed is the PSD of the response, which might be stress, deflection, or acceleration in a component. In order to determine the structural integrity, the PSD needs to be interpreted in terms of the probability that the response will exceed some allowable, such as yield stress. Common practice is to use the three standard deviations or 3σ value of the response as the peak load. From statistics, the integral of the PSD is the mean square value of the function, and the square root of this integral is the root mean square (RMS), or one σ value.

This definition of peak is based on the probability density function for a Gaussian random process. The selection of 3σ as peak is equivalent to acceptance of the statistical probability of exceeding the allowable value by 0.27%. This stress is only a statistical quantity, and there is no phasing information associated with the RMS value. Any combination of the predicted response from a random analysis with the predicted responses due to other types of loads needs to take into account the statistical meaning of the random analysis and the fact that the phasing of each stress is unknown. Therefore, one should consider multiple stress cases with stresses added with different phasing possibilities in each case.

Shock Response Spectra (SRS)

Some shock loads are defined by a time domain description, such as a half sine pulse with amplitude and pulse width. A shock caused by a drop from a helicopter is shown in Figure 3, which is a haversine pulse with a width of 10 msec, which could be directly used in a transient analysis. However, for many complex shocks, such as missile launch, the environment is represented by a shock response spectrum (SRS). This frequency domain analysis definition consists of the peak amplitude of response of a single degree-of-freedom system, tuned to each frequency on the frequency axis, when subjected to the environment. The analysis method that corresponds to this definition assumes that a real structure can be represented by a finite number of single degree-of-freedom systems, which are its modes. The analysis process just scales the response of each mode to the amplitude of the SRS. If there is more than one mode in the frequency range of interest, the peak responses for each mode need to be combined with the peak responses for all other modes in some way to get the response of the structure without knowledge of phasing information in each mode.



UH-60 (Blackhawk) Random Env.		
Freq, Hz	PSD (g^2)	PSD($(in/s^2)^2$)
10	0.002	298.1
100	0.02	2981.5
300	0.02	2981.5
500	0.002	298.1

Figure 3. Blackhawk random environment from Mil-Std-810F.

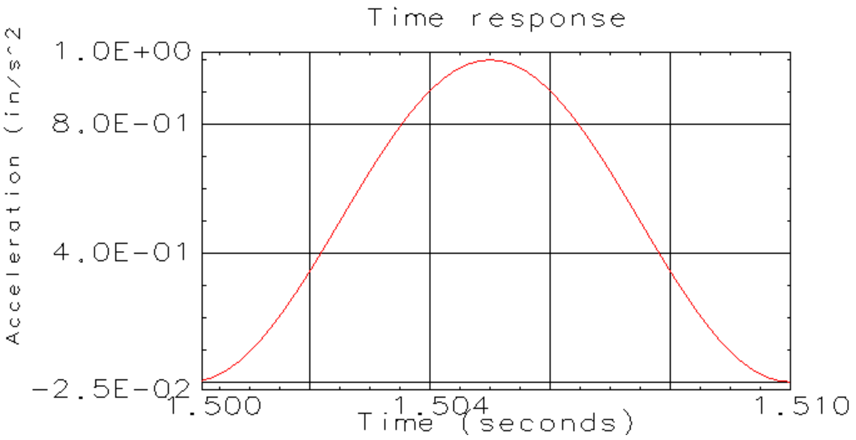


Figure 4. A sample drop shock excitation of haversine waveform and 10 msec pulse width.

In structural design for the environment specified above, each of the above frequency domain responses can be very valuable during the assessment and development of the structure for survival in the complex environment. Consideration of the combination of these results often leads to a very hard-to-interpret and possibly very conservative assessment if the sine, random, and shock peak values are added or if the number of cycles of load needs to be determined for fatigue. Other approaches to combining loads are considered next.

EQUIVALENT TRANSIENTS

The frequency domain versions of the environmental definitions are very useful in that the raw acceleration data has been processed into a more easily understood form showing the separate sine, random, and shock components. In all cases, conversion to the frequency domain condenses and cleans a great deal of what might be noisy data from flight and provides a condensed description of what may have started as hours of flight data. In the condensed frequency domain form, the data can be directly used with standard analysis methods or for control of test equipment. However, once this condensation has taken place, an equivalent transient version can be a very useful since it has the important characteristics of the overall original data yet can be cleaner and more manageable in size; as such, it is more amenable to use in a transient analysis than the original acceleration time histories. Tools are readily available for this data conversion between frequency and time domain. Siemens NX I-deas 12[1] was used for the demonstration in this paper.

Transient From Random

An example of a transient generated from the PSD definition is illustrated in Figure 5. This time signal is not unique. The method used to generate the transient is to multiply the amplitude of the PSD by the frequency bandwidth and take the square root of the product. The result is taken as the amplitude at the particular frequency. This amplitude is associated with random phase using a random number generator for phase between ±180°. The result is a frequency response function with amplitude and phase which can then be Fourier transformed to an equivalent transient with the exact PSD as the original signal. The result is a Gaussian random process if there are enough spectral lines. The statistical validity of this is described in a paper by Rice,[1] and this application of the equivalent transient is described by Engelhardt.[3] In Siemens I-deas response analysis, this conversion is handled by a single icon.

Each time this is done, a different transient will be obtained which is consistent with the statistical nature of the process. As one creates the transient, one can control the resolution of the PSD spectral lines and time points and thus the length of the transient. The transient could be as long as the original flight data, but the value of this approach is to limit the time to a more manageable size that can be easily used in a simulation. The recommended guidelines are long enough to include the longest wavelength response of interest and enough points to include a peak that is 3σ or 3 times the RMS value of the initial PSD. Although these guidelines are arbitrary, they are consistent with the acceptance of 3σ as the peak of a random variable and probably consistent with the algorithms for shake table control.

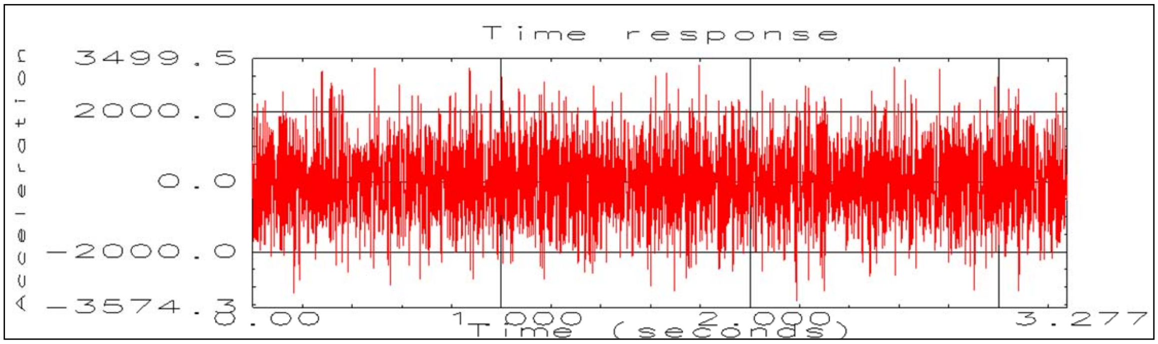


Figure 5. Transient generated from Blackhawk random excitation.

Transient from Sine

The four sine tones from the helicopter environmental specification were also turned into a transient by using Siemens NX I-deas signal processing (function tool kit) to generate a sine sweep that started and stopped at the tone frequency. The length of the random signal was used as a guideline for how long a transient was needed. However, an additional consideration is that there should be enough cycles of each transient to be sure that any resonant response will be able to develop and dominate any start-up transient that might be present. Four transient functions, one for each tone, were generated and added. Since this is a generic study, exact sine tones from Figure 1 were used. Half of a second of the combined sine excitation is plotted in Figure 6. Note that in this special case for a helicopter, each of the sine tones have the same source and could thus start and remain in phase. The rotor frequency, blade pass, and harmonics will all be multiples of the same initial sine function. Thus, unlike sine tones of random phase, these helicopter sine tones can add to form a new periodic excitation of higher amplitude than any single tone but not as high as the sum of the amplitudes. Note that in Figure 6, there is a new periodic wave form that is close in frequency (17 Hz) to the second sine tone but has am amplitude of 7.27 G, which is less than the sum of the sine tones (8.69 G) but much greater than any of the individual sine tones (.11 G, 2.58 G, 3.75 G, 2.25 G).

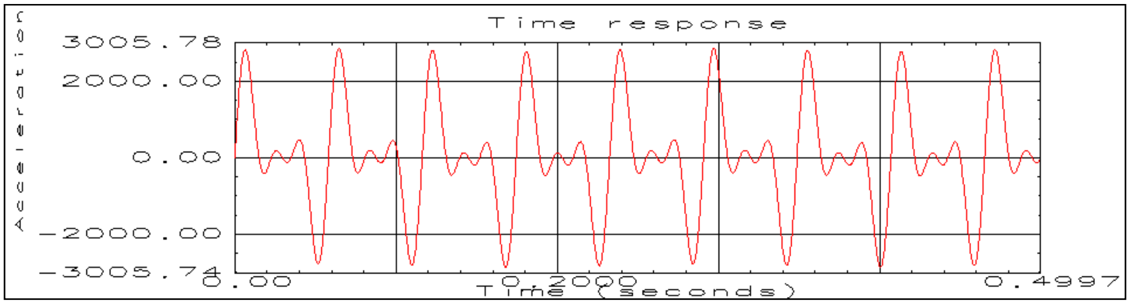


Figure 6. The sine tones are generated and summed.

Transient From SRS

If shock loads are defined by a pulse description such as half sine pulse 11 msec, the shock definition is already a transient, and this transient input can be directly added to other loads occurring simultaneously.

If the shock is only defined by a shock response spectrum (SRS), the equivalent transient for analysis can be obtained using the same methods used in the test lab to determine the real physical motion of the shock test equipment. This process is a curve-fitting technique for which a particular transient wave form is selected with arbitrary parameters which are used to minimize the error between the SRS for the synthesized transient and the original specified SRS. Wave forms used are damped sinusoids, wavelets, chirp, or burst random with fitting parameters such as stop and start time, frequency, amplitude, and damping for each element of the wave form.[4] These approaches are more challenging than the equivalent transient for the PSD in that the transient created will not provide the exact SRS. Many iterations may be required to get a transient with an SRS close enough to the specification. It is important to match the SRS of the synthesized function to the original SRS, particularly at the frequencies of important modes that will be driven by the shock.

Routines to do this are available from the shock and vibration website of Tom Irvine. [5] The wavelet and damped sinusoid algorithms have been implemented in Siemens NX I-deas.

Combined Transient

These separate excitations can be directly added once they are in the time domain, which is appropriate in the case of sine and random loads since these do occur simultaneously throughout the operation of the helicopter. Note that during addition, sine and random excitations sometimes interfere and sometimes add. The interference can prevent full resonance while the addition may cause higher instantaneous loads. The combined transient can be used for a more realistic evaluation than simply adding stresses from each environment. To get a better understanding of the combined load, the transient generated from adding the random equivalent transient and the four sine tones was converted back to the PSD form, as shown in Figure 7. Note the RMS of the combined transient is higher than the original PSD. Discrete sine tones are marked with the peaks at 4.3, 17.2, 34.4 and 51.6 Hz. The PSD plot was created for visualization purposes but is not appropriate for use directly in a random vibration analysis. Random analysis assumes that the environment includes a Gaussian random process with zero mean, conditions which are not met by our sine on random transient.

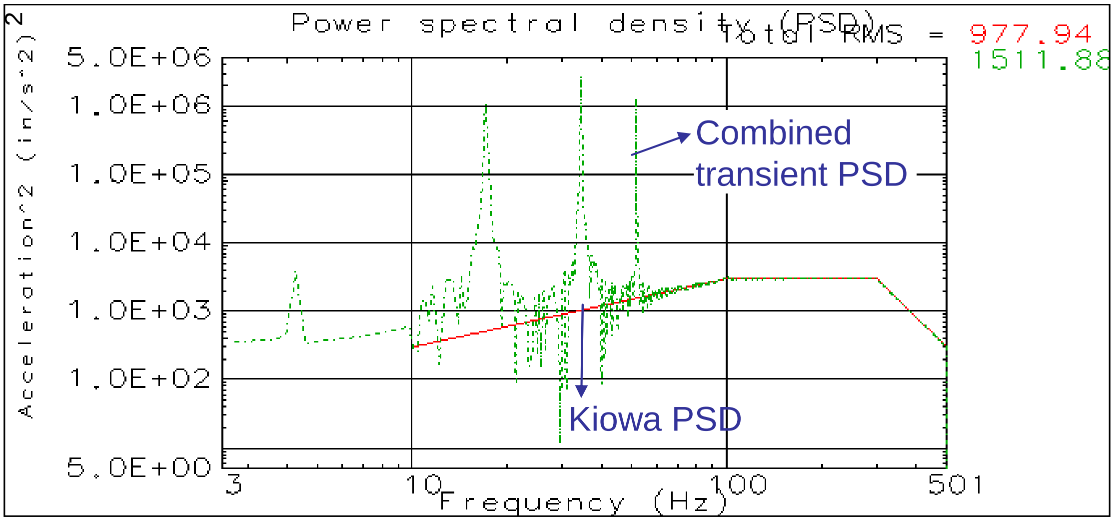


Figure 7. During the summation of the random and sine excitation, some waves cancel and some add.

Equivalent Transient Analysis Approach

The transient excitation for sine and random can be used directly in a transient analysis. Although the input is not unique, it is representative of the input excitation that would be seen in the flight test and is as accurate as any sine and random test that could be performed from the specification.

One concern in this approach is whether or not the start-up transient is affecting the results. Siemens NX I-deas provides two ways to address this concern. One way includes the option that the analysis can be started at quasi-steady state. Thus, initial conditions are not zero but the static response to the initial load levels. This moves the simulation closer to steady state. Another way to address this concern is to restart a dynamic event from the previous event. This allows the analyst to be sure the time segment is truly representative of one of many samples of a random process rather than dominated by the start-up.

Because the analysis is in the time domain, the deformed shape can be obtained and will help in understanding the design impact of the load. Even though the deformed shape is not unique, it will be typical of deformation associated with the dynamic loading. The ability to animate the motion and display deformed shape associated with peak stress is very helpful in design and is not possible in the frequency domain where the deformations are not real physical shapes. Sometimes in the design it is helpful to know if high stress responses are due to a particular motion that can be prevented. For example, a frame might be redesigned to reduce or eliminate an observed twisting or bending to carry load in tension, thus greatly reducing stress without resizing members.

An added flexibility in this approach is that the resulting transient signal can be used for fatigue analysis using tools like Siemens NX I-deas durability module[1] for cycle counting to determine the number of cycles at each amplitude. Even though the transient is a short segment of the random or sine environment, the actual operating life can be simulated by multiplying the damage by the number of time segments of each environment in the actual duty cycle.

EXAMPLES OF STRUCTURAL RESULTS

The approach presented in the paper is demonstrated on a generic payload designed for captive carry by a helicopter. The generic payload, shown in Figure 8a was modeled with shells. At the two mount locations, the circular perimeters of the payload were connected with spider rigids, and the master nodes of the spider rigids were connected to the fixed nodes using springs. The two fixed node locations are shown in the figure. The excitation is applied to these nodes during dynamic analysis.

The modal analysis of the payload was conducted in Siemens NX I-deas. The first ten modes are listed in Table 1, and the first mode shape is shown in Figure 8b. Note that the first mode is between the third and fourth sine tones and within 7% of the third sine tone in the Blackhawk sine environment.

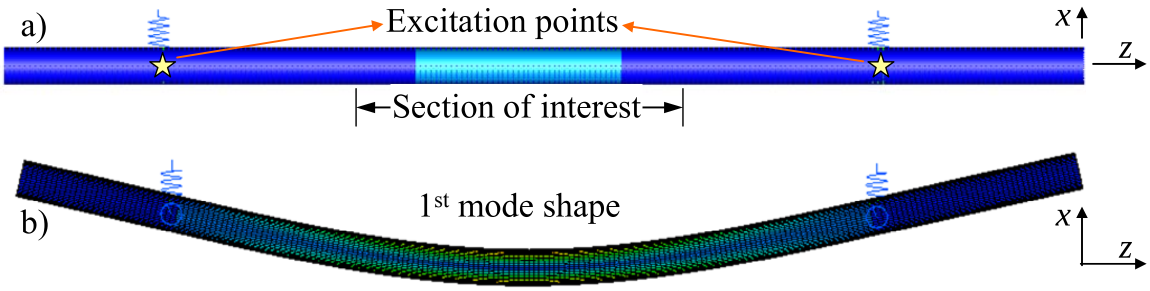


Figure 8. a) Payload excitation points, and b) 1st mode shape of the payload.

Table 1. First ten modes of the payload.

Helicopter Payload Modal Frequencies			
Mode	Freq.	Mode	Freq.
1	36.77	6	238.56
2	75.54	7	238.56
3	160.87	8	303.50
4	160.87	9	306.25
5	202.22	10	404.86

Blackhawk sine and random excitations, shown in Figure 1 and Figure 2, are applied on the two restraint points along the *x* direction in the frequency domain in I-deas. Modal damping of 5% was used for all modes. The excitations are applied at the two points in phase. This damping level was measured in a modal test on a previous structure carried in the same way on the helicopter.

The Von Mises stresses at each excitation frequency are contour plotted in Figure 9. As expected, the sine excitation at 34.4 Hz, which is closer to the first mode frequency, creates much higher stresses. Note that during sine analysis in the frequency domain, the stresses due to each spectral line are computed separately without combining results from other frequencies. This is the meaning of frequency domain analysis. Thus each sine tone stress is reported separately, but the combination of sine stress can occur and needs to be addressed in some way.

If the sine excitation is broad band in frequency content, the peak stresses would most likely occur at the modes of the structure which would allow one spectral line to show the peak stress. However, in this case, stress occurs at each sine tone, and the peak stresses are not clear in the frequency domain. For any narrow band sine excitation, the possible addition of stresses from different sine tones needs to be considered even though these tones might have unknown phase. For random phase, resonance may not occur; however, beating frequencies and sine tones from a common source raise the possibilities that stresses at different sine frequencies can add.

For a helicopter such as the Blackhawk, where the vibration for the externally carried payload is assumed to be dominated by the main rotor, the sine excitations are all from the same source; they have a fixed phase angle, and the frequency of the excitations are multiples of each other. In developing the forcing function shown in Figure 2, it is clear that the four sine waves create a new period function with an amplitude higher than any of the individual tones. A conservative way to take this phenomenon into account is to sum the peak loads from individual sine tones. The sum of peaks for the results displayed in Figure 9 is 30,514.6 psi. In order to verify this approach in the time domain, all four sine tones were converted to the time domain and added. The resulting transient was applied on the payload. Von Mises stresses in Figure 8a show that the peak stress after steady state has been reached is much higher than the peak stress of any single sine tone but slightly less than the combination of all sine tone stresses.

The PSD function in Figure 3 was applied on the payload in the frequency to calculate the loads due to the random environment. Since there were two locations where the helicopter supported the payload, it was also important to apply correlation functions which correctly represented the phasing of the two input locations. In this case, the two inputs where assumed to be in phase with a phase angle of zero or correlation of one. The RMS of the Von Mises stresses are plotted in Figure 11a. Siemens NX I-deas correctly computes Von Mises Stress from a random analysis using the methods developed by Segalman et al[6]. The typical peak stress from random might be achieved by multiplying the RMS stress by three; the probability density distribution for the Von Mises stress is not known, yet the three sigma value is still commonly used. The case where the two functions are uncorrelated was also simulated but not shown. The results for uncorrelated input at the two locations are lower by a factor of square root of two for the RMS stress.

To utilize the equivalent transient analysis approach, the PSD function in Figure 3 is converted to time domain using I-deas function tools. A transient with a max peak of 3.15σ and the exact PSD was created. Since we have assumed that the inputs at the two locations are perfectly correlated and in phase, we used the same transient at each input location. Had we wanted to represent random phase, we could have generated two separate transients and applied a different one at each input location. The Von Mises stresses caused by the correlated transients are plotted in Figure 11b.

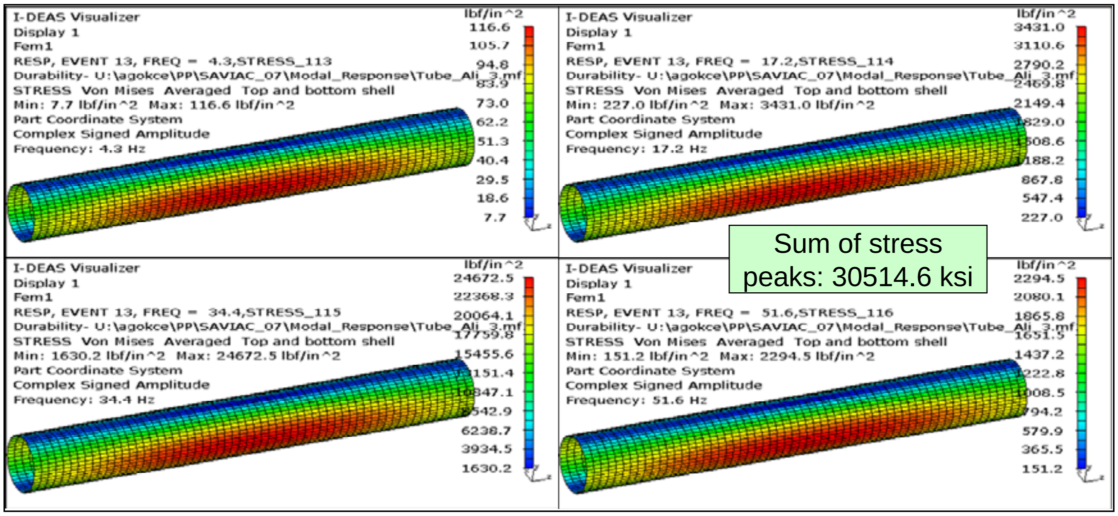


Figure 9. Von Mises stresses at excitation frequencies.

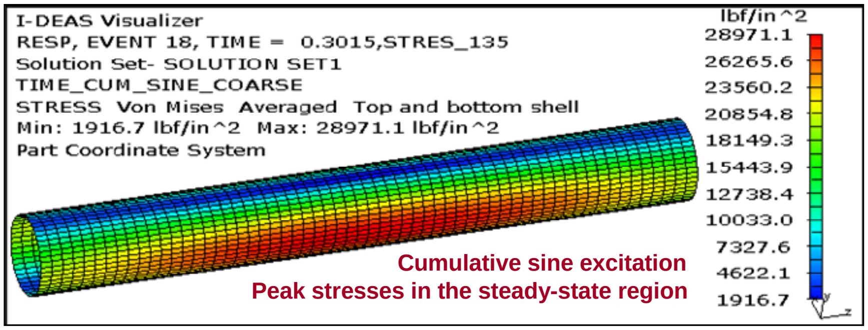


Figure 10. Steady state peak Von Mises stresses due to the transient input
of sine tones added with 0 phase angle in the time domain.

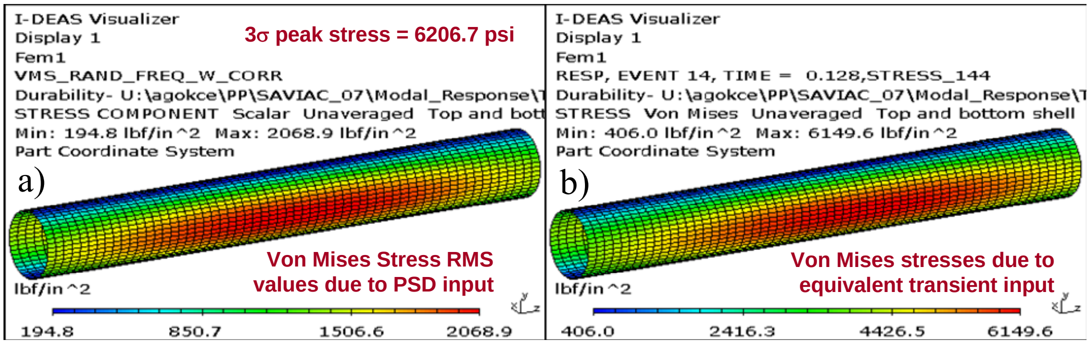


Figure 11. a) Von Mises stress RMS plot due to PSD input, and b) Von Mises stresses due to transient input.

In a real environment, sine and random excitation takes place simultaneously. If margins of safety are small, a direct combination of worst case of all loads may be too conservative. In this particular example, the sum of the peak stresses from four sine tones plus 3s stress from the random input makes 36,721 psi. Particularly for fatigue, it is helpful to predict the rate at which stresses occur under the combined loading and to understand how probable the peak combined load is.

Since we already have converted the sine and random environments to the time domain, we can calculate the loads due to the combined loading by adding the equivalent transients for the sine and random environments and applying the resulting signal to the payload as an input. In doing this it is important to compute a number of transients from random to increase the confidence that the results are representative. It is also important in the case of the time loads to use the sine transient response after the initial transient has died out. The I-deas function tools help to create the appropriate transients.

Figure 12 shows the contour plot of peak Von Mises stress as calculated by the equivalent sine on random combined transient analysis approach along with the associated deformed shape. The peak stress of 32,796 psi is 10% lower than the sum of all peak stresses from individual analyses in the frequency domain. The solid shape in the figure is the undeformed model. An advantage of the equivalent transient analysis approach presented in this paper is the availability of representative physical deformation for design insight. Depending on the nature of the deformation, twisting, bending, etc., the designer may be able to achieve an acceptable design using less material. Frequency domain analyses often do not provide physically reasonable deformations due to the lack of information on the phasing of displacements throughout the structure.

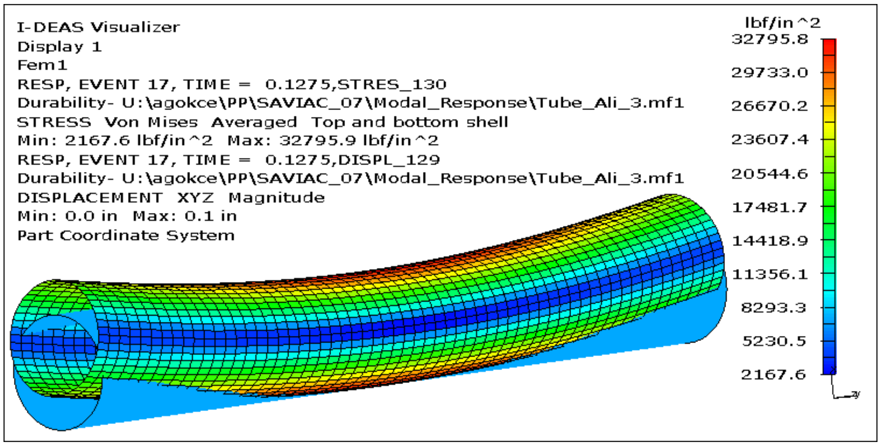


Figure 12. Von Mises stresses due to the combined transient input and the deformation of the payload.

Another advantage of the approach presented here is that fatigue-related post-processing, such as stress cycle counting and calculation of damage caused by each stress level, becomes straightforward using software like I-deas. Figure 13 shows the stress cycle counts and the damage ratio due to each stress level for the case study. The part is predicted to have 89 hours of life under the transient loading used for the analysis.

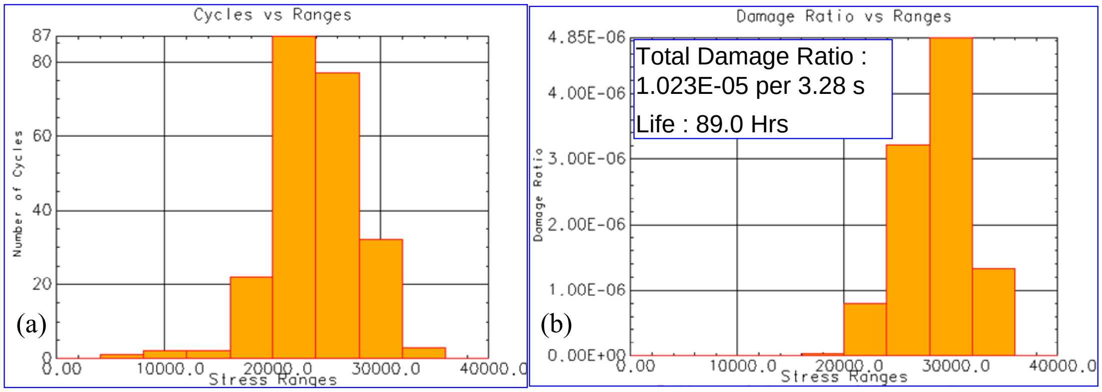


Figure 13. a) Stress cycle count due to loading, and b) damage ratios due to stress cycles.

A helicopter duty cycle may include several stages with different load types, such as flight, firing, landing, etc., as illustrated in Figure 14. A representative transient could be developed for each of these stages using the methodology outline here. The total damage from a full duty cycle can be estimated by scaling and summing the damage due to each stage, as shown in the figure.

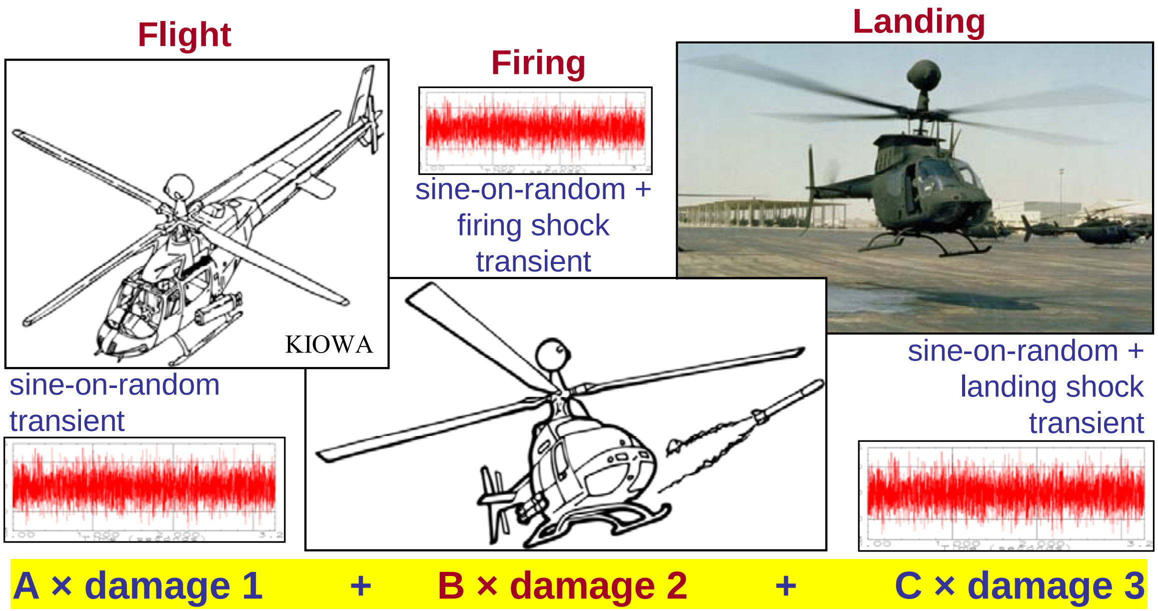


Figure 14. Total damage due to a duty cycle can be calculated by using appropriate transients.

CONCLUSIONS

It is often necessary to take structural loads due to sine, random, and shock environments defined in the frequency domain and reach a conclusion on the total damage expected for the actual flight environment in which all occur simultaneously. Methods for sine, random, and shock in the frequency domain do not address the issue of how the loads from these analyses are combined.

However, using readily available tools, one could convert these abstracted environments to the time domain following the precautions stated in this paper and combine them to create a realistic and manageable transient analysis that provides margins of safety for the combined environments. Moreover, with a constructed transient, one can obtain representative structural deformed shapes for design insight, conduct nonlinear analyses if needed, and perform cycle counting, fatigue analysis, and life predictions.

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