THE VIBRATION RESPONSE OF A PANEL TO AN ACOUSTIC PRESSURE FIELD
Revision B

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Introduction

Consider an acoustic pressure field which excites a homogenous panel. Assume that the pressure field has random incidence upon the panel.

As an example, a satellite is mounted inside of a launch vehicle nosecone fairing. The satellite's solar panels must withstand the liftoff acoustic field. The fairing attenuates a great deal of this energy. Nevertheless, a portion of the acoustical energy is transmitted to the satellite.

The purpose of this report is to derive a relationship between the acoustic pressure and the panel vibration response.

Critical Frequency

The critical frequency is the frequency at which the airborne acoustic wavelength matches the panel bending wavelength. The critical frequency f_c for a homogeneous panel is

$$f_c \approx \frac{c^2}{1.8\, c_L\, h} \qquad (1)$$

where

c	is the speed of sound in the surrounding air medium
c_L	is the longitudinal wave velocity
h	is the panel thickness

Equation (1) is taken from Reference 1, chapter 6, equation (56a).

Panel Velocity Response to Acoustic Pressure

Let V_{RMS} be the velocity root-mean-square.
Let P_{RMS} be the pressure root-mean-square.

The velocity and pressure are related by the following semi-empirical equation.

$$\left[V_{RMS}\right]^2 = \beta \left[P_{RMS}\right]^2 \tag{2}$$

Equation (2) is taken from Reference 1, chapter 8.

Note that each term in equation (2) is a function of frequency.

The coefficient β is

$$\beta = \begin{cases} \dfrac{1}{2\pi^2 f^2 m^2}\left[1 - \dfrac{\rho c}{\pi f m}\arctan\left(\dfrac{\pi f m}{\rho c}\right)\right], & \text{for } f < f_c \\[2ex] \dfrac{c^2}{8\pi f^2 m^2 c_B^2 \ \eta}, & \text{for } f \geq f_c \end{cases} \tag{3}$$

where

f	is the excitation frequency
m	is the mass per area
c	is the speed of sound in the surrounding air
ρ	is the mass per volume of the surrounding air
c_B	is the phase speed of the bending waves in the panel
η	is the loss factor

Equation (3) is taken from Reference 1, chapter 8, section c for frequencies below the critical frequency.

Equation (3) is derived from Reference 1, chapter 8, section d, equations (101) through (103) for frequencies above the critical frequency.

Note that the phase speed c_B and the loss factor η each vary with frequency.

Panel Phase Speed

Consider a panel with thickness h and a longitudinal wave velocity of C_L. The longitudinal velocity is a constant for a given material. The phase speed is

$$C_B \approx \sqrt{1.8 \ C_L h f} \tag{4}$$

Equation (4) is taken from Reference 1, chapter 3.

Mass Law

Equation (2) essentially follows the "mass law" below the critical frequency. A doubling of either the mass density or the frequency causes the velocity to decrease to one-half its value, which is equivalent to a 6 dB drop.

Equation (2) is complicated above the critical frequency due to the variation of the phase speed and loss factor with frequency.

Loss Factor

The loss factor η is an empirical parameter. The loss factor is assumed to be due mainly to acoustic radiation above the critical frequency.

A mean loss factor curve for aluminum panels is shown in Figure 1. The frequency scale is normalized in terms of the critical frequency. The panels were suspended from a frame by elastic cords such that the boundary conditions were essentially free around the perimeter. The data was taken from two panels. One panel was 1/4 inch thick. The other was 1/2 inch thick. Each panel was 4.0 by 6.5 feet.

The data in Figure 1 is thus approximate data for a particular material, geometry, and boundary condition case. Loss factor measurements on a particular structure of interest must thus be taken in order to accurately use the calculation methods in this report.

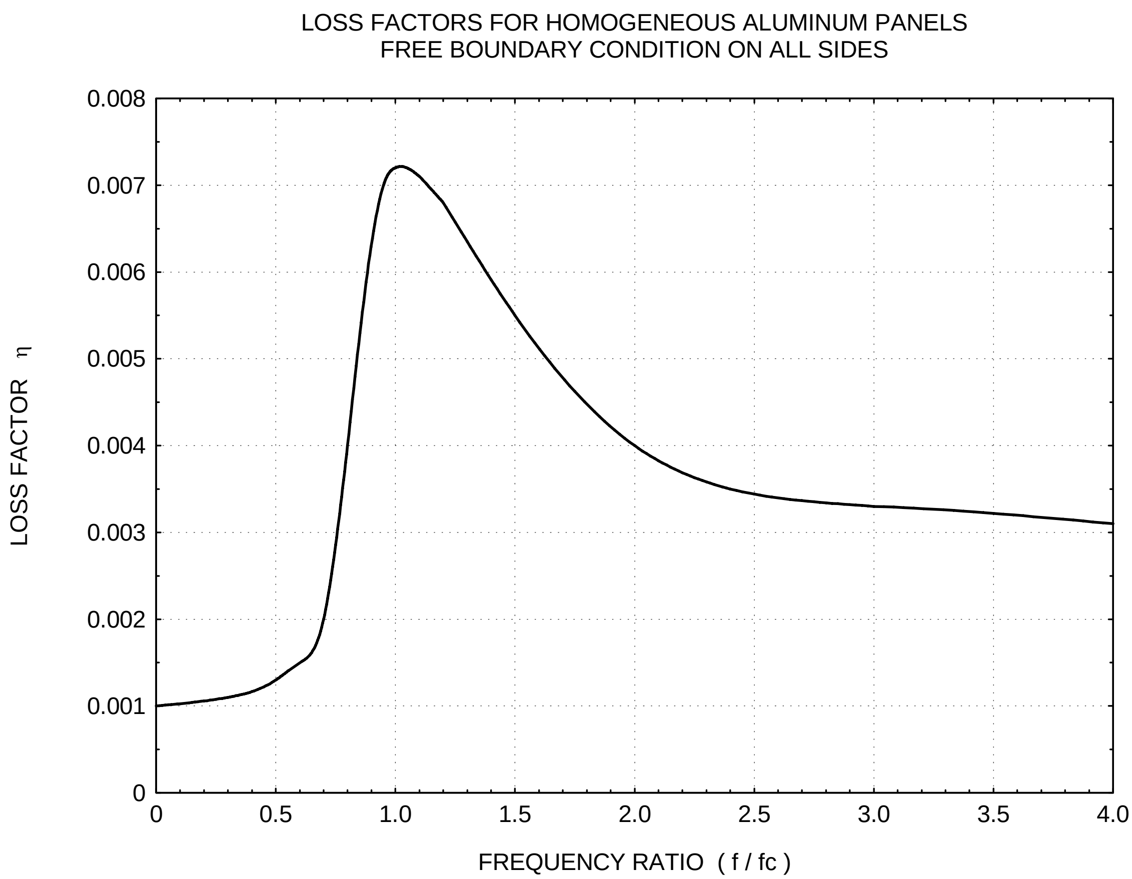
LOSS FACTORS FOR HOMOGENEOUS ALUMINUM PANELS
FREE BOUNDARY CONDITION ON ALL SIDES
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
LOSS FACTOR η
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
FREQUENCY RATIO (f / fc)

Figure 1.

Power Spectrum Format

Acoustic pressure fields are typically represented in one-third octave band power spectra.

Let V_{PS} be the velocity power spectrum.

Let P_{PS} be the pressure power spectrum.

Equation (2) can be modified as follows

$$\left[V_{PS}\right] = \beta \left[P_{PS}\right] \tag{5}$$

The acceleration is often of greater interest than velocity for structural vibration.

Let A_{PS} be the acceleration power spectrum.

$$\left[A_{PS}\right] = \beta \left[2\pi f\right]^2 \left[P_{PS}\right] \tag{6}$$

Reference Data

The parameter ρc is called the "characteristic impedance." The values for air are shown in Table 1. The speed of sound for gases and solid is given in Tables 2 and 3, respectively.

Table 1. Characteristic Impedance ρc

Gas and Temperature	Impedance ($Pa \cdot s/m$)	Impedance ($lbf \cdot s / ft^3$)
Air at 0° C	428	2.73
Air at 20° C	415	2.64

Table 2. Gases at a pressure of 1 atmosphere					
Gases	Molecular Mass (kg/kgmole)	Temperature (°C)	Density (kg/m^3)	Ratio of Specific Heats	Speed of Sound (m/sec)
Air	28.97	0	1.293	1.402	332
Air	28.97	20	1.21	1.402	343
Oxygen (O_2)	32.00	0	1.43	1.40	317
Hydrogen (H_2)	2.016	0	0.09	1.41	1270
Steam	-	100	0.60	-	404.8

Table 3. Solids						
Solid	Density (kg/m^3)	Elastic Modulus (Pa)	Shear Modulus (Pa)	Poisson's Ratio	Speed of Sound (m/sec)	
					Bar	Bulk
Aluminum	2700	$7.0 (10^{10})$	$2.4 (10^{10})$	0.33	5100	6300
Brass	8500	$10.4 (10^{10})$	$3.8 (10^{10})$	0.37	3500	4700
Copper	8900	$12.2 (10^{10})$	$4.4 (10^{10})$	0.35	3700	5000
Steel	7700	$19.5 (10^{10})$	$8.3 (10^{10})$	0.28	5050	6100
Ice	920	-	-	-	-	3200
Glass (Pyrex)	2300	$6.2 (10^{10})$	$2.5 (10^{10})$	0.24	5200	5600

<u>Alternate Method</u>

An alternate equation based on statistical energy analysis is given in Appendix A.

<u>References</u>

1. L. Cremer and M. Heckl, Structure-Borne Sound, Springer-Verlag, New York, 1988.

2. L. Beranek & I. Ver, Noise and Vibration Control Engineering, Principles and Applications, Wiley, New York, 1992.

3. Byam & Radcliffe, Statistical Energy Analysis Model and Connectors for Automotive Vibration Isolation Mounts.

APPENDIX A

<u>Diffuse Sound Field Driving a Freely Hung Panel</u>

The mean square, space-time average velocity $\left\langle v^2 \right\rangle$ of the panel is

$$\left\langle v^2 \right\rangle = \left\langle p^2 \right\rangle \frac{\sqrt{12}\,\pi\, c_{air}{}^2}{2(\rho c)_{air}\, h\, c_L\, \rho_s \omega^2} \left\{ \frac{1}{1 + \left[\dfrac{\rho_s \omega \eta}{2(\rho c)_{air}\, \sigma_{rad}} \right]} \right\} \qquad \text{(A-1)}$$

where

σ_{rad} is the radiation efficiency
ρ_s is the mass per surface area of the panel
η is the loss factor of the panel
$\left\langle p^2 \right\rangle$ is the mean square, space-time average pressure

Equation (A-1) is taken from Reference 2, section 9.8.

The radiation efficiency for supersonic bending waves in structures is

$$\sigma_{rad} = \frac{1}{\sqrt{1-\left(f_c / f\right)}} \qquad \text{(A-2)}$$

Equation (A-2) is taken from Reference 3.

The radiation efficiency of a vibrating body generating sound energy in air is

$$\sigma_{rad} = \frac{W_{rad}}{\left\langle v^2 \right\rangle (\rho c)_{air} S} \qquad \text{(A-3)}$$

Equation (A-3) is taken from Reference 2, section 9.6.