Force-limited Acceleration Spectra Derivation by Random Vibration Analysis: Methodological Cases and Industrial Application

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Abstract

The *standard methods* used for the computation of Random Load Factors are based on Miles' formula. Nevertheless, standard methods have the following limitations:

- Do not allow taking into account the flexibility of the supporting structure.
- Assume that the acoustic load and the mechanical vibration at interface are statistically independent
- Do not allow taking into account spatial variations of the acoustic field.

These limitations can be overcome by performing a system Random Vibration Analysis, and deriving the interface force *resultant*. This methodology has been verified with simple test cases and applied in the frame of SOLAR CPD project.

Introduction

Payload flight equipment and secondary structures are designed to sustain a defined set of loads, called design load factors and usually given in terms of equivalent accelerations (expressed in g's). These accelerations are imposed on the payload equipment mass to generate reaction forces at the payload attachment points.

The design load factors are mainly originated by the following two typical sources:

- Quasi Static & Low Transients (QSL)
- Random Vibration (RVLF) and Acoustic Load Factor (ALF), assembled into the Random Load Factor (RLF)

RVLF are typically calculated from the applicable random vibration criteria using Miles' Equation, which is based upon statistical analyses of induced acceleration spectra with a 3-sigma distribution. Miles' Equation determines a load factor by assuming that the fundamental (first system) mode in each orthogonal direction will provide the primary response:

$$\mathrm{RVLF} = 3 \times \left(\frac{\pi}{2} \times \mathrm{Q} \times \mathrm{f_n} \times \mathrm{ASD_n} \right)^{1/2} \qquad (1)$$

where Q = Amplification factor, f_n = System fundamental frequency (Hz),
ASD_n= Acceleration Spectral Density at f_n (g^2/Hz)

For most components, a Q of 10 for all three directions should be used if not test data are available. Component frequencies are determined either by Normal Mode

Analysis or sinusoidal sweep test. The ASD values are determined from the component natural frequency, f_n, and the design random vibration environment, which envelopes the maximum input spectra for a particular mounting location. This method is reasonably accurate for systems with dominant fundamental modes, considering it approximates a component's response using only a *single degree of freedom* spring-mass system to represent loading over an entire frequency spectrum (20 to 2000 Hz).

For frequencies above 2000 Hz, an approximate RVLF may be obtained by multiplying the overall g_{rms} value by 3 (for 3-sigma statistical distribution). g_{rms} is the "composite" or "overall" level of the input acceleration ASD. For complex components in which a single dominant mode in each direction cannot be identified, the modal mass participation method may be used to provide a more realistic, less conservative load factor.

As a first step, a Normal Mode Analysis of the Finite Element Model (FEM) of the component with the boundary constrained in the flight configuration is performed and the mass participation of the model in each mode for each orthogonal direction is extracted. The sum of the participating in all of the significant modes in each direction should add up to *at least 80 percent of the total mass* of the component.

Essentially, this method allows the use of multiple modes in the calculation of the RVLF by multiplying the contribution of each significant mode ($RVLF_i$) by a ratio of the effective mass participating in the mode to the total component mass. Each mass-weighted $RVLF_i$ is then root-sum-squared with the others to obtain the composite RVLF for that orthogonal direction.

$$RVLF_{i,N} = 3 \times \frac{Meff_{i,N}}{M} \times \left(\frac{\pi}{2} \times f_{i,N} \times Q \times ASD_{i,N} \right)^{1/2} \qquad (2)$$

$$RVLF_N = \left(\sum_i RVLF_{i,N}^2 \right)^{1/2} \qquad (3)$$

where N = direction (X, Y, Z), i = mode number.

ALF are calculated with the same approach.
The flight acoustic spectrum Sound Pressure Levels (SPL) expressed in [dB] are converted in rms [Pa].

$$p_{rms} = p_{ref} \times 10^{SPL/20} \qquad \text{where } p_{ref} = 2 \times 10^{-5}\ \text{Pa} \qquad (4)$$

The rms [Pa] are converted in pressure PSD [Pa^2/Hz]

$$p_{PSD} = \frac{p_{rms}^2}{\Delta f} \qquad (6)$$

where $\Delta f = 0.233 \times f_c$ for 1/3 octave bands and $\Delta f = 3 \times 0.233 \times f_c$ for 1/1 octave bands, f_c =band central frequency.
The pressure PSD at the fundamental frequency is converted in input ASD with the following formula:

$$ASD[g^2/Hz] = \frac{p_{PSD}}{m^2 \times 9.81^2} \qquad (5)$$

where m = mass per unit area.

Then the Miles' formula is applied.
For each orthogonal direction, the RLF is computed as the RSS of the ALF and the RVLF, that is:

$$RLF_N = \left(ALF_N^2 + RVLF_N^2\right)^{1/2} \qquad (6)$$

where N =direction (X, Y, Z)

Standard methods have the following limitations:

- Do not allow taking into account the flexibility of the surrounding structure.
  In fact, the analysed structure shall be in hard-mounted condition.
- Assume that the direct acoustic load impingement and the mechanical vibration due to the surrounding structure are statistically independent
- Do not allow taking into account spatial variations of the acoustic field, but only foresee uniform acoustic pressure distribution on the analysed structure.
  In reality, for acoustic wave in air, the acoustic wavelength is an inverse function of frequency, that is:

$$\lambda_a = \frac{c}{f}$$

Nevertheless, the assumption of infinite acoustic wavelength is generally valid and conservative in the low frequency range, in which for common spacecraft structures the acoustic wavelength greatly exceeds the structural flexural wavelength. In particular, a methodology has been developed by Alenia to perform the Random Vibration Analysis of a spacecraft structure subjected to flight acoustic load with MSC/NASTRAN program. Spatial variations of the acoustic field can be represented.

These limitations can be overcome by performing a Random Vibration Analysis, using the FEM model of the component integrated in the FEM model of the spacecraft. In particular, a methodology has been developed by Alenia to perform the Random Vibration Analysis of a spacecraft structure subjected to flight acoustic load with MSC/NASTRAN program. This approach let represent spatial variations of the acoustic field. The drawback of Random Vibration analysis, however, is that the equivalent acceleration at C.o.G is not provided as direct output.

RLF derivation from Random Vibration Analysis

This kind of analysis is already used to define the Random Vibration Environment and test levels for secondary structures and equipment units. In fact, local responses, in terms of PSD and *root mean square (rms)*, are provided.

Nevertheless, Miles' formula can be also written as:

$$RVLF_N = \frac{\left(\sum_i F_{i,N}^2\right)^{1/2}}{M \times g} \quad (7)$$

where:

$$F_{i,N} = 3 \times Meff_{i,N} \times g \times \left(\frac{\pi}{2} \times f_{i,N} \times Q \times ASD_{i,N}\right)^{1/2} \quad (8)$$

and where N = direction (X, Y, Z), i = mode number.
$F_{i.N}$ is the *resultant* of the interface forces.

Random Vibration Analysis provides, instead, the single interfaces forces, in term of FSD and Frms. The resultant of the interface forces cannot be directly computed from the single interface forces: in fact, by simply performing a sum of the single FSD, the resultant is overestimated, since the phase differences between the single interface forces are not taken into account.

Nevertheless, since the Random Vibration Analysis is only a post processing of the Frequency Response Analysis, the resultant interface forces can be conveniently computed in this first step, where the phase information is available, and then squared (as the other transfer functions) to derive the resultant force spectral densities and the associated *rms* values.

Therefore, the following procedure has been set-up.

1. A frequency response analysis is performed, with unitary input
2. The interface forces at the constrained nodes are required, for each frequency, as real and imaginary part
3. The interface forces resultant is derived, for each frequency, as the RSS of the SUM of the real parts and the SUM of the imaginary parts, that is:

$$Fres_{k,N} = \left(\left(\sum Freal\right)_{k,N}^2 + \left(\sum Fimag\right)_{k,N}^2\right)^{1/2} \quad (9)$$

where k = frequency step number, N = direction (X, Y, Z)

4. The resultant FSD can be computed as:

$$FSDres_{k,N} = \left|Fres_{k,N}\right|^2 \times S_{k,N} \quad (10)$$

where $S_{k,N}$ = power spectral density of the input load

Test cases

Two simple test cases have been developed to verify the method:

1. Rectangular homogeneous panel clamped at the 4edges (symmetric case)

2. Rectangular homogeneous panel clamped at 3 edges (asymmetric case)

Both have been loaded with an homogeneous pressure field. The obtained results are compared hereinafter for the two cases.

SYMMETRIC

MODE	FREQ. (Hz)	M11	M22	M33
1	79.07	0	0	84.12
2	166.20	0	0	0
3	208.45	0	0	0
4	270.59	0	0	7.5
5	393.76	0	0	0
6	427.96	0	0	0
7	485.01	0	0	3.97
% OF TOTAL		**0**	**0**	**95.59**

Table 1a: Modal basis

ASYMMETRIC

MODE	FREQ. (Hz)	M11	M22	M33
1	43.74	0	0	64.47
2	103.34	0	0	22.58
3	175.74	0	0	0.22
4	246.52	0	0	5.66
5	306.58	0	0	0.67
6	414.96	0	0	0
7	439.75	0	0	1.44
% OF TOTAL		**0**	**0**	**95.05**

Table 1b: Modal basis

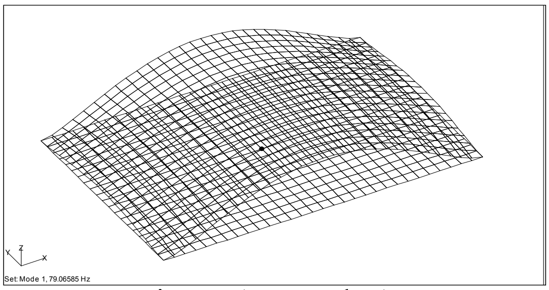


Figure 1a: Mode 1

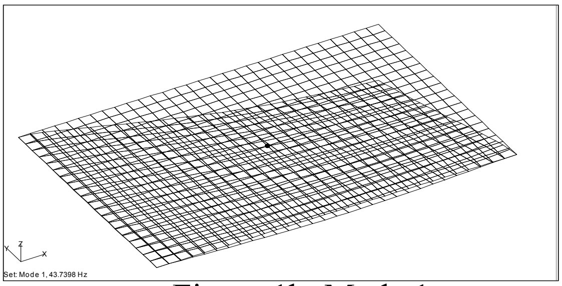


Figure 1b: Mode 1

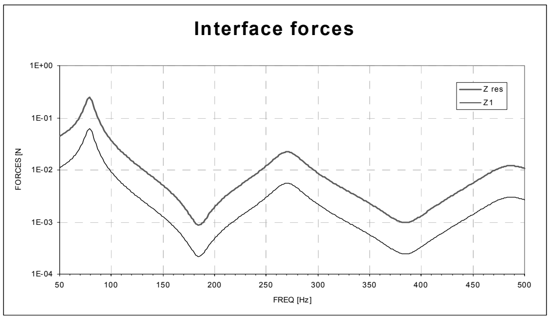


Figure 2a: Frequency response - single I/F compared with resultant

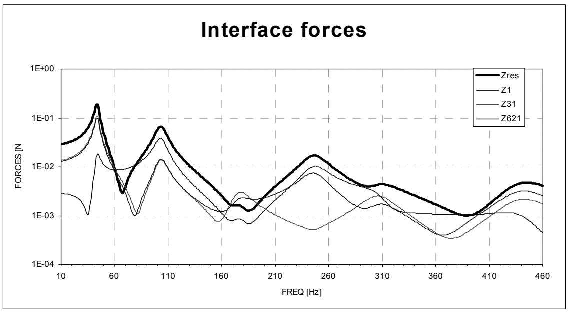


Figure 2b: Frequency response - single I/F compared with resultant

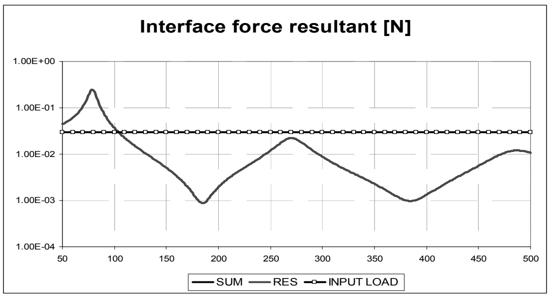


Figure 3a: Frequency response - resultant, sum and input load

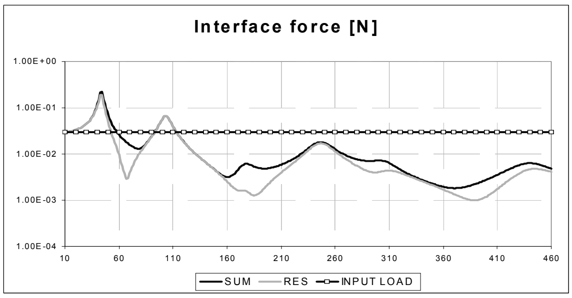


Figure 3b: Frequency response - resultant, sum and input load

SYMMETRIC

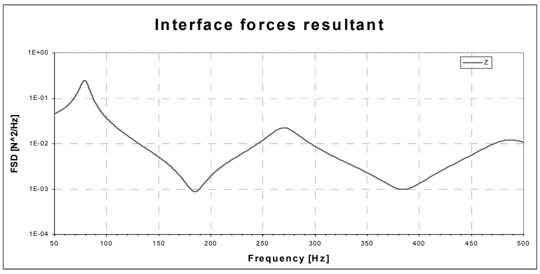


Figure 4a: Random response –resultant

Frms [N]	M [kg]	RLF [g]
8.423	1.102	2.3

MODE	f [Hz]	PSD [Pa^2/Hz]	PSD [g^2/Hz]	g rms	m eff X	geffx
1	79.1	91.2	0.0	**0.9**	0.8	**0.8**
2	166.2	145.4	0.0	**1.7**	0.0	0.0
3	208.5	182.6	0.0	**2.1**	0.0	0.0
4	270.6	182.6	0.0	**2.4**	0.1	**0.2**
5	393.8	28.9	0.0	**1.1**	0.0	0.0
6	428.0	28.9	0.0	**1.2**	0.0	0.0
7	485.0	28.9	0.0	**1.3**	0.0	0.1
						RLF Z
						2.4

Table 2a: RLF computation (resultant and Miles)

ASYMMETRIC

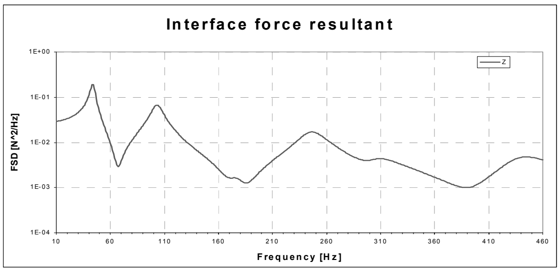


Figure 4b: Random response - resultant

Frms [N]	M [kg]	RLF [g]
6.084	1.102	1.7

MODE	f [Hz]	PSD [Pa^2/Hz]	PSD [g^2/Hz]	g rms	m eff X	geffx
1	43.7	115.1	0.0	**0.8**	0.6	**0.5**
2	103.3	145.4	0.0	**1.3**	0.2	0.3
3	175.7	145.4	0.0	**1.7**	0.0	0.0
4	246.5	182.6	0.0	**2.3**	0.1	**0.1**
5	306.6	182.6	0.0	**2.6**	0.0	0.0
6	415.0	182.6	0.0	**3.0**	0.0	0.0
7	439.8	182.6	0.0	**3.1**	0.0	0.0
						RLF Z
						1.8

Table 2b: RLF computation (resultant and Miles)

From the comparison of the obtained results, it can be noticed that, while in the symmetric case there is no difference between resultant and sum of interface forces, in the asymmetric case the sum overestimates the response.

This behaviour can be explained looking at the modal reaction forces: while in the symmetric case the reaction forces are always concordant (significant effective mass) or discordant (null effective mass), in the asymmetric case hybrid conditions occur. The complete agreement with Miles' formula confirms the reliability of the method. In fact, in this simple cases, the structure is hard-mounted and therefore the same boundary conditions assumed by Miles' formula are realised.

In case of soft-mounted conditions, instead, the actual interface acceleration can be automatically considered for RLF estimation, avoiding the conservatism relating to the adoption of an envelope ASD. In addition, if a direct acoustic load is applied, its effect can be directly combined with the contribution of interface ASD.

This methodology has been successfully applied in the frame of SOLAR CPD project.

Development tests and analysis activities

In the project of the SOLAR Integrated CEPA Payload, the flight configuration with CEPA frame (soft mounted – Figure 5) differ from the RV Test configuration by the absence of the CEPA plate (hard mounted condition – Figure 5). As a consequence, the random input spectra specified at CEPA frame bottom I/F cannot be directly applied to CPD without CEPA, since this input is subjected to such a modification/amplification due to CEPA dynamics. The I/F Forces method, presented in the above paragraphs, is then utilised in order to derive, for CPD soft mounted configuration, the I/F forces at CEPA to CPD I/Fs, the RVL factor at CPD CoG, instruments and ball bearings, injecting the predefined Random Flight levels at CEPA I/Fs. Using the above results, new opportunely notched random input spectra for CPD test configuration are derived, in order to avoid the exceedance of the RVL defined for the whole CPD project. The integrated FEM models of SOLAR CPD with CEPA (soft mounted condition) and without CEPA (hard mounted condition) have been used, as presented in Figure 5.

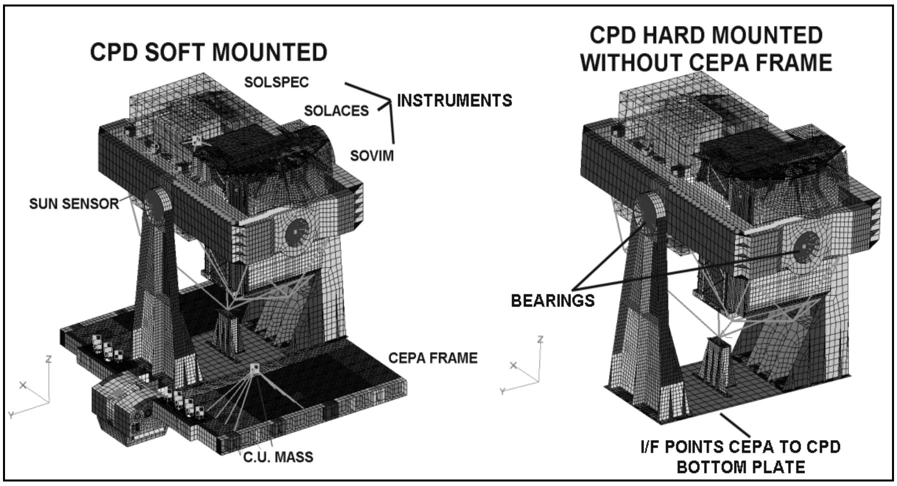


Figure 5: F.E.M. Model – CPD soft and hard mounted configuration

Qualification input spectra and environment evaluation

PSD spectra given by ESA for CPD soft mounted configuration at CEPA frame active/passive bottom I/F are different spectra in X, Y, and Z directions (Figure 6). The CPD input spectra are the maximum expected ones and are induced from the acoustic environment of 141 dB and so are used as qualification levels.

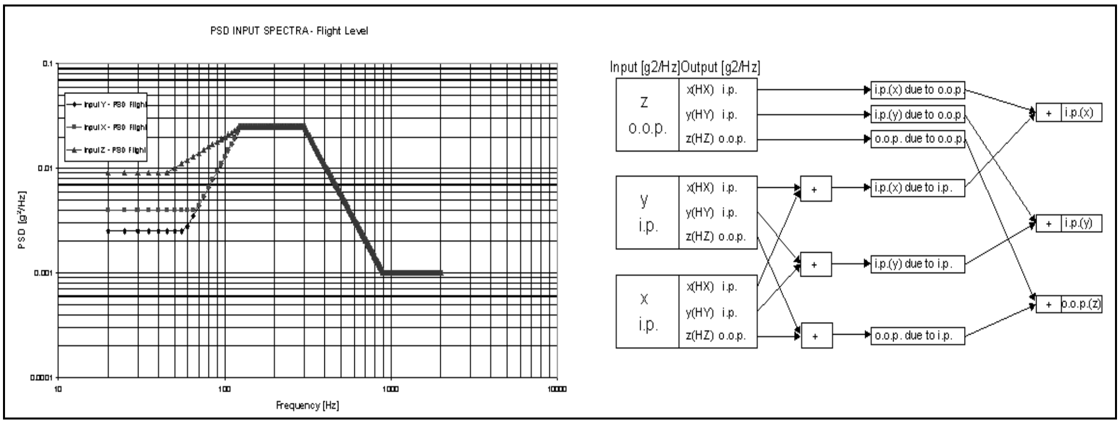


Figure 6: Input PSD spectra and acceleration combination schema

In Figure 3 the RVE derived at CEPA to CPD IFs are reported, obtained by the averaged environment from the 72 I/F points (Figure 5) between the CPD base-plate and CEPA. The spectrum levels derived are obtained injecting the ESA defined flight input levels at the CEPA I/F points and following the acceleration combination schema (as per Figure 6).

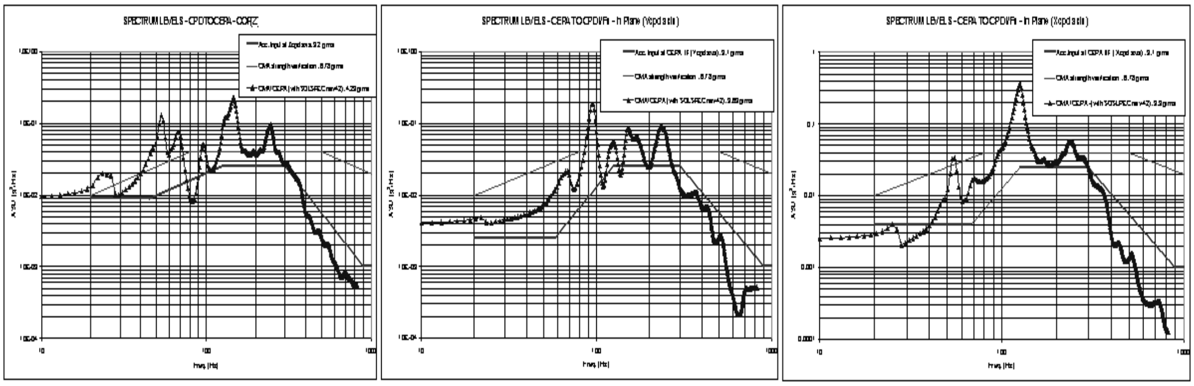


Figure 7: RVE at CEPA to CPD I/F - Z_{CPD} ,Y_{CPD} ,X_{CPD} direction

As can be seen from Figure 7 the RVE evaluated at CEPA to CPD interface is much higher in all the three axis directions, resulting also in higher g_{rms} values, with respect to the input level applied to CEPA (0.025 g^2/Hz). As a consequence, the input random spectra specified at CEPA I/F cannot be directly applied to CPD without CEPA, since this input is subjected to such a modification/amplification due to CEPA dynamics that should induce a different interface forcing function scenario.

I/F forces comparison

In order to derive the forcing functions and the RVLFs (as per equation 9), a frequency response analysis has been performed applying the acceleration of 1g for the entire range of frequencies. In this case, the FSDs and g_{rms} are evaluated, for each of the three principal CPD directions (X_{CPD}, Y_{CPD} and Z_{CPD}), as the squared of the *I/F Transfer Function (Apparent Mass in terms of [N/g])* of SOLAR CPD both hard and soft mounted. These transfer functions (in terms of *Squared Apparent Mass Function),* multiplied by ESA defined input acceleration spectra (Figure 6) and by derived RVE in terms of [g^2/Hz] for CPD hard mounted (Figure 7), give respectively the equivalent FSD for CPD soft and hard mounted configuration. The relation used is as follow:

$$\text{Squared\_Apparent\_Mass\_CPD}\left[{}^{N^2}\!/_{g^2}\right] * \text{Acce\_Input\_Spectra}\left[{}^{g^2}\!/_{Hz}\right] = \text{FSD\_CPD}\left[{}^{N^2}\!/_{Hz}\right] \quad (11)$$

For CPD hard mounted a comparison with the derived Miles RVLF shows a similarity to the g_{rms} obtained using the I/F force method:

RVLF			
METHOD	X [g_{rms}]	Y [grms]	Z [grms]
MILES EQ.	5.99	3.17	11.5
I/F FORCES	5.87	3.1	11.44

Table 3: Comparison RVLF with Miles and I/F Forces

Method for RVE derivation

The definition of the new input RVE for the CPD without CEPA is based on the comparison of the I/F forces between the two configurations studied (Figure 5). The application of the derived environment at CEPA to CPD I/F (Figure 7) for CPD hard mounted lead to a too conservative environment in terms of FSD, resulting also in high value of RVLF (Figure 8 shows as example the FSD comparison for X direction).

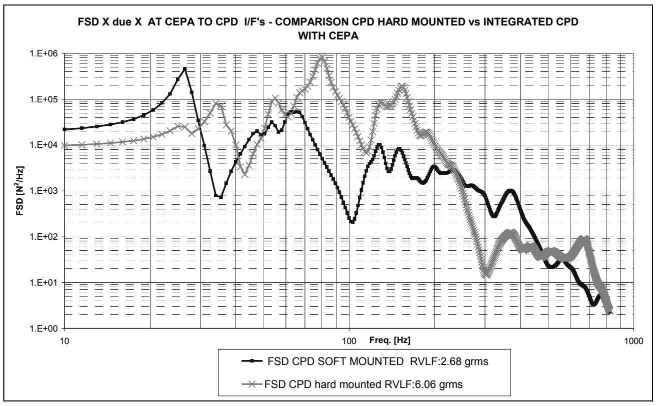


Figure 8: FSD soft and hard mounted comparison - X_{CPD} direction

A preliminary approach based on the equivalence of the I/F forces at each frequency band between the two configuration using relation(1) is corrected from an analytically point of view but could lead to a high conservative level in the definition of the new spectra. In facts, as appears in Figure 8, at 300 Hz in X direction is necessary to inject an high level of acceleration to the hard mounted model in order to make an equivalence of force with the soft mounted that is at least one order of magnitude lower than the maximum value achieved by the soft mounted model at lower frequency. Based on the results achieved and in order to reduce the conservatism a new approach has been defined based on the correspondence of the I/F forcing function at each equivalence of the principal mode shapes of both configuration. A Modal Assurance Criterion (MAC) has been used in order to evaluate the equivalence of the modal behaviour of the two models. Through the MAC criteria a list of correspondence modes for the 3 principal direction has been identified. The coupled mode shapes have been split in the three principal axis X, Y and Z using the results of the effective masses of the CPD+CEPA and CPD with-out CEPA configurations.

X DIRECTION						
CEPA+CPD			CPD HARD MOUNTED			MAC
MODE	CYCLES	T1	MODE	CYCLES	T1	
NO.	FREQ.	FRACTION	NO.	FREQ.	FRACTION	VALUE
2	26.5	28.68%	1	35.2	22.0%	0.6
1	23.0	0.47%	2	38.8	7.0%	0.61
6	61.8	5.06%	4	62.9	1.3%	0.59
7	63.9	1.97%	5	66.8	2.9%	0.54
9	66.6	5.36%	6	70.9	0.9%	0.57
14	80.1	0.09%	9	80.1	33.9%	0.83
18	93.6	0.002%	13	93.7	0.1%	1
23	114.3	0.01%	18	114.4	0.01%	1
44	203.6	0.0001%	37	184.6	2.2%	0.61

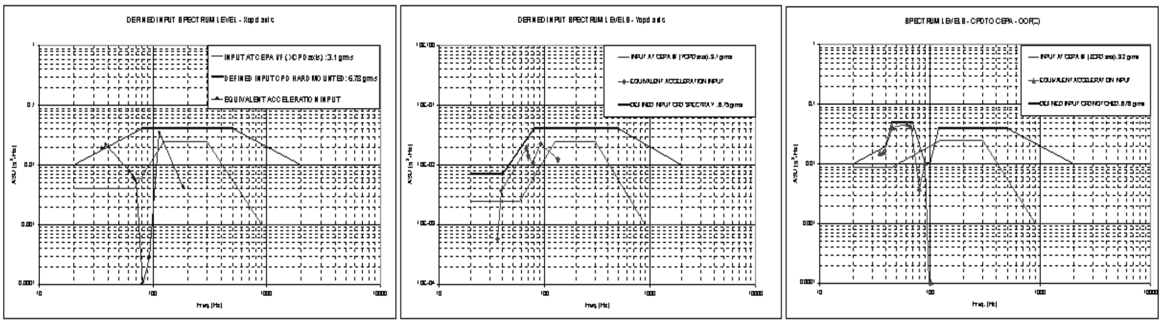
Y DIRECTION						
CEPA+CPD			CPD HARD MOUNTED			MAC
MODE	CYCLES	T2	MODE	CYCLES	T2	
NO.	FREQ.	FRACTION	NO.	FREQ.	FRACTION	VALUE
2	26.5	0.4%	1	35.1681	17.6%	0.6
1	23.0	42.1%	2	38.78	49.0%	0.61
7	63.9	6.4%	5	66.79	1.5%	0.54
8	65.0	0.3%	6	69.84	5.2%	0.59
12	75.8	0.4%	7	77.20	1.0%	0.85
10	69.4	4.8%	12	90.26	3.9%	0.52
31	133.6	0.01%	20	133.24	1.3%	0.88

Z DIRECTION						
CEPA+CPD			CPD HARD MOUNTED			MAC
MODE	CYCLES	T3	MODE	CYCLES	T3	
NO.	FREQ.	FRACTION	NO.	FREQ.	FRACTION	VALUE
2	26.5	5.6%	1	35.17	0.5%	0.6
1	23.0	0.9%	2	38.78	1.6%	0.61
3	45.49	0.8%	3	45.61	0.2%	1
6	61.8	9.3%	4	62.87	0.5%	0.59
7	63.9	0.5%	5	66.79	0.4%	0.54
9	66.6	13.0%	6	69.84	0.2%	0.57
14	80.1	0.01%	9	80.42	1.2%	0.83
10	69.4	7.9%	12	90.26	9.8%	0.52
21	101.73	0.01%	16	101.83	0.1%	1

Figure 9: CPD modal coupling using MAC - X_{CPD} Y_{CPD} and Z_{CPD} direction

New RVE input spectra derived

For the CPD RV test unit qualification, using the ratio between the squared apparent mass function and the known value of spectral force in terms of [N^2/Hz] (equation 11) the equivalent acceleration input for X_{CPD}, Y_{CPD} and Z_{CPD} has been derived, in order to reach the same I/F force at each principal equivalent mode shape (Figure 9). The equivalent acceleration input obtained is an uneven curve based on the modal equivalence; the new RVE spectra is obtained enveloping these curves. For CPD hard mounted a notching approach vs. the input at the base of CEPA at 90 Hz has been necessary for the Z direction, due to the presence of a mode which gives an high force level contribution and have no correspondence with the soft mounted configuration.



Using these new defined environment, it has been possible to derive the RVLF for each of the instruments mounted on CPD structure, acquiring the resultant I/F force. The comparison of the RVLF derived from the new spectra with the design RVLF allows to confirm or to change and optimise again the results achieved.

Conclusions

These successful experiences highlight the suitability of the design verification process developed in order to verify and control the RVE requirements. Evidence :

- Less conservative method
- Flexibility due to unitary runs – derivation of the apparent mass functions
- Rigorous modal correspondence due to MAC criteria
- Direct investigation of RVLF for soft mounted structures
- This industrial success qualifies Alenia as a primary player in the manned space systems field.

Acronyms

ASD	Acceleration Spectral Density
CEPA	Columbus Express Pallet Adapter
CoG	Center of Gravity
CPD-SOLAR	Coarse Pointing Device-Solar version
ESA	European Space Agency
FEM	Finite Element Method
FSD	Force Spectral Density
I/F	Interface
RVE	Random Vibration Environment

References

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[2] T.D. Scharton, Force Limited Vibration Testing Monograph, NASA RP-1403, May 1997.