SRS ALGORITHM LIMITATIONS FOR A RECTANGULAR BASE INPUT
Revision D

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February 4, 2012

Idealized rectangular pulses are used in dynamics analyses to excite all the modes of a structure. The shock response spectrum (SRS) is essentially intended for single-degree-of-freedom systems. But the same numerical engines used in the SRS calculation may be applied to the modal transient analysis of a multi-degree-of-freedom system where the system has been decoupled into modal coordinates via its normal modes.

Introduction

The SRS of a rectangular base input pulse can be calculated exactly using a Laplace transformation.

The SRS calculation for arbitrary pulses is performed using a numerical algorithm. Some of the candidate algorithms have a slight shift error during a rectangular pulse, as shown in the response acceleration time history.

This shift error is almost negligible for acceleration. The shift becomes significant, however, when acceleration is integrated to velocity in the time domain.

The purpose of this paper is to test five algorithms with respect to this potential error.

The cause of the error will be addressed in a future revision of this paper.

Candidate Methods

The five methods are:

1. Convolution integral converted to a nested series for digital data
2. Kelly-Richman digital recursive filtering relationship
3. Smallwood ramp invariant simulation digital recursive filtering relationship
4. Newmark-beta implicit numerical integration method
5. Runge-Kutta fourth order method[1]

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[1] The Runge-Kutta fourth order method may give unstable results for stiff systems with higher natural frequencies. Therefore, it may be unsuitable as a general purpose SRS tool.

The Convolution method is not commonly used because it is numerically inefficient.

The Kelly-Richman algorithm is a historical method which is sometimes still used.

The Smallwood algorithm is the current, preferred method for analyzing shock data.

The Newmark-beta method is favored in structural dynamics books for multi-degree-of-freedom systems. It can be adapted for an SRS calculation.

<u>Test</u>

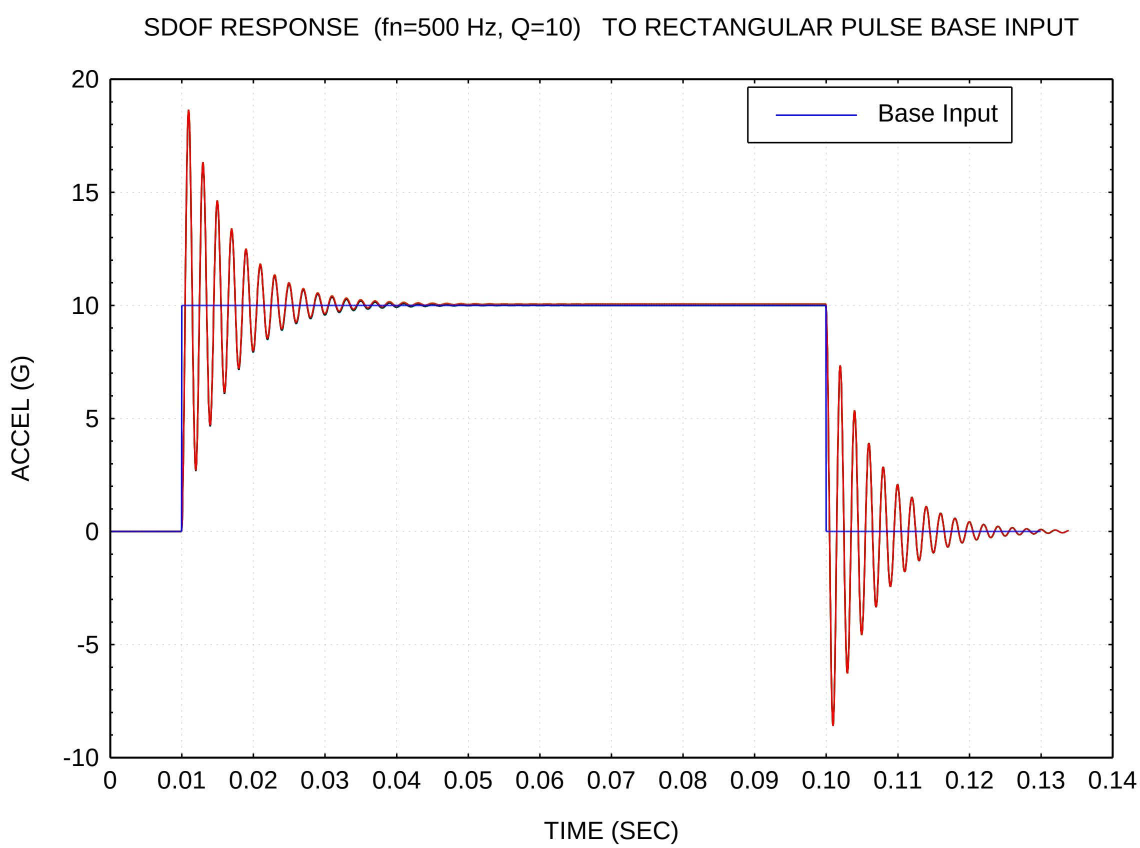

Figure 1.

A single-degree-of-freedom system has a natural frequency of 500 Hz and an amplification factor of Q=10.

It is subjected to a base excitation rectangular pulse with amplitude of 10 G and duration of 0.090 sec. Furthermore, there is a 0.010 sec pre-pulse and a 0.030 sec post-pulse, each with zero amplitude.

The responses via the five methods are shown along with the input pulse in Figure 1. The legend for the five response curves is omitted because the curves are very similar.

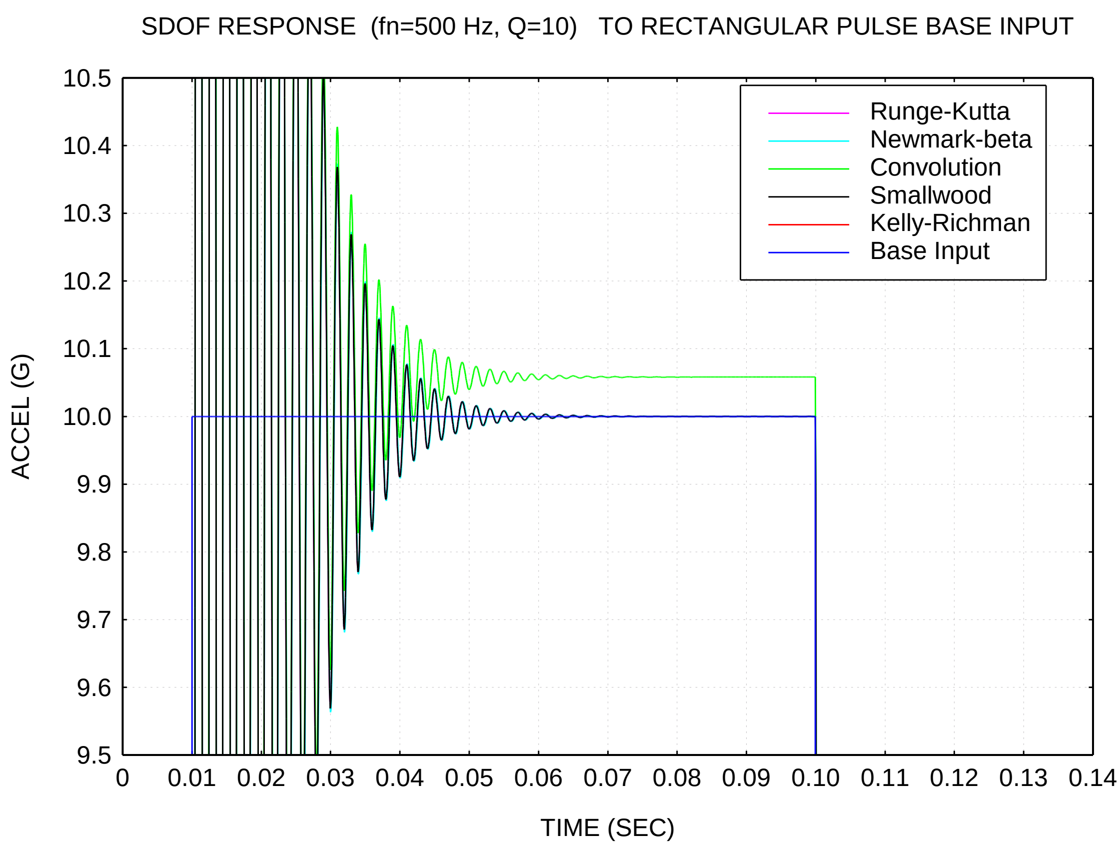

Figure 2.

The close-up view shows the error. The Kelly-Richman and Convolution curves are essentially identical, and both have an offset error. (Note that the Convolution curve is hidden behind the Kelly-Richman curve.)

The Smallwood, Newmark-beta & Runge-Kutta curves are very nearly equal, and each correctly converges to the base input plateau amplitude.

Conclusions

The results of the special case considered in this paper support the current industry preference for the Smallwood method.

The Laplace transform results, which are exact, were omitted for brevity. The Laplace response was nearly identical to the Smallwood curve. The Laplace and the Newmark-beta curves also had excellent agreement.

Further investigation is still needed.

References

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