



CALCULATION OF SHOCK RESPONSE SPECTRUM

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Abstract: *As it is stated in the ISO 18431-4 Standard, a Shock Response Spectrum is defined as the response to a given acceleration acting at a set of mass-damper-spring oscillators, which are adjusted to the different resonant frequencies while their resonance gains (Q-factor) are equal to the same value. The maximum of the calculated responses as a function of the resonance frequencies compose the Shock Response Spectrum. The paper will deal with employing Signal Analyzer, the software for signal processing, for calculation SRS. The theory will be illustrated by examples.*

Key words: *shock response spectrum, shock, single degree of freedom system, measurements*

1. INTRODUCTION

Shock Response Spectrum (SRS) analysis was developed as a standard data processing method in the early 1960's. Firstly SRS was used by U.S. Department Of Defense. Now this signal processing method is standardized by ISO 18431-4 Mechanical vibration and shock — Signal processing — Part 4: Shock response spectrum analysis [1].

Let it be assumed that many small instruments or parts as substructures are mounted to the base structure of a product [2]. The mounting elements are flexible and could be described by stiffness and damping parameters. The mass of the mentioned substructure creates the mechanical oscillator. Now, we suppose that a vibration transient is excited on the base structure as a consequence of either its environment or normal operation. Many sources of excitation could be described: Drop impact during handling, explosive bolts on aerospace structures, bolted joints suddenly opening and closing with an impact, reciprocating engine fuel explosions inside cylinders, etc.. For example we could assume that vibration transient results from the product dropping onto a concrete surface while producing an impact load on one corner of the base structure. The impulse force acting to substructures excites lightly damped oscillations. The peak value of the substructure acceleration may cause destruction of the substructure due to the large inertia force. Two main parameters specifying the oscillation are the substructure natural frequency and damping. For a normalized value of the damping parameter Q the key role plays the substructure natural frequency f_n . The individual frequency components of the force, acting to the base structure, excite the various peak values of accelerations of the substructure. The dependence of the peak values (maximum or minimum) on the natural frequency in the form of a diagram is called Shock Response Spectrum (SRS).

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SRS is a tool for evaluating the effect of the impact load of the base structure to the mounted substructures differing in the resonant frequencies. It is required that the mentioned substructures, in fact equipments and instruments, have to survive mechanical shocks. Shock exposure has to be under control and verified by shock tests.

As it was stated above, mechanical shock pulses are analyzed in terms of shock response spectra. The shock response spectrum assumes that the mechanical shock pulse is applied as a common base input to a group of independent single-degree-of-freedom systems, see figure 1. The shock response spectrum gives the peak response of each system with respect to the natural frequency of each system. Damping is typically fixed at a constant value, such as 5%, which is equivalent to an amplification factor of $Q=10$.

Natural frequencies for SDOF system

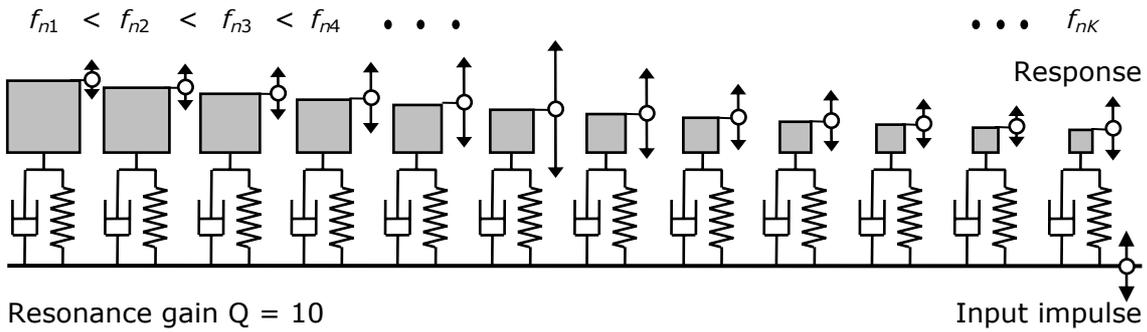


Figure 1: Shock Response Spectrum Model

2. SINGLE-DEGREE- OF-FREEDOM MODEL

The equation of motion, which is describing oscillation of the single-degree-of-freedom (SDOF), shown in figure 2, is written in the form

$$m\ddot{x}_2 = -c(\dot{x}_2 - \dot{x}_1) - k(x_2 - x_1) \quad (1)$$

where x_1 is a base input displacement and x_2 is a response displacement of a single-degree-of-freedom system, m , c , k are mass, damping parameter and stiffness, respectively. The corresponding velocity as a time function is designated by v_1 and v_2 and acceleration is designated by a_1 and a_2 .

The Laplace transfer function relating the base displacement to the substructure mass displacement is as follows

$$G(s) = \frac{X_2(s)}{X_1(s)} = \frac{cs + k}{ms^2 + cs + k} \quad (2)$$

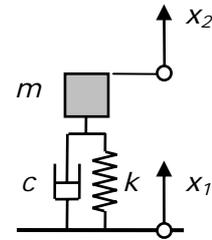


Figure 2: Single-degree-of-freedom system

Assuming the zero initial condition, the relationship between the Laplace transform of displacement and velocity and relationship between the same transform of velocity and acceleration are as follows

$$V_1(s) = sX_1(s), V_2(s) = sX_2(s), A_1(s) = sV_1(s), A_2(s) = sV_2(s) \quad (3)$$

where

$$\begin{aligned} L\{x_1(t)\} &= X_1(s), & L\{v_1(t)\} &= V_1(s), & L\{a_1(t)\} &= A_1(s) \\ L\{x_2(t)\} &= X_2(s), & L\{v_2(t)\} &= V_2(s), & L\{a_2(t)\} &= A_2(s) \end{aligned} \quad (4)$$

Alternatively the Laplace transfer function relating the base velocity to the substructure mass velocity and the base acceleration to the substructure mass acceleration are as follows

$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{cs + k}{ms^2 + cs + k} \quad G(s) = \frac{A_2(s)}{A_1(s)} = \frac{cs + k}{ms^2 + cs + k} \quad (5)$$

All the transfer functions are of the same form. Introducing the SDOF system natural frequency f_n , Q value (resonance gain) and damping ratio ξ (fraction of critical damping), we obtain

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}, \quad Q = \frac{\sqrt{km}}{c}, \quad \xi = \frac{1}{2Q} = \frac{c}{2\sqrt{km}}. \quad (6)$$

The Laplace transfer function relating the base acceleration to the substructure mass acceleration could be written in the form

$$\frac{A_2}{A_1} = \frac{\frac{\omega_n s}{Q} + \omega_n^2}{s^2 + \frac{\omega_n s}{Q} + \omega_n^2} \quad (7)$$

For a given damping ratio Q , natural frequency ω_n and base structure acceleration a_1 in the form of the time function, it is theoretically possible to calculate the acceleration response a_2 in the form of the time function as well and consequently to determine the peak value (either maximum or minimum) of this time function. The problem is that the input acceleration signal is in the form of the sampled signal with the sampling interval $T_s = 1/f_s$, not in the form of a continuous signal. The transfer function (9) has to be approximated by the Z-transform function, i.e. to be transformed into a discrete system (digital filter).

3. DISCRETE APPROXIMATION OF CONTINUOUS TRANSFER FUNCTION

Approximation of a complex function in s-plane by a function in the z-plane, i.e. approximation of the differential equation by the difference equation, is a problem of the numerical integration. The continuous function between two adjacent samples may be approximated either by a constant value or by a ramp or by any suited function. We need find a mapping of the s-plane to the z-plane saving the important properties of the continuous system after transformation into a discrete system of the same order. There are two important properties, namely the resonant frequency and resonant gain.

The digital filter corresponding to SDOF system responses is the second order filter IIR (Infinite Impulse Response), with the general z-transform expression

$$H(z) = \frac{A_2(z)}{A_1(z)} = \frac{\beta_0 + \beta_1 z^{-1} + \beta_2 z^{-2}}{1 + \alpha_1 z^{-1} + \alpha_2 z^{-2}}, \quad (8)$$

where $\beta_0, \beta_1, \beta_2, \alpha_1, \alpha_2$ are the filter parameter.

The approach to the approximation, which is preferred in the ISO 18431-4 Standard, is based on the Ramp Invariant Method, which was described in [3]. The transfer function corresponds to the difference equation, which is enabling to calculate the response function in the time domain

$$x_2(k) = \beta_0 x_1(k) + \beta_1 x_1(k-1) + \beta_2 x_1(k-2) - \alpha_1 x_2(k-1) - \alpha_2 x_2(k-2). \quad (9)$$

The digital filter parameters are calculated by formulas given below

$$\beta_0 = 1 - \exp(-A) \sin(B) / B \quad (10)$$

$$\beta_1 = 2 \exp(-A) [\sin(B) / B - \cos(B)] \quad (11)$$

$$\beta_2 = \exp(-2A) - \exp(-A) \sin(B) / B \quad (12)$$

$$\alpha_1 = -2 \exp(-A) \cos(B) \quad (13)$$

$$\alpha_2 = \exp(-2A) \quad (14)$$

where

$$A = \frac{\omega_n T_s}{2Q}, \quad B = \omega_n T_s \sqrt{1 - \frac{1}{4Q^2}}. \quad (15)$$

4. TRANSFER FUNCTIONS FOR RELATIVE VELOCITY AND DISPLACEMENT RESPONSE

For some substructure, the difference of the velocity or displacement between the base structure and the mentioned substructure, which is excited by the base structure impulse load, is more dangerous than the substructure acceleration. The other transfer functions, which are relating this velocity or displacement difference to the base structure acceleration, are shown in table 1. The corresponding filter coefficients could be found in [1].

Tab. 1 Laplace transfer function

Relative velocity response spectrum	Relative displacement response spectrum
$\frac{V_2 - V_1}{A_1} = \frac{-ms}{ms^2 + cs + k}$	$\frac{X_2 - X_1}{A_1} = \frac{-m}{ms^2 + cs + k}$
Pseudo velocity response spectrum	Relative displacement response spectrum expressed as equivalent static acceleration
$\frac{X_2 - X_1}{A_1} \omega_n = \frac{-m\omega_n}{ms^2 + cs + k}$	$\frac{X_2 - X_1}{A_1} \omega_n^2 = \frac{-m\omega_n^2}{ms^2 + cs + k}$

5. SOFTWARE TOOLS FOR SRS CALCULATION

There are many software tools for signal analysis, for example Matlab®, which is a high-performance language for technical computing. The disadvantage is that the high frequency sampling is not integrated in the unified environment and a user has to import the input data. On the other side there are software tools specialized only on signal measurements and recordings without signal processing.

Signal Analyzer extended by Scripts, the VSB - Technical University of Ostrava indoor software is an experiment how to overcome the mentioned disadvantage. Signal Analyzer is software intended to support laboratory measurements. The recorded data may be

```
'Shock Response Spectrum';
'CrLf';
ymax=[];
ff=[];
fmin=1;
fmax=1000;
n=90;
qv=(fmax/fmin)^(1/n);
T=1/get(input1,'freq');
Q=10;
for(i=0;i<n;i=i+1)
{
fn=fmin*qv^i;
ff=[ff,fn];
wn=2*pi*fn;
A=wn*T/2/Q;
B=wn*T*sqr(1-1/4/Q/Q);
b0=1-exp(-A)*sin(B)/B;
b1=2*exp(-A)*(sin(B)/B-cos(B));
```

immediately tested by an analyzer virtual instrument. The script language is described by a paper [4].

The script for calculating SRS is shown in figure 3. The input signal is arranged into a vector `input1`. The other parameters are included into the script code. The frequency scale is logarithmic containing 30 values of SRS per decade. The output data are arranged in the frequency vector `ff` and the vector `ymax`, which is containing the response maxima.

```

b2=exp((-2)*A)-exp(-A)*sin(B)/B;
a1=(-2)*exp((-1)*A)*cos(B);
a2=exp((-2)*A);
BB=[b0,b1,b2];
AA=[-a1,-a2];
y=filter(input1,BB,AA);
yy=max(y);
ymax=[ymax,yy];
};
save(ff);
save(ymax);

```

Figure 3: Signal Analyser script for calculation SRS

6. HALF-SINE AS AN INPUT FUNCTION

The popular testing function for SRS is a half-sine function. The input signal and the result of calculation are shown in figure 4. The maximum value of the acceleration response is for the substructure natural frequency of 100 Hz.

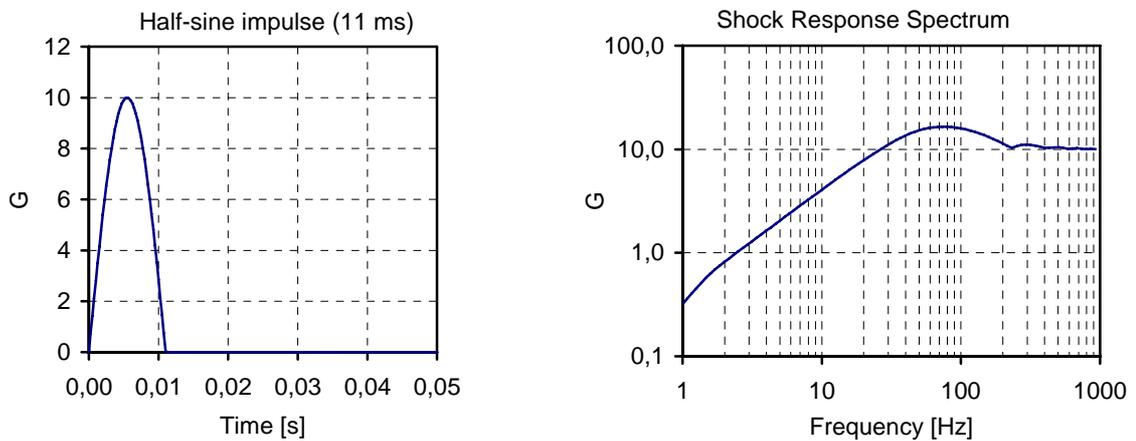


Figure 4: Shock Response Spectrum of a half-sine impulse (interval of 11 ms)

7. MEASUREMENT EXAMPLES

Two examples of shock measurements and calculation SRS are added to demonstrate the described methods. The first measurement is focused to the acceleration signal produced during impact test of a car front light. The result is shown in figure 5.

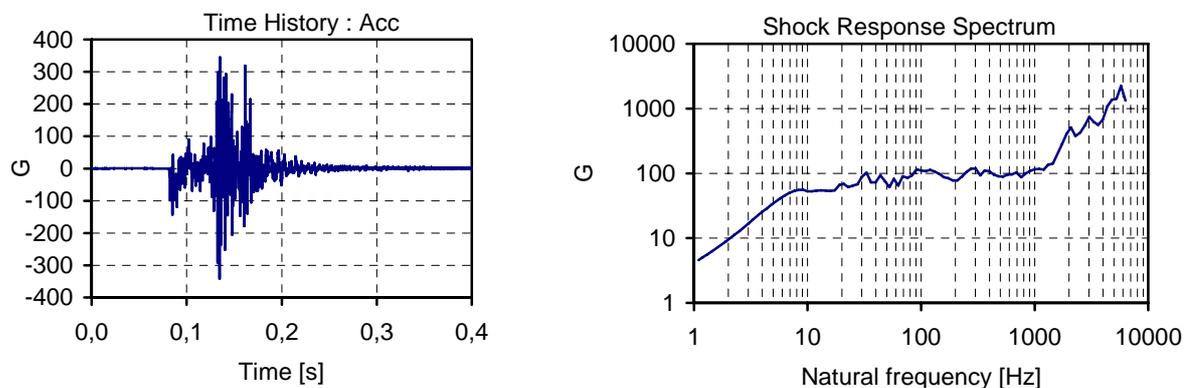


Figure 5: Acceleration during impact of a car light system and corresponding SRS

The second measurement shows the result of the impact test of the light plastic rear shield by a steel hammer. This measurement verifies the computer simulation of the crash test.

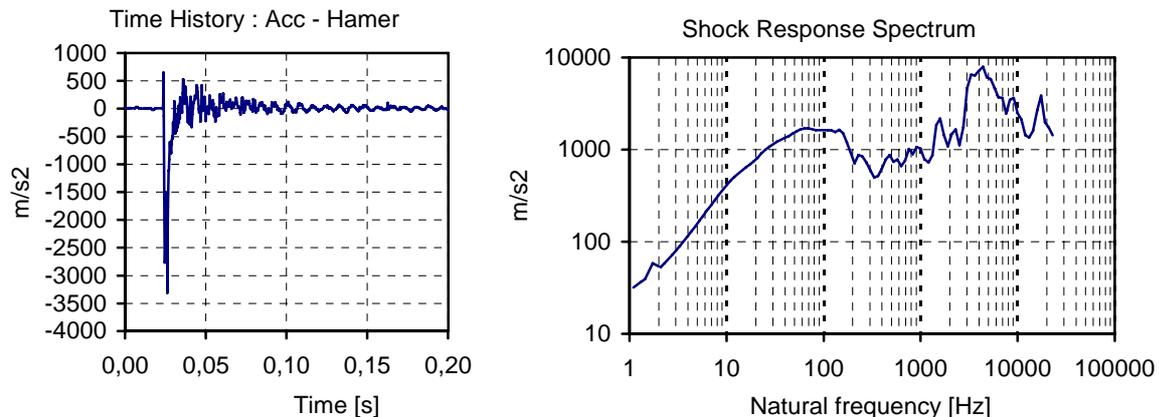


Figure 6: Acceleration during impact of a steel hammer to the plastic shield and corresponding SRS

8. CONCLUSION

The paper presents the method of calculation the shock response spectrum, which is corresponding to an acceleration signal exciting the resonance vibration of substructures. SRS determines the maximum or minimum of the substructure acceleration response as a function of the natural frequencies of a set of the single-degree-of-freedom systems modelling the mentioned substructures.

The vibration (or shock) is recorded in digital form, commonly as acceleration signal. The single-degree-of-freedom systems are approximated by an IIR digital filter and the filter response to the sampled acceleration signal may be easily calculated. This shock response spectrum shows how the individual component of the impulse signal excites the mechanical structure to resonate.

9. REFERENCES

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10. ACKNOWLEDGEMENT

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