INTRODUCTION

Definition of the Shock Response Spectrum
Shock tests are performed to verify that a structure or a device can support transient vibrations encountered during its life in real environmental conditions. One of the shock testing formats is the Shock Response Spectrum. Shock Response Spectrum (SRS) analysis is, by definition, the maximum response of a series of Single Degree Of Freedom (SDOF) systems of same damping to a given transient signal.

The degree of freedom is defined as the ability to move along or around only one axis. These SDOF systems are selected as a reference to analyse transient phenomena. Instead of analysing them by FFT process, the SRS uses another mathematical tool provided by this SDOF reference. Indeed, the FFT algorithm is not suited to non-stationary signals of short duration and one states the hypothesis that due to the shortness of the vibrations, the severity can be characterised by their maximum effects on the SDOF set.

Equivalence principle
Even if the selected reference (SDOF systems) does not represent the equipment to be tested, we can assume that if two different excitations have the same severity (SRS), they will induce equivalent effects on this equipment. This notion of equivalence is at the origin of the using of the SRS because a synthesised pulse can replace a complex excitation whose severity is the same. Typical applications are: simulation of gunfire, launching, seismic, military aircraft taking off or landing etc…

SRS testing provides the ability to synthesise a complex waveform that can be applied to electrodynamic or electrohydraulic exciters in a controlled manner with consistent repeatability avoiding the using of shock machines that limit the shape of the excitation pulse. The severity of the test is maximised using SRS techniques.

Advantage of a generalised qualification
The qualification of equipment is often performed using a record of its specific vibration environment. This procedure must take into account the particular conditions of the records:

- Possible evolutions of some parameters
- Uncertainties on the source of excitation
- Location of the equipment
- *Etc…

In other words, the time domain transient signal used must not be considered as unique whichever are the conditions of recording.

If the environment is characterised by the SRS of the transient it becomes possible to average or envelope different spectra of different records as to extend the frequency bandwidth or increase the level in particular areas of uncertainties. Then the resulting calculated SRS provides a reference for a global qualification of the equipment, simulating different conditions of use.
**TEST PROCEDURE**

The SRS test procedure is very close to any transient testing format; the main difference being that in an SRS test, a time domain signal is synthesised from the RRS (Required Response Spectrum). We can summarise the procedure as follows:

1) Shock response synthesis  
2) Calculation of the transfer function of the test facility including amplifier + shaker + table + fixture + structure to be tested.  
3) Calculation of one or several drive signals for single or multi axes testing  
4) Feasibility control to verify that the test facility limits are not exceeded  
5) Sending and measurement  
6) Control of the validity of the test by comparing RRS with TRS (Test Response Spectrum) of the control points  
7) Correction if necessary after each transient sent to the shaker in closed loop

**SRS SYNTHESIS**

Shock Response Spectrum synthesis is a process where a time history is created whose SRS is exactly the same as the required one.

In the definition of SRS, only the maximum responses of the SDOF systems are conserved. It appears then that some data is lost in the information. It is easy to understand that the generation of a time domain signal involves the selection of different unknown parameters.

1) Selection of components: shape and frequencies  
The two most common techniques of waveform synthesis are summation of damped sine terms or wavelets created at each frequency “f” placed at intervals of 1/N Th octave (1/3, 1/4, 1/6, 1/12,…) in the frequency bandwidth of the RRS. Although there is no special obligation for the selection made to perform the test, damped sine is closer to the physical behaviour of any structure. Indeed, the excitation applied to a device or a structure is influenced by the response of adjacent structures that react with their own dynamic characteristics, resulting in a sum of damped sine excitations.
2) **Delay and number of cycles of components**

The number of cycles in each term and the placement in time (the delay of appearing within the total time domain window of synthesis) is user defined.

In fact it is recommended to follow rules of “good sense”:

- Wavelet: same delay for all components provides a better accelerogram shape.
- Damped sine: the components of heavy weight are placed at the beginning of the time record for a waveform coherent with natural events.

Heavyweight processing is necessary to create a synthesised time domain record meeting the requirements of the SRS test. Today’s powerful processors help the user in the process of automatically calculating optimal selection of delays and the number of cycles to observe the required rules.

The only user decision concerns the total duration of the waveform. Normally this parameter should be part of the test requirements given automatically with the RRS. If not, the choice is guided by the lowest frequency of the RRS and by the type of the environment to be simulated.

Another parameter can help in the choice of this duration: it is named ZPA (Zero Period Acceleration). It corresponds to the asymptotic value of the RRS and should be also the maximum value of the waveform. When the duration is coherent with the RRS, one can verify this redundancy.

3) **Compensation of the damped sine components**

Wavelets defined by acceleration versus time, carefully applied with an odd number of cycles, involve zero velocity and displacement at the end of the transient.

On the contrary, damped sine components don’t involve a non-zero value for velocity and displacement at the end of each send. It naturally follows that the same must be true for the sum of several components. This particularity can be dangerous for both the shaker and for the structure under test; some compensation must be performed to avoid this problem. Different techniques exist such as:

- Compensating each component of the synthesised waveform
- Using the “ZERD” method, which does not excessively modify the physical damped sine shape and leads to a zero value for both velocity and displacement of the synthesis.

4) **Weight of components**

Individual compensated components are summed and input to the SRS calculation. Then, their amplitudes are adjusted in an iterative manner until the SRS of the synthesised waveform matches the RRS as closely as possible. A technique to achieve good synthesis with a natural waveform shape is to use alternate positive and negative component start polarities.
**SRS FEASIBILITY**

The ability to perform the entire SRS test is limited by several factors.

1) **Dynamics of the RRS**
   A classical problem is encountered when the dynamics of the spectrum (for example, the ratio between the maximum part and the asymptote) is incoherent with the mathematical equation of SRS and especially when it is greater than it should be.

   This situation can be due to different processes applied to basic Shock Response Spectrum calculations (average, envelop, increasing of some areas) to define the RRS reference for the test. In this case, the synthesis matches the maximum part of the spectrum with a higher level than expected for the asymptote value.

   The problem is also encountered when the RRS includes several maximum areas separated by low levels involving a high dynamic range. One must keep in mind that the SRS of only one component defined at one specific frequency, influences all the adjacent frequencies: even if the synthesis implies zero level for the adjacent components, the spectrum will not be zero and sometimes higher than the requirement.

   Introducing negative levels does not solve the problem.

   Engineers performing the test most often think that the structure is overtested in this area, but the situation cannot be avoided due to incoherent data.

2) **Incoherent parameters**
   Another current problem encountered concerns the lower frequencies of the RRS. The specification of the lowest frequency implies a minimum value for the duration of the synthesis (say 1Hz implies at least 1s of duration). If this is incompatible with the test specification, the RRS must be modified to a frequency compatible with the maximum duration. Indeed, this parameter is normally of higher importance because it characterises the type of event to be reproduced.
3) Shaker limits

Another current problem concerns the feasibility of the test in terms of the shaker displacement, velocity and acceleration limits:

Specifications for seismic application as for simulation of the taking off or landing of a military aircraft on a ship most often involve much too high displacement requirements to enable the reproduction of the RRS at its lowest frequencies using any available shaker in the world.

This restriction forces the user to limit the RRS by suppressing one or several frequency components in this area. On many occasions, suppressing only the lowest component allows a test to pass the feasibility requirements. One can verify that just this first component, without any other, implies already a too high displacement; modifying delays and/or cycles usually will not lead to an improvement in the feasibility when the difference between the shaker’s limit and the displacement induced by this first component is important.

Specifications for pyrotechnic shock very often involve a very high level of acceleration. This case is difficult to manage when the necessary acceleration is incompatible with the shaker. The approach to achieve a feasible test is to “play” with the shape of the components, or if permitted with the duration.

Indeed, the amplification of SDOF systems depends on the shape of the transient and also on its duration. In other words, the necessary acceleration to match the maximum part of the RRS can be reduced in terms of these parameters.
SRS CLOSED LOOP CONTROL

When the synthesised waveform is performed as its feasibility checking, the resulting signal must be reproduced on the testing facility with one or several shakers and DOF (Degrees Of Freedom). As for any vibration testing, a pre-test is necessary to get a first estimation of the transfer function or the transfer function matrix. Then the drive signal(s) can be computed and reproduced.

If we could assume that the functions obtained during the pre-test are perfect (non-noisy data), and unique (linear behaviour), we could output the drive signal(s) directly and work in open loop without any correction. In reality the pre-test must be performed at low level first; the obtained functions can be noisy as also different from those of higher levels and in particular of the nominal level.

For these reasons, control and correction must be performed during the entire test and corrections applied to update the drive signal(s) after each send.

The control is affected more efficiently if different types of corrections are provided. The correction, which seems the more obvious, involves modifying the drive(s) in terms of the differences between RRS and the obtained TRS because really RRS is the reference to match. One must be reminded that the qualification of the structure submitted to a RRS is valuable if and only if the TRS is greater than the RRS in the entire frequency bandwidth of interest: just one frequency having an amplitude below the RRS leads to a requirement to redo the test.

In practice, other corrections must be provided to manage particular situations.

- The reference might be in the form of a time history corresponding to a captured record or to the design department result of a FE calculation.
- The RRS must be modified because non feasible completely (low frequencies suppressed or digital filtering applied for example)

In these two cases the correction must be performed in time domain comparing the external or synthesised waveform(s) with the obtained signal(s) at the control points.

- The whole facility and structure behaves non-linearly.

This difficult case must be managed using coefficients of correction, which can be different within the total bandwidth. Sometimes user-defined corrections are applied to force amplification or reduction of the component weights in specific frequency areas.
CONCLUSION

The technique of SRS testing is very convenient to reproduce complex signals accurately using the power of today’s controllers, respecting the aspect of a generalised qualification.

Laboratories involved in the seismic qualification of equipment installed in a Nuclear Power Plant have used this technique for many years. They have probably been pioneers using SRS, usually in a multi-shaker control manner.

Within the last 10-15 years, other industrial fields have understood the advantages of the technique and have caused the testing technique to evolve in different directions. Today, the range of applications using SRS is becoming very large including the extremes from seismic qualification (low frequencies, long duration, high displacement), to pyrotechnic testing involving very high frequencies and accelerations and to launch, Gun Fire and aircraft take off and landing.

All are possible if we use flexible tools for synthesis and control to adjust the requirements to make the test feasible.

The needs for SRS testing will continue to evolve: the tendency today being to reproduce as closely as possible, the real conditions of the environment.

An example of the evolution of SRS testing is to consider the new requirement for mixing SRS with a random sequence.

Random sequence

![Random sequence graph]

Random + Synthesised waveform

![Random + Synthesised waveform graph]

When a military aircraft is taking off or landing on a ship, a high level and complex shock is induced in all the equipment while the engine is working. This combination of random and transient excitation is best reproduced in the lab using SRS on Random.