TECHNICAL PUBLICATION
NO. SSA-3
4-73

UNDERSTANDING AND MEASURING THE SHOCK RESPONSE SPECTRUM

by

Wayne Tustin
Tustin Institute of Technology

George M. Hieber
Dyne Engineering

and

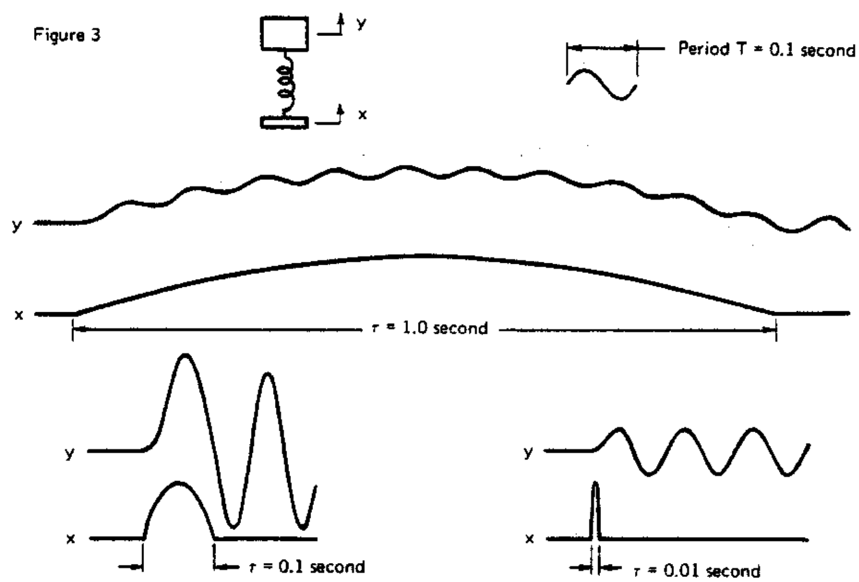
The Applications Engineering Staff
Spectral Dynamics Corporation of San Diego

SPECTRAL DYNAMICS CORPORATION OF SAN DIEGO
POST OFFICE BOX 671, SAN DIEGO, CALIFORNIA 92112
TELEPHONE 714—278-2501 • TWX 910-335-2022

the resulting motion may be very uncomfortable. At 80 mph, the motion might cause less discomfort, but might damage the auto's suspension. We see that the same "forcing function" (the roadbed) has different effects, depending upon vehicle velocity (the time during which the forcing function acts).

Although an automobile is certainly not a single degree of freedom (SDF) system, its response motion closely resembles the response motion of simple SDF systems to impulses or shocks. If an input is long-lasting, a system will follow it with a minimum of oscillation. If an input lasts from ½ to 1½ times the natural period "T" of a system, relatively violent oscillation can result. If an input lasts only a small fraction of T, it gives only a small response. This is shown in Figure 3 for an undamped SDF system whose natural frequency $f_N = 10$ Hz and whose period $T = 1/f_N = 0.1$ sec. The foundation is subjected to half sine input motions "x" of various durations and the resulting motions "y" of the mass are examined. We see that, for an input pulse whose time duration "τ" is close to the natural period T of the system, the response "y" will be large.

Figure 3



The situation of Figure 3 (an idealized undamped SDF system, an idealized half sine excitation pulse) is easy to analyze. The resulting dynamics are well known and extensively documented. But what happens in real life? Even if we model a device to have only one or two known critical f_N's, how does a designer select his dynamic load factors if his input shock time history resembles one of the complex pulses of Figure 4?

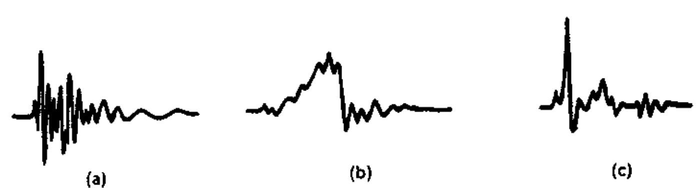


Figure 4

Furthermore, how does a specification writer call out a test when test machines that duplicate such time histories are not generally available? Finally, how do test personnel determine that a

Introduction

Figure 1 shows a representative late model Shock Spectrum Analyzer (SSA) costing over $13,000. Should you buy it or should you spend more – perhaps $50,000 – because you need greater capability? By contrast, Figure 2 shows an SSA you could perhaps duplicate for $1,000.

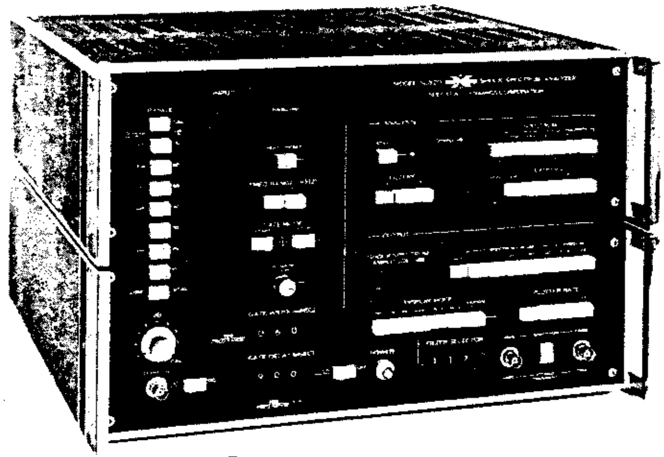


Figure 1

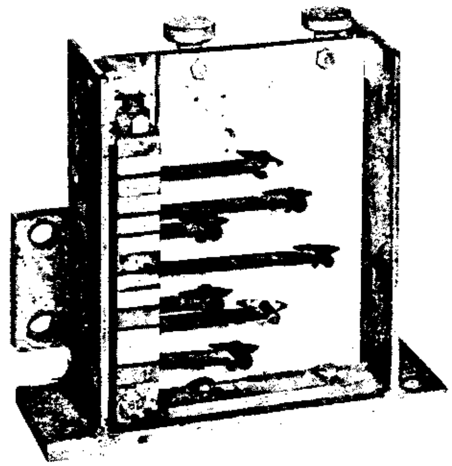


Figure 2

Your selection should relate to your particular problems. We hope this article will help you understand the Shock Response Spectrum (SRS) and its measurement. Then your choice of an SSA will be rational.

The SRS and Benefits Derived from Its Use

The SRS is one of the two most commonly used methods of analyzing mechanical shock. The other method is by use of the Fourier Spectrum. In either case, the time history of the transient is "spectrally decomposed"; that is, the amplitude vs. time picture of the transient is converted into an amplitude vs. frequency picture, or spectrum. These spectral analysis methods provide great power for understanding and working with mechanical shock.

A relationship exists between SRS and Fourier Spectrum analysis and is discussed later. At this point, however, it is important to realize that the two forms of analyzers are supportive, not competitive, in application.

SRS analysis is used to spectrally decompose transient rather than periodic (stationary) signals; Fourier analysis decomposes either. If this is the case, why is SRS used at all? The answer is that SRS provides unique information to help engineers quickly visualize the effects of mechanical shock upon a physical system not obtained by Fourier analysis and offers two popular and useful benefits:

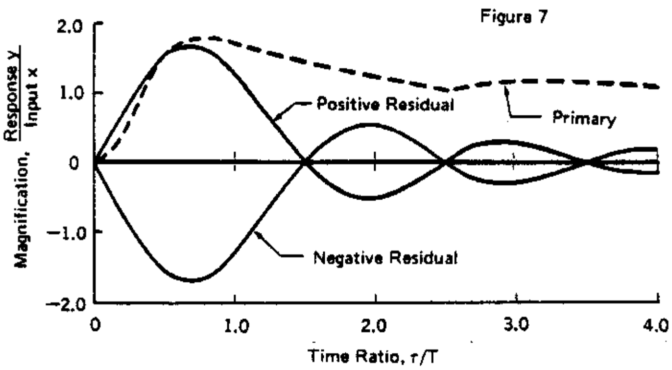
1. The SRS gives a designer a concise indication of the maximum dynamic loads various parts of his equipment will experience (as a function of frequency), to aid in predicting damage potential; and,
2. The SRS gives test engineers a very sensitive technique for helping insure repeatability of shock tests.

SRS is frequently misunderstood and misused. Perhaps we can dispel some misconceptions. We will describe the SRS without mathematics and we will explain the principles and use of electronic SSA's that capitalize on the advantages of SRS techniques.

What is Shock?

Mechanical shock defies precise definition. Generally, it is the rapid transfer of energy to a mechanical system. The word "rapid" is vague. For instance, if an automobile travels over a short section of uneven road at 5 mph, the resulting motion may be insignificant, but at 25 mph, Two recording pens are used: "x" for the input base motion, and "y" for the response motion of the mass. The recording paper moves at constant velocity. Although displacement is shown here, identical results are obtained if "x" and "y" both represent velocity or both represent acceleration.

Figure 7 identifies several different SRS that appear in technical literature. This gives an overview of the response behavior of a half sine input.



The vertical axis measures magnification while the horizontal axis measures the "rapidity" of the pulse. For analytically tractable pulses (half sine, versed sine, square, triangular, sawtooth and trapezoidal) the axes are normalized; that is, they are made non-dimensional. Response amplitude "y" of each SDF system is divided by input amplitude "x" so as to show magnification ratio vertically. Similarly, the horizontal axis is an indication of the ratio τ/T of the time duration τ of the pulse to the natural period T of the SDF system. As natural frequency f_N of SDF's increases, T decreases and τ/T increases toward the right. Examples: If τ is 0.025 second, the response of an SDF system with f_N = 40 Hz or T = 0.025 second would appear at 1.0 on the τ/T axis while the response of an SDF system with f_N = 140 Hz or T = 0.00713 second would appear at 3.5 on the τ/T axis.

The PRIMARY (sometimes called INITIAL) spectrum is the peak response which occurs *during* application of the pulse. The RESIDUAL spectrum is the peak response *after* the response has subsided (while the system is "ringing"). Note that the negative residual (peak downward response) is a mirror image of the positive residual; that is not true with damped systems . . . see Figure 16. The MAXIMAX spectrum is an envelope of either spectrum (primary or positive residual) whichever is greater at each τ/T ratio along the horizontal axis.

Now let us develop the SRS of Figure 7, using the mechanism of Figure 5 and the input pulse of Figure 6. We will vary the mechanism's natural frequency f_N by changing its spring stiffness. Time histories of input "x" and response "y" are shown at Figure 8(a). The first three points of our SRS will lie in Figure 8(b) at the abscissa τ/T = 0.1 Sec./ 0.4 Sec. = 0.25.

a. Peak response during the pulse (primary spectrum)

b. Peak positive response after the pulse has ended (the positive residual spectrum—note that in this case the response is greater after than during the pulse; this occurs with low f_N systems) and

c. Peak negative response after the pulse has ended (negative response spectrum).

specified test has been met, aside from "eye-balling" a shock time history via oscilloscope/camera or recording oscillograph? These difficulties can largely be satisfied by use of the SRS.

The Shock Response Spectrum (SRS)

In the early 1930's, M. A. Biot[1] was trying to devise a method for determining the resistance of buildings to earthquakes. He proposed that, rather than be concerned with the shape (time history) of input shock pulses, we should use a method of describing the *response of systems* to those pulses. We would then no longer be concerned with the complex shape of a pulse, but merely with its *effect.* This can be done analytically or experimentally before a product or equipment is designed. Biot's method was to determine the response of a series of undamped SDF systems (each with a different f_N) to an input pulse. If we can determine the peak response of each SDF system to this input pulse, we can then plot peak response vs. f_N. This plot is our *shock response spectrum* (SRS).

Accepted Definition of SRS

A *Shock Response Spectrum* is a plot of the peak responses of an infinite number of single degree of freedom systems to an input transient. Unless otherwise specified, the systems are undamped. Some authors specify that the systems are also massless, which is merely to emphasize that the systems do not "load" the input, as they are imaginary.

Generating Shock Response Spectra

Let's look at a detailed example using the undamped SDF system of Figure 5.

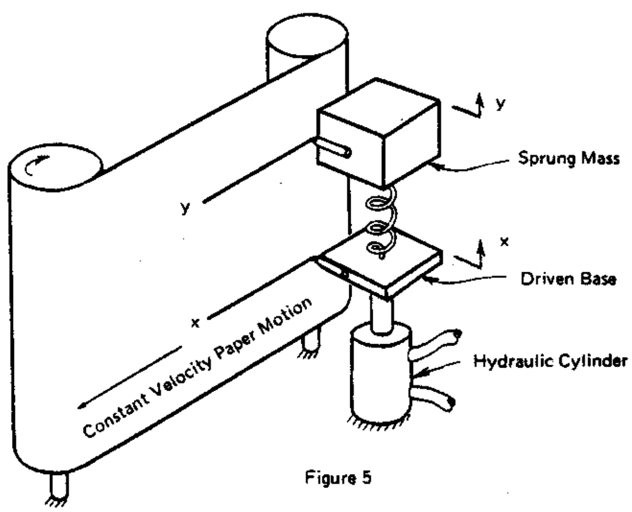
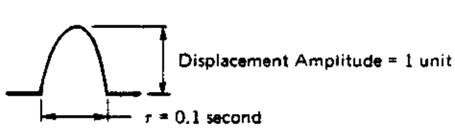


Figure 5

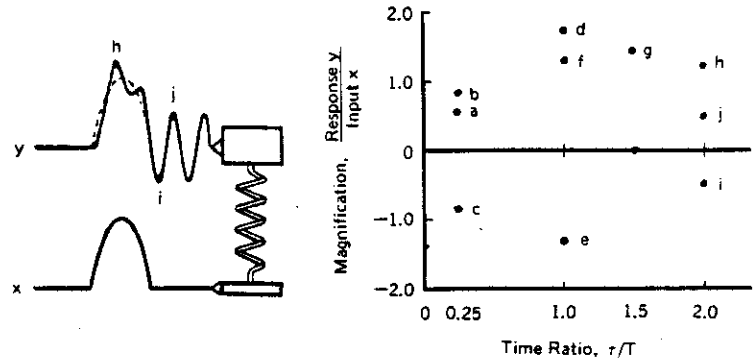
The motion of the sprung mass and driven base are constrained to move only vertically. The hydraulic cylinder is programmed to give half sine displacement pulses per Figure 6. We chose that input pulse because it is simple, it typifies many test specifications, experimental results are easily corroborated mathematically, yet it yields a relatively complex SRS which illustrates several important points.

Figure 6



If we now change f_N to 15 Hz ($\tau/T = 1.5$), an interesting result is shown by Figure 10: zero residual response. How can this be? Pulse energy must be present as kinetic energy KE (proportional to mass velocity squared), as potential energy PE (proportional to spring deflection squared) or as a combination of KE and PE. In this example, when the mass comes to rest (at a negative peak of oscillation, so KE = 0), the pulse ends, returning the spring to its original length (so PE = 0). No energy remains; thus, no motion! This circumstance, "holes in the spectrum," occurs at τ/T ratios of 1.5, 2.5, 3.5 etc. for a half sine pulse. It also occurs for other symmetrical pulses  but at different τ/T ratios. It does not occur with non-symmetrical pulses  (see later discussion in connection with the sawtooth pulse SRS).

Figure 11

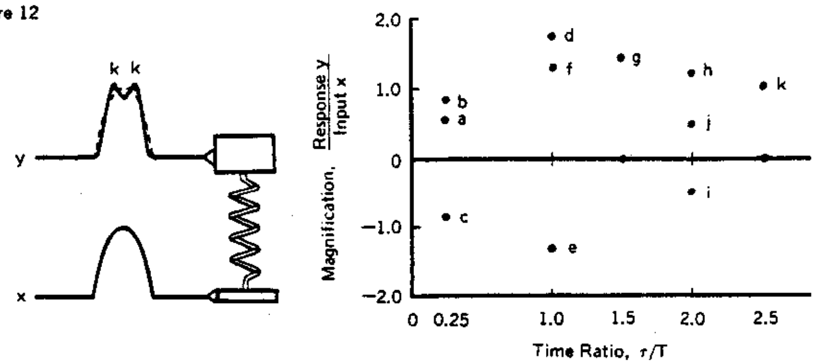


(a) TIME HISTORY (b) SPECTRUM

We now change our system's f_N to 20 Hz, ($\tau/T = 2.0$). The result is shown at h, i, and j in Figure 11(b).

Finally, we record in Figure 12 the response of a system whose f_N is 25 Hz ($\tau/T = 2.5$) and in Figure 13 of a system whose f_N is 30 Hz ($\tau/T = 3.0$). Notice that as f_N goes higher, responses tend to follow inputs more closely.

Figure 12



(a) TIME HISTORY (b) SPECTRUM

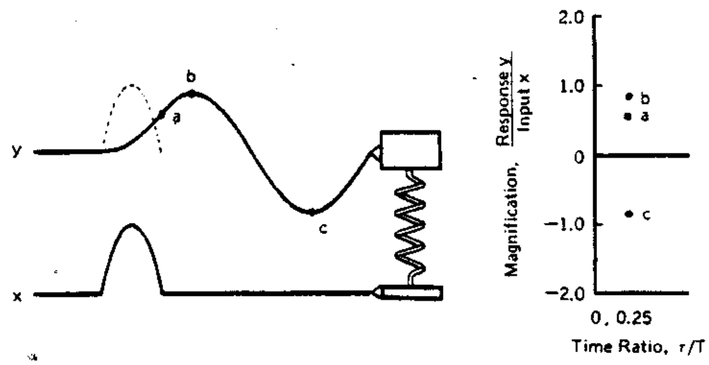
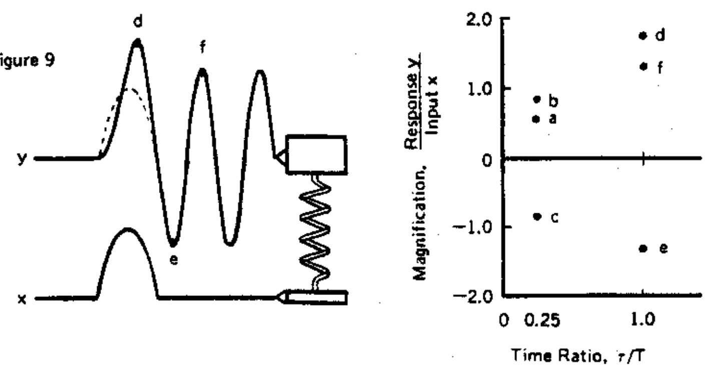


Figure 8 (a) TIME HISTORY (b) SPECTRUM

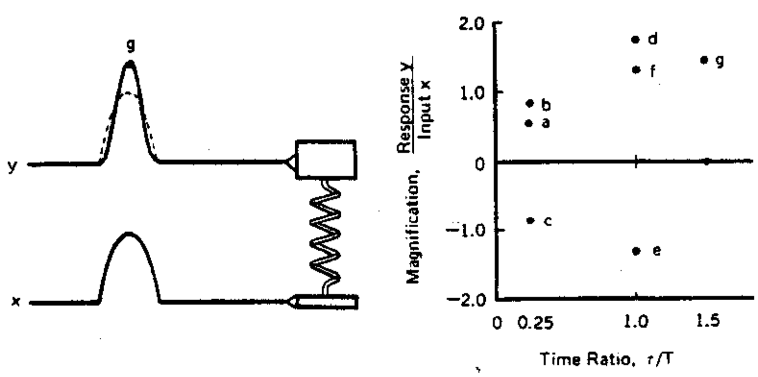
We now change f_N to 10 Hz, so that τ and T both equal 0.1 sec. and develop three additional points on our SRS at the abscissa $\tau/T = 1$ in Figure 9(b). Note the greater "y" responses with this f_N. Note also that primary response "d" is greater than positive residual response "f"; therefore, "d" is also a point on the maximax spectrum.

Figure 9



(a) TIME HISTORY (b) SPECTRUM

Figure 10



(a) TIME HISTORY (b) SPECTRUM

Magnitude Terms of SRS Analysis

Although we have been discussing displacements, practically all field measurements utilize accelerometers (easy to use, absolute measurements referenced to Earth; also, acceleration is significant in determining forces). In order to simplify our measurements and make them more meaningful, we divide our acceleration readings by 386 in/sec^2 or 980 cm/sec^2 so our numbers will be multiples of the Earth's gravitational constant and will be stated in *g*'s. Thus, the supports of an element experiencing an acceleration of 25 *g*'s must withstand 25 times that element's weight.

Designing with SRS

The SRS can be used by the designer to quickly estimate expected loads and stresses. For instance, given a half sine maximax SRS as in Figure 15, we can easily compute the response of (and thus the force on) any SDF system to any half sine pulse. An example: a structure must withstand a half sine shock pulse whose peak acceleration "x" is 50 *g* and whose duration τ is 10 milliseconds. A critical element in this equipment has an f_N of 35 Hz; another has an f_N of 250 Hz. What will be their peak responses? For the 35 Hz system, $\tau/T = f_N = 0.01\ (35) = 0.35$; an SRS shows that at time ratio $\tau/T = 0.35$ the response ratio y/x is 1.25. Its response will be 1.25 times input or 63 *g*. Similarly, the 250 Hz system, whose $\tau/T = f_N = 0.01\ (250) = 2.5$ has a response ratio y/x of 1.1, leading to a response of 1.1 times input or 55 *g*. If the 35 Hz element weighs 10 lbs., the load on its attachments would be 10 x 63 = ±630 pounds.

If structural stress is of interest, the relative deflection of a particular mode is found from a relative deflection SRS*, at the mode frequency. The stress is found by dividing the stiffness by this deflection.

In estimating natural frequencies, assume all systems to be SDF, uncoupled and undamped. These assumptions are usually reasonable but "undamped" may be too conservative. In order to reduce this conservatism, SRS of damped SDF systems are sometimes used, as described below.

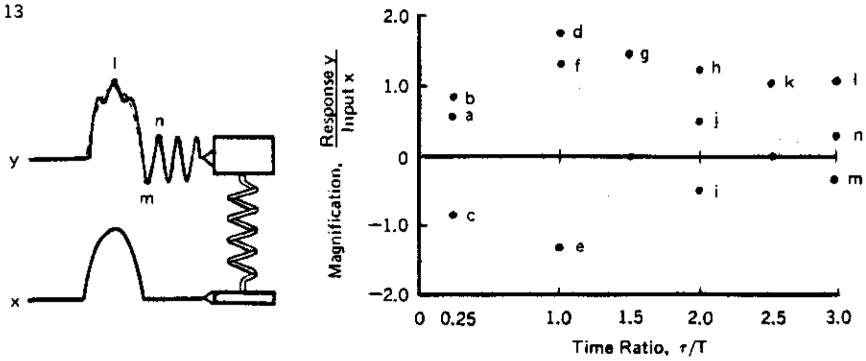
Damping & SRS

In the special case of a step input shock pulse, we find the peak response of an undamped SDF system will be double the input peak (Fig. A-1). (This particular pulse's SRS has a flat "primary" peak response over a wide frequency range and is frequently used as an SRS calibration signal.) Its 2 to 1 magnification of the input signal points up the fact that SRS amplitudes frequently are considerably higher than input peaks. This may surprise those new to SRS analysis, but should be expected since an "undamped" system means "infinite Q" or amplification. Why doesn't it go to infinite amplitude? Because it would require excitation at its f_N for an infinite length of time to reach infinite amplitude, even with "infinite Q."

All the SRS previously presented have dealt with undamped systems – an unlikely real life condition, but handy for theoretical or mathematical work. As noted, use of undamped SRS analysis results in very conservative predictions of damage potential – on the high side of reality! The use of damping in SRS analysis is intended to more closely relate predicted dynamic loading (and, thus, damage potential) to anticipated structural conditions. A damped SRS is generated the same way as the undamped SRS, the difference being that each SDF system is damped. For a particular damped SRS, every SDF system is given the *same* damping. A damped spectrum is shown in Figure 16. Note that the "residual" spectrum is composed of the first positive and negative peaks after the termination of the pulse and that they are not mirror images.

*See Appendix C

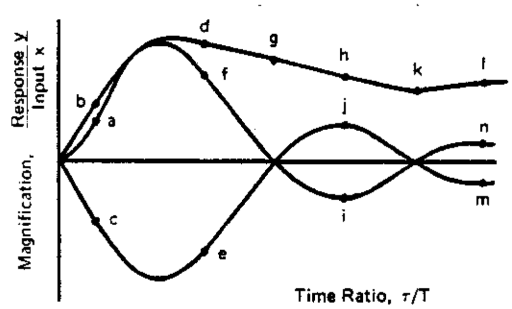
Figure 13



(a) TIME HISTORY (b) SPECTRUM

With many more measurements, using f_N's chosen to fill the gaps between our earlier measurements, the spectrum of Figure 14 will appear.

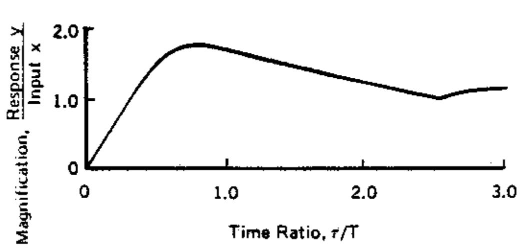
Figure 14



Remember that the SRS exists *only* in our imagination. It is not a picture (time history) of the input, nor of any responses. It does not necessarily give the frequency content of the input (see later discussion of the Fourier spectrum). It simply "bounds" the peak responses of all possible SDF systems to a particular transient.

Some writers represent an SRS merely by its maximax values, as in Figure 15, for a half sine pulse. Note that maximax SRS equals primary SRS beyond $\tau/T = 0.5$, because the most severe responses of high f_N systems occur during transients, not after them. The reason we often consider only the maximax SRS is that maximum motion is usually most critical when we design against malfunction or fracture of a structure. However, if fatigue is a potential mode of failure, then we must also examine the residual spectrum.

Figure 15



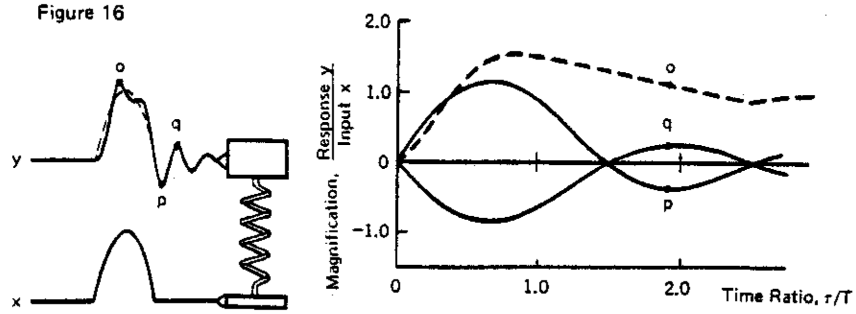
SRS vs. Fourier Analysis

We have reviewed the means of decomposing a shock transient into dynamic loading terms vs. frequency using SRS analysis, thus providing directly usable information to designers and establishing a sensitive measurement system to insure repeatability of shock environments. During the process, however, we have not retained phase relationship and, therefore, are not capable of reconstructing the time history of the event.

Decomposing the same input by Fourier Transform gives us the frequency content of the Shock Transient, plus phase relationships of the components at those frequencies; this enables us to reconstruct the transient. On the other hand, it does not provide primary or maximax SRS information which is of prime concern in predicting damage potential.

It is quite apparent that there is plenty of work for both Fourier and Shock Response Spectrum Analyzers in their roles as data reduction devices. In another section, we will deal with the practical application of SRS and Fourier Transform devices in support of Test Laboratory shock simulation as well as analysis and will review equipment currently available to do the work.

Figure 16



(a) TIME HISTORY (b) SPECTRA

The Fourier Spectrum

The Fourier Spectrum is another method of changing a time history into an amplitude vs. frequency plot.

The famous mathematician, Fourier, showed that any realizable transient consists of an infinite number of sine waves, each of specified amplitude and phase relationship to one another. A Fourier analysis, then, is a means of determining these constituent parts of a given transient. The results of this analysis are usually given as two spectra; one, the amplitude vs. frequency content (the Fourier "amplitude spectrum") and two, the phase vs. frequency content (the Fourier "phase spectrum").

The Fourier Spectrum analyzes the shock pulse *itself*, but the SRS determines the peak *response* of systems subjected to the shock pulse. Thus, these two analysis methods are completely different in concept.

Even so, an interesting (and fundamental mathematical) relationship between SRS and Fourier analysis has been shown.[2] The Fourier *magnitude* spectrum is identical to the undamped residual *velocity* SRS. Therefore, if an undamped relative displacement SRS is available, simply multiply all magnitudes by $2\pi f_N$ to get a Fourier magnitude spectrum. Or, if an undamped *acceleration* spectrum is available, simply divide all magnitudes by $2\pi f_N$ to get a Fourier magnitude spectrum. Amplitudes of all frequencies present in the transient will be shown, but phase information will be missing. However, if we perform a Fourier Transform on a given transient, we are provided information on the frequency content of the pulse *plus* the phase relationships of the components at these frequencies and have the ability to *reconstruct the transient.*

Figure 17 shows a very simple example of the importance of phase; 1 volt at 10 Hz is combined with 0.5 volt at 30 Hz. Two possible phase relationships yield two time histories; note the difference in peak values.

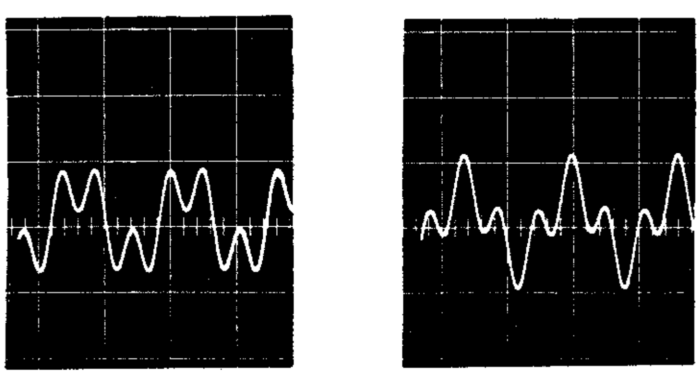


Figure 17

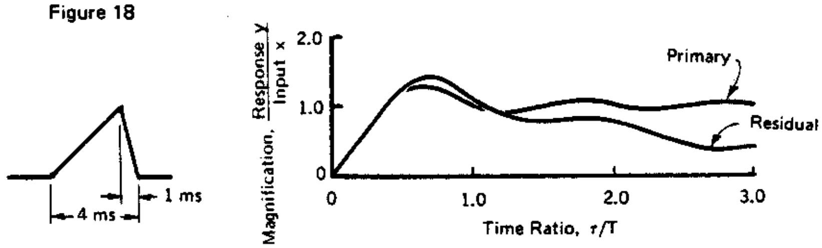
Shock Testing

The SRS can be very helpful in testing and, when properly used, has led to more uniform test results.

Frequently, shock test specifications have called for the generation of half sine pulses. This has led to problems because shocks of varying duration will always be encountered in the field. Thus, if a laboratory used a symmetrical pulse such as a half sine with a notch in its residual spectrum just where a critical f_N happened to fall and the equipment passed the test, the equipment might subsequently fail in the field. A field shock with a different τ would not have a notch at the same τ/T. Smart designers sometimes take advantage of "holes in the SRS" in order to pass tests. The fact that there are notches or "holes" in the spectra of unsymmetrical pulses eventually led USA specification writers to require terminal peak sawtooth pulses for environmental testing. (In Europe, initial peak sawtooth pulses are preferred.) With a sawtooth pulse, any system with a τ/T ratio > 1.0 effectively gets the same "treatment" and it was reasoned that test results should be more reliable.

While the terminal peak sawtooth is thus highly desirable, difficulties arise in the generation of ideal pulses with vertical segments in their time histories. With any slope $< \infty$, the SRS degenerates badly. As has been illustrated (5), a terminal peak sawtooth with a non-zero dropoff time yields the SRS of Figure 18.

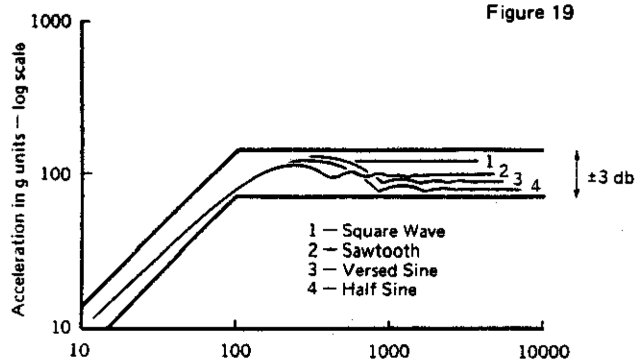
Figure 18



(a) TIME HISTORY (b) SPECTRUM Negative Residual not shown

These practical difficulties have led to the conclusion that we should not specify time histories or pulse shapes for tests. Rather, the SRS should be specified which gives better control and repeatability because the *effect* of a shock on equipment will be duplicated. Perhaps the reader has encountered specifications resembling Figure 19 which give a required spectrum along with allowable tolerances.

Figure 19



the scope of this article. Many present day specifications call for meeting SRS requirements so some type of SSA is necessary in testing as well as in field data analysis.

As mentioned previously, designers sometimes want the SRS of damped systems because they need to more accurately predict loads or stresses. (This desire presupposes, rather optimistically, that designers know how much damping will exist in their structures.) Damped SRS are also used in testing in an attempt at realism. Some of the previously described methods of synthesis have obviously abandoned any attempt to generate exciting frequencies within the time history of a real life shock. In fact, a 5 millisecond pyrotechnic shock might be synthesized by a pulse train 350 milliseconds long! Because of this, a damped SRS can be specified for the test – this reduces the possibility of overtesting components having significant damping. Thus, Shock Spectrum Analyzers, for maximum utility, should provide for various amounts of damping.

But what if we desire to synthesize a "real time" event on a shaker system? The answer is that Fourier analyzers must be incorporated into the control system, assuming that the amplitude requirements are within the shaker capabilities.

Shock Spectrum Analyzers (SSA)

The only way to take advantage of the benefits of the SRS is to have methods of analysis available which enable us to quickly take the measure of complex transients. Fortunately, SSA methods have been devised to meet our needs; these methods range from very simple to very sophisticated. Table I summarizes the various methods.

TABLE I
CLASSIFICATION OF SHOCK SPECTRUM ANALYZERS

Shock Response Spectrum Analyzers		Fourier Transform Analyzers	
Analog	Digital	Analog	Digital
Mechanical per Figure 2			
Electrical			
Mathematical	Duhamel integral	Stepping oscillator	Classical
Physical	Filter		FFT

As mentioned earlier, the main advantage of the Fourier Transform is that a shock time history can be reconstructed or synthesized from the Fourier magnitude and phase spectra. The disadvantage is that the primary SRS and, thus, the maximax SRS cannot be obtained using Fourier analysis.

One can obtain Fourier Transform (amplitude and phase) data from Shock Response Analyzers (SSA) but it must be done before data is presented in SRS format. When an SSA is set to measure residual response (but that response is still in the time domain . . . not yet transformed into the frequency domain), phase information can be extracted. This is done by comparing the phase of a residual response frequency to a reference sinusoid of the same frequency; the latter is triggered at zero time.[6]

If you don't have to synthesize time histories (the usual situation), then an SRS analyzer is needed. If you have to synthesize a time history, you need a Fourier Analyzer but it is usually an integral part of a sophisticated and expensive test machine control system; it does not stand alone.[7]

In the types of test programs discussed, need for both Fourier and SRS analyses would be rare. Fourier analysis is needed only if shock time histories are to be synthesized; if this is done, there is no need for examining the SRS other than to confirm that the job was done correctly by use of an independent SRS analyzer. Conversely, if we test by matching an SRS, we have no need for time history other than for record purposes. (It is possible that future test specifications may

Observe that the axes are now dimensioned; the reasons are significant. Earlier we considered classical, simple pulses. With a simple shape, we can calculate τ/T ratios and know where on the SRS to pick off magnification ratios y/x. We must recognize that field pulses are neither classical nor simple. For instance, refer to Figure 4 and try to assign to each pulse a single amplitude and a single duration.

Any SRS which might be specified can be met with (theoretically, at least) an infinite variety of pulse shapes! This rather startling statement can be supported by examining the SRS's of half sine, versed sine, square and terminal peak pulses. By slight adjustments of their height and duration, all can meet the specification as shown in Figure 19.

We can go even further. It is not even necessary to generate a shock pulse to meet the requirements of the SRS specification – it can be done with a sinusoidal sweep as illustrated in Figure 20(a)! After all, a series of SDF systems can be excited to a particular response by suitably adjusting input sweep rate and amplitude. The slower the sweep rate, the lower the input amplitude needed to excite a system to the required response because, with a slow sweep rate, more cycles of excitation are present to build up the response. This approach is the basis of most shock testing using electromagnetic shakers. As shakers have displacement, velocity and force limits, it is not always possible to duplicate a short duration, high amplitude pulse. In this case, no attempt is made to simulate the original shock *pulse* but, rather, the *spectrum* requirements are met by a low amplitude longer duration sine sweep, which allows the shaker to be used within its limitations. (In actual practice, closer simulation to a shock pulse is often obtained by combining a series of exponentially decaying sine waves.)

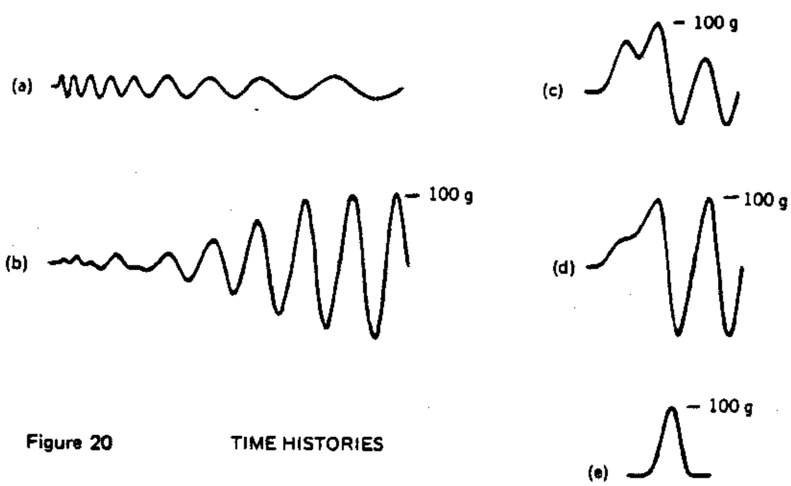


Figure 20 TIME HISTORIES

A typical lightly damped SDF system would respond as in Figure 20(b) to a sine sweep. If its maximum response, say 100 *g*, is the same as found with a half sine pulse per Figure 20(c) *or* a sawtooth pulse per Figure 20(d) *or* a square pulse having a τ/T ratio of 1.0, per Figure 20(e), then the specification will be satisfied with any of these transients. This type of test specification is not concerned with *how* all the SDF systems achieve their specified maximum response, but only that all SDF systems *do* so.

Surely the residual spectra (not shown in Figure 19 nor in typical specifications) are vastly different. We could speculate rather negatively on the validity of a test in which residual amplitudes vary so widely, particularly if fatigue could be a problem but that question is beyond the Fast Fourier Transform (FFT), developed by Cooley and Tukey in 1965[13] reduces computer time about 300:1; Computation is accomplished in less than 1 second, essentially "real time." Special purpose computers with the FFT wired in are commercially available.

Commercially Available Shock Spectrum Analyzers

SSA is not for the faint of heart or weak in purse. Analysis of extremely complex, very short duration signals makes great demands upon instrumentation. With present SRS specifications, analysis should cover 1-1,000 and/or 10-10,000 Hz. (This requirement alone eliminates mechanical reed gages.) Let us assume that accelerometers and related equipment are available. Selection of SSA technique will depend mainly upon allowable time after an event before analysis is needed.

If analysis is not needed for a day or two, then demands upon the SSA are not as stringent as if analysis is needed within minutes or seconds. If shock analysis is not performed often, the galvanometer array method will give results with a few hours of data manipulation. Special f_N galvanometers must be ordered and a tape recorder is usually required.

Better, use a tape recorder and an A–D converter to present data to a computer (probably time-shared). No data manipulation is needed, as the computer can plot results directly, often within an hour. You will probably need to buy and "debug" a program; few computer services are properly equipped for shock analysis programs.

If immediate spectral analysis is required, three commercial units are presently available; all require an oscilloscope for viewing and a plotter for printout. Two require repetitive data input for each different SRS analysis. One stores the shock transient in digital memory for all succeeding analyses.

At the heart of these SSA's is a series of electrical "filters" —— really *analog models of SDF systems,* whose center responses are spaced over the analyzer's frequency range. Unfortunately, the models are commonly referred to as "filters" rather than as "SDF models." It is true that these electrical models behave and can be readily described as *low pass filters.* When one wishes to specify SSA low pass filters, he should not select conventional analyzer bandpass filters which may be offered in terms of 1/3, 1/6, 1/12 or other octave-related spacings. Both types may be specified in terms of constant percentage bandwidth but they have different shapes, as shown by Figure 21. SSA filters have selectable amplification factor "Q" which changes the percentage bandwidth, while most conventional analyzer filters have fixed Q. (For example, acoustic 1/3 octave filters have a bandwidth approximately 23% of their center frequency, with Q approximately 4.4.) Note that the Q values in the graphs of Figure 21 are not related to each other in any way.

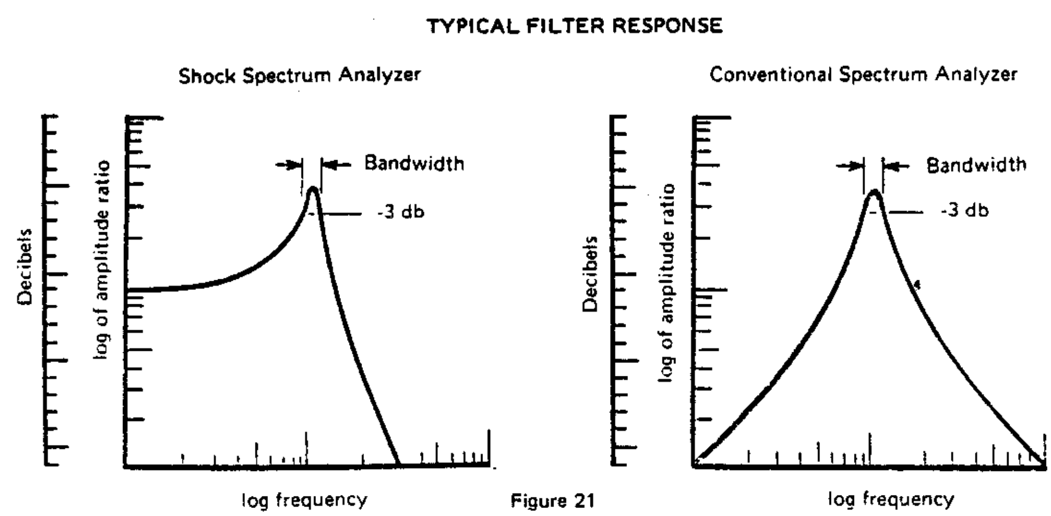


Figure 21

Amplitudes of these curves are not related.

require that SRS's be developed by time histories more closely approximating the "real time" shock event; such is not presently the case.) Let us now examine the various types of analyzers (Reference Table I):

Analog-Mechanical SSA—Highly direct is the Navy Reed Shock Gage of Figure 2. Individual reeds are adjusted to prescribed f_N's. A stylus on each reed scratches a wax paper when the structure to which the instrument is attached receives a shock. The length of each scratch is divided by 2 and plotted vs. f_N, giving a maximax displacement SRS. (Multiplying the length of each scratch by $4\pi^2 f_N^2$ and plotting vs. f_N gives a maximax acceleration SRS.) It represents relative displacement because stylus deflection is measured relative to the frame rather than to Earth. An advantage: no electrical power is needed. Disadvantages: (1) the Gage is heavy and can only be used on relatively massive foundations; (2) only low f_N reeds give scratches of useful length; (3) data must be manually transferred and graphed; and (4) the number of reeds is limited – 7 is typical.

Another scheme uses a recording oscillograph with an array of different f_N galvanometers to model an array of SDF systems.[8] This is better than the reed gage because tape recorded accelerometer signals can be analyzed, but data must still be manually transferred and graphed.

Analog-Electrical SSA—"Mathematical types" utilize an analog computer which solves the differential equations of SDF systems. These are not popular, mainly due to the long time required to perform the many calculations and/or the expense of providing many computer channels. Input electrical signals usually originate in accelerometers. Output electrical oscillatory responses can be recorded on an oscillograph, then manually transcribed to SRS plots; more commonly, an array of peak storage meters is used. All types of SRS can be obtained.

"Physical" types use direct electrical analogs. SDF systems are modeled by inductors, capacitors and resistors; an electrical transient input from an accelerometer is applied and peak values of output voltages are read out on meters and are hand plotted. (Alternately, voltmeter circuits can be automatically interrogated and peak values plotted vs. f_N.) Any type of SRS can be obtained. All commercially available SRS analyzers are of this type.

Digital SSA—Two types of computer programs have been devised to compute SRS; both are similar to the analog techniques. One (the "mathematical" type) solves the equation of a system's response to an impulse by using a convolution, or Duhamel, integral. The other (the "physical" type) synthesizes a series of filters and determines their outputs to an input pulse. Both utilize recursion programs which reduce computer time; otherwise, the response at each time increment, taking into account the complete time history from time zero, would have to be determined. With recursion, the solution of the previous time interval is used as the initial value for each computation, saving much computer time.[9] A program using recursive integration generates relative SRS,[10] whereas two programs, one for generating relative SRS[11] and the other for generating absolute SRS[12] have been written for recursive filtering. The Fourier Transform may be obtained by a simple extension of these programs.

Fourier Transform SSA – Analog—An oscillator with two outputs 90° out of phase is set at a frequency of interest. An incoming transient is directed to two amplifiers where it is multiplied separately by the two oscillator signals. This gives sine and cosine Fourier components which can be manipulated to give the amplitude and phase of that particular frequency component. Oscillatory frequency is stepped to give a series of points which form Fourier magnitude and phase spectra.

Fourier Transform SSA – Digital—Computer programs written to perform the classical Fourier transform are little used in shock analysis because of excessive computer time required. However,

SUMMARY

The primary use of Shock Response Spectrum in Shock Testing is to establish test specifications in terms that are not dependent on how the shock is generated. This will insure a more accurate duplication of the *effects* of a shock on a structure.

Analyses of test results fall into three categories:

1. Analysis of the input Shock Response Spectrum to a test article to prove compliance with specifications.
2. Analysis of the Shock Response Spectrum within a structure at its critical sub-system mounting points.
3. Analysis as part of an iterative procedure to quickly change the input time history signal to a shaker system in order to obtain the required Shock Response Spectrum.

Whether SRS should be done by Shock Response Spectrum analysis programmed in a computer or by a dedicated, stand-alone SRS analyzer will usually be resolved on a Cost Effectiveness basis. Stand-alone SRS analyzers are powerful tools for cutting costs of test setup and anlysis times. They are compatible with the dynamic range of data acquisition and recording systems now available in most laboratories and results are available in seconds. They are excellent for screening data prior to computer processing.

The analyzer shown in Figure 1 provides a complete shock response analysis system for any transient stored in memory. The time history can be plotted out and peak "g" scaled accurately. Also, the + or - primary and residual or maximax analysis at any, or all, of 5 damping factors can be performed and plotted in a matter of minutes while the same data is displayed on visual monitors in seconds.

In short, it is a very versatile and cost effective instrument.

On the other hand, a computer based Shock Response Spectrum analyzer which is part of a closed loop control system is not only slower, but is often not available for shock response analysis if its primary job is simulation. This is a serious limitation, particularly if any volume of data reduction is required.

Finally, it should be clearly remembered that a dedicated SRS analyzer yields unique structural dynamic loading information not available from Fourier type computer-based analyzers.

Table II illustrates typical SSA filter bandwidths as a function of Q or Zeta (damping).

TABLE II
FILTER BANDWIDTH vs AMPLIFICATION OR ZETA

*%Bandwidth (Approx.)	Amplification	Damping, Zeta
1%	100	.005
2	50	.01
5	20	.025
10	10	.05
20	5	.1

**Measured at -3 dB points.*

The response of a possible Series of SSA filters is shown in Figure 22.

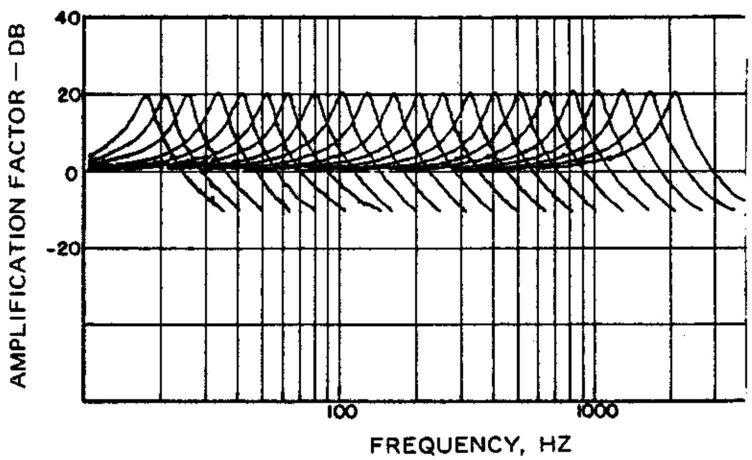


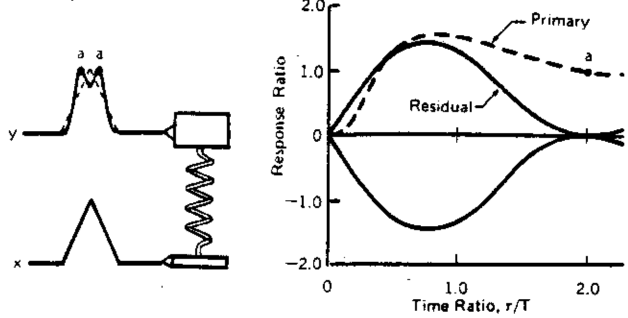
Figure 22

SSA Filters Simulating SDF System Responses — Flat SRS Sprectrum
(1/3 octave center frequency spacing; Q = 10)

For practical reasons, the damping factor "Zeta" is the same for all filters regardless of f_N, and the Q of each filter is changed simultaneously to represent different amounts of damping. This turns out to be convenient — if we define Q as $f_N/\Delta f$, where Δf is filter bandwidth at its 70.7% response level. If Q remains constant, filter bandwidth must increase as f_N increases. An array of such filters is usually called a set of "constant percentage bandwidth" filters. With f_N plotted logarithmically, as in Figure 22, all filter response curves are the same shape; therefore, the analyzer's "resolving power" is uniform at all f_N's and the resulting plot has equal data point spacing.

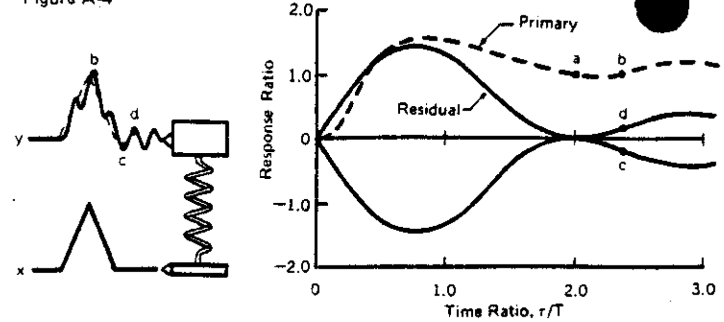
One of the important criteria in evaluating an analyzer is its resolving power. Always conscious of cost vs. performance, manufacturers first marketed SSA's having only 30 to 40 filters. As previously noted, "holes" can be found in many shock spectra. The need for increased resolution resulted in SSA's having 60 to 80 filters; a recently developed SSA (Figure 1) has 120 filters.

Figure A-3



(a) TIME HISTORY (b) SPECTRUM

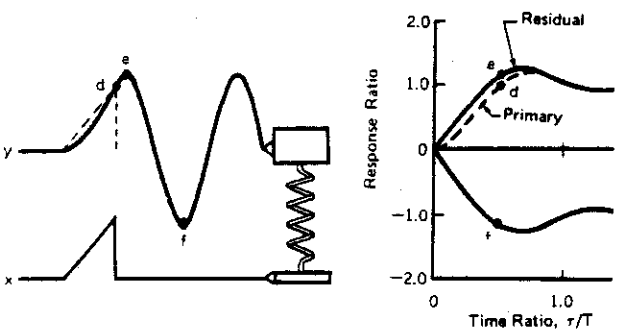
Figure A-4



(a) TIME HISTORY (b) SPECTRUM

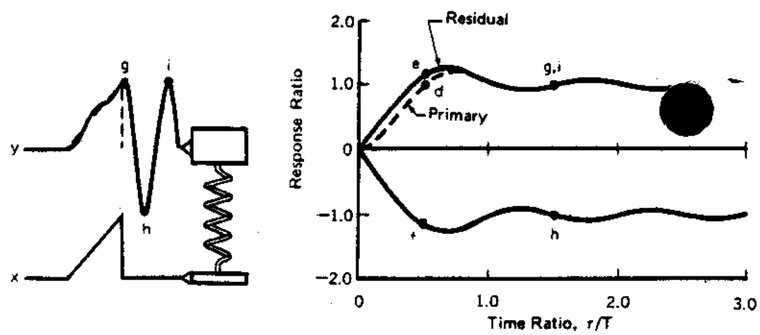
Sawtooth

There are no "dead beat" phenomena and no "holes" in the spectrum. Such pulses are thus desired for testing. Also, the maximax SRS is identical to the residual SRS at all τ/T ratios.



(a) TIME HISTORY (b) SPECTRUM

Figure A-5



(a) TIME HISTORY (b) SPECTRUM

Figure A-6

If the leading edge of the sawtooth pulse can be made parabolic, SDF systems are not disturbed during the rise of the pulse, and the residual and maximax spectra are even smoother.

APPENDIX A

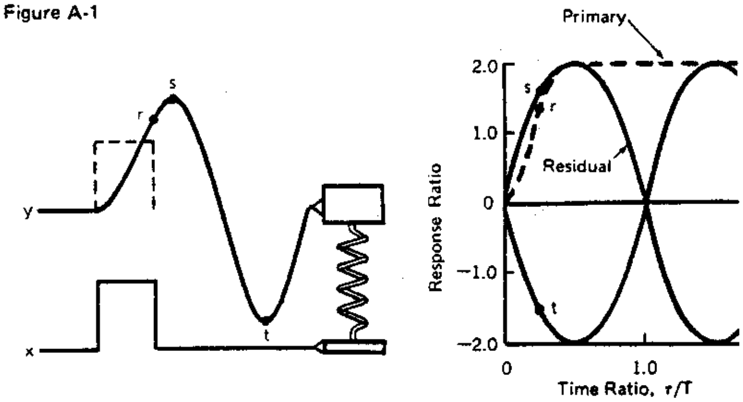
Additional SRS

Several other shock spectra for different shock pulses are shown in Figures A-1 to A-6. The mechanism of Figure 5 is used.

Square Wave

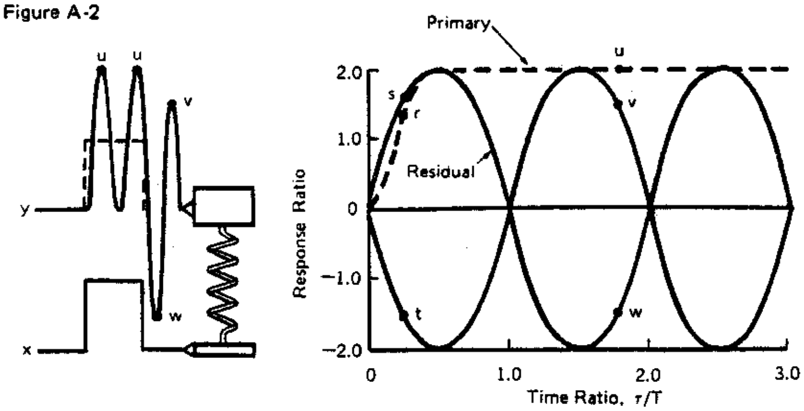
Note that the primary response "y" is twice the input "x" wherever $\tau > \frac{T}{2}$. Also, the "dead beat" phenomena occurs in the residual spectrum at τ/T = 1.0, 2.0, 3.0, etc.

Figure A-1



(a) TIME HISTORY (b) SPECTRUM

Figure A-2



(a) TIME HISTORY (b) SPECTRUM

Triangular

The triangular pulse has "holes" in the spectrum at τ/T ratios of 2.0, 4.0, etc.

APPENDIX C

ABSOLUTE SRS VS. RELATIVE DEFLECTION SRS

The main body of this first article describes spectra based on absolute measurements; all motions (whether displacement, velocity or acceleration) are referred to Earth; e.g., our displacement motions were recorded on paper connected to Earth.

When dealing with shock-sensitive devices (a human in an automobile, electronic equipment in an aircraft or guidance equipment in a ship), we want to know the expected *g forces* these devices will be exposed to. For this, the absolute SRS is most useful as it provides the information directly. We measure the input pulse with an accelerometer. The resulting output accelerations of the (imagined) SDF masses form our absolute SRS.

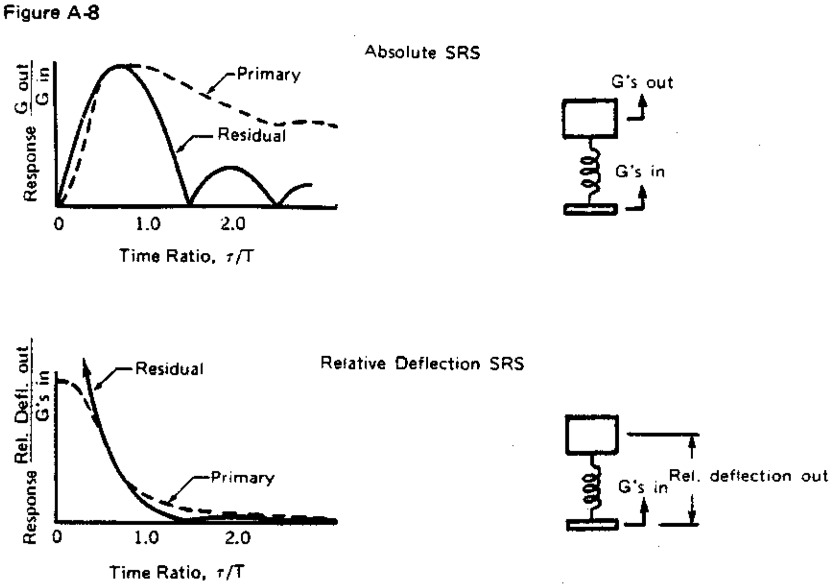
If we are concerned with structure, however, we are interested in the *stress levels* caused by transients. To determine stress, we need to measure spring *deflections* when our imagined SDF systems are subjected to an acceleration pulse, rather than measure the acceleration of the imagined mass as in the absolute SRS.

Knowing the stiffness of a structure (spring constant) and its deflection, we can easily compute stress. Such measurements interest structural designers who are concerned with earthquake resistant buildings; for instance, acceleration of the top floor is less informative than determining stress at the base.

As with the absolute SRS, the relative deflection SRS is usually derived from an input pulse measured by an accelerometer. In this case, however, the accelerometer signal must be integrated twice.

Figure A-8 illustrates the two types of spectrum, utilizing a half sine input pulse:

Figure A-8



Examination of the relative deflection SRS shows that structural stresses are most affected by low frequency content of a shock pulse.

Although the spectra of Figure A-8 appear considerably different, there is a simple relationship between them: the factor $\omega_N{}^2/386 = 39.48\ f_N{}^2/386 = 0.1\ f_N{}^2$. If we happen to know the relative

APPENDIX B

Note that the low frequency region of every spectrum shown rises at a constant slope, proportional to the SDF f_N. (Such a constant slope will exist when both axes are structured the same; i.e., *both* linear or *both* logarithmic – this article shows examples of each.) The reason is that low f_N systems cannot respond to a "rapid" pulse. As such a slope (proportional to frequency) is equivalent to constant velocity, this low frequency region is called the velocity sensitive part of the spectrum. Low f_N systems, therefore, respond to velocity change (the area under the acceleration vs. time curve). If various shaped "rapid" pulses have the same area, then the velocity change of each pulse is the same and all low f_N systems respond as if the pulses were identical in shape. Fig. A-7 illustrates how two different low f_N systems would respond to a given pulse. The peak output velocity responses are the same; thus, these systems are velocity sensitive. Note, however, that as the natural frequency increases, peak acceleration response also increases. This causes the constant slope (or + 6 db per octave in log-log coordinates) of this portion of the acceleration SRS.

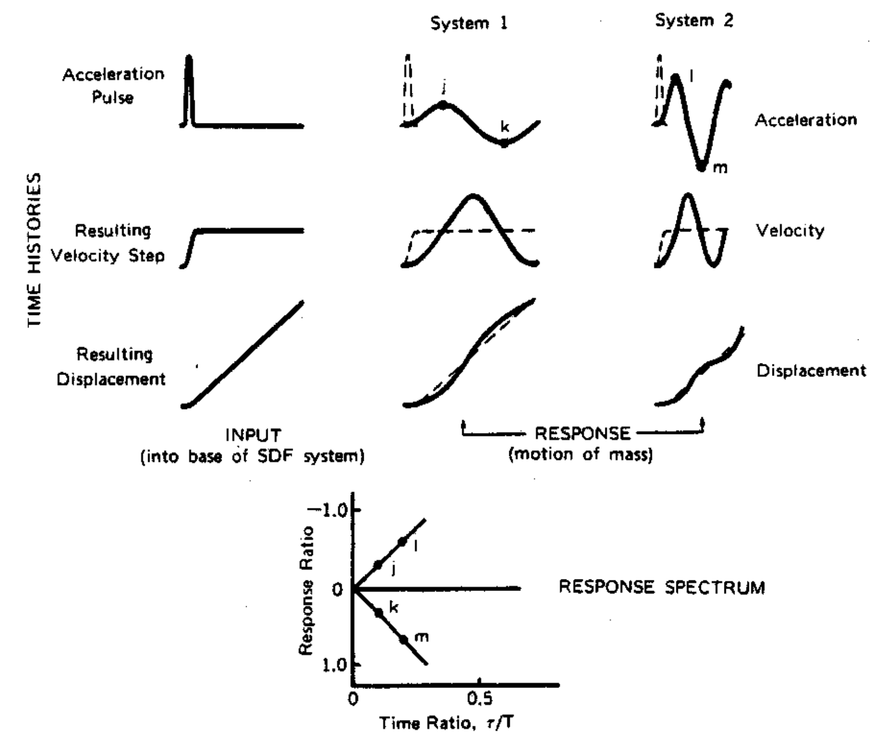
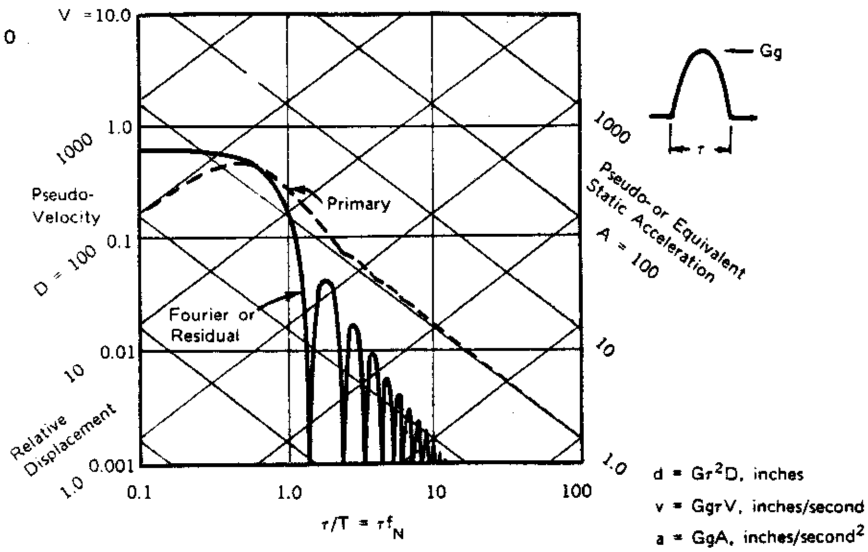


Figure A-7

Figure A-10



Now why is the velocity response spectrum called a "pseudo–velocity" spectrum, and why is the acceleration response spectrum called a "pseudo-acceleration" or "equivalent static acceleration" spectrum? The reason lies in the difference between real-life systems, which always have some damping, and the zero-damped SDF systems we have been considering. When working with a zero damped system, it is possible to obtain absolute velocity of the mass by multiplying relative displacement by ω_N and to obtain absolute acceleration by multiplying relative displacement by $\omega_N{}^2$. This is not true for damped systems (although most structures have so little damping that errors are small) and the labels pseudo-velocity and equivalent static acceleration are used to account for this subtle difference.

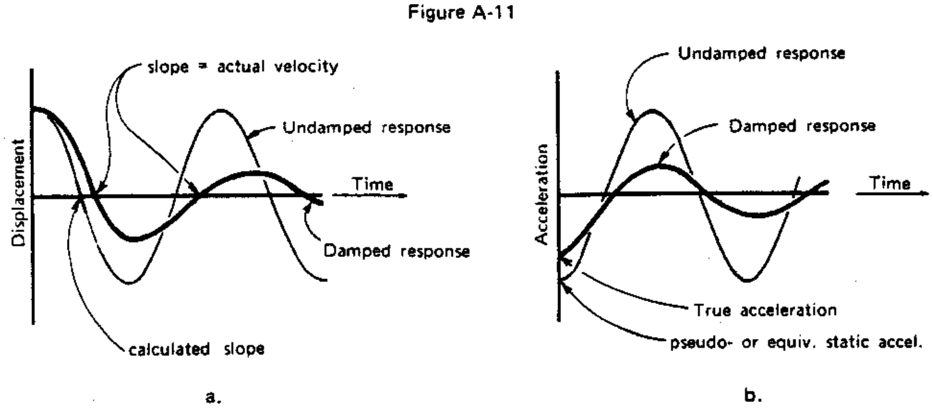
The error in the velocity calculation can be seen from the exaggerated curves in Figure A-11a.

It can be seen that, for an undamped system, the true velocity can be obtained. When damping is present, however, errors are inherent because

1. response is not sinusoidal, and
2. the actual frequency is slightly lower than the undamped f_N.

The same comments apply to acceleration computation as to velocity computation. The error in acceleration computation can be seen in Figure A-11b.

Figure A-11



deflection response at a given frequency, we can calculate the absolute g response at that frequency by multiplying relative deflection by $0.1 f_N{}^2$.

A format recommended by Vigness[3] for relative displacement SRS involves four coordinate paper. The relative displacement SRS is computed, but before plotting is performed, each response point is multiplied by ω_N. This gives a plot of in/sec vs. frequency – a so-called "pseudo velocity" spectrum. Comparative results are shown in Figure A-9.

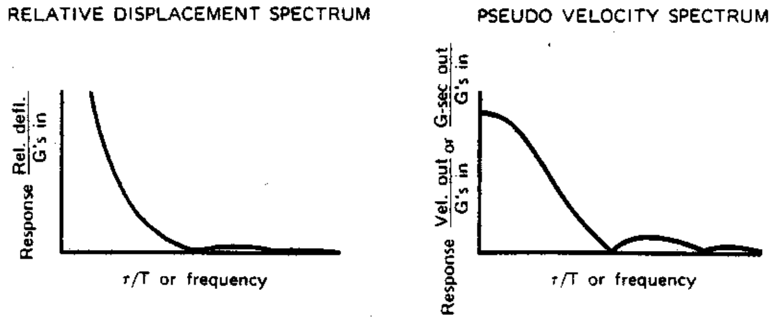


Figure A-9

The use of four coordinate paper allows relative displacement velocity *and* acceleration SRS to be read off the same graph. In the main text, it was stated that multiplying a relative displacement spectrum by ω provides a spectrum equivalent to a Fourier magnitude spectrum. Thus, the terms "*residual* pseudo-velocity spectrum" and "Fourier magnitude spectrum" can be used interchangeably. Note that this does not apply to the *primary* pseudo-velocity spectrum.

The vertical scale of this Fourier/pseudo-velocity spectrum is sometimes dimensioned as "g-seconds." The reason can be explained as follows:

If one starts with acceleration and divides by ω,

$$\frac{a}{\omega} = \frac{\text{in}}{\text{sec}^2} \times \frac{\text{sec}}{1} = \frac{\text{in}}{\text{sec}} \quad \text{(pseudo-velocity)}$$

If one starts with acceleration in the form of g's,

$$\frac{g}{\omega} = g \times \frac{\text{sec}}{1} = g\text{-sec}$$

as "g" is non-dimensional.

A four coordinate plot is shown in Figure A-10. The acceleration is called the "pseudo acceleration" or the "equivalent static acceleration." It is obtained by properly placing the coordinates of acceleration on the graph, effectively multiplying the relative displacement coordinate values by $0.1\ f_N{}^2$ as specified earlier.