ACOUSTIC POWER SPECTRA Revision B

By Tom Irvine
Email: tomirvine@aol.com

April 10, 2006

Introduction

Acoustic energy is measured by microphones in terms of a pressure time history. The data is typically digitized by a data acquisition system. The purpose of this report is to describe the process by which this data is converted into acoustic power spectra, with a frequency domain in terms of octave bands.

Fourier Transform

The digitized time history can be converted to the frequency domain via a Fourier transform. The Fourier transform is typically implemented in terms of a Fast Fourier transform (FFT) as described in Reference 1.

Amplitude

The Fourier transform yields complex spectral amplitude components. The amplitude units are in terms of peak pressure. The data is processed as follows:

1. Multiply the pressure values by their respective complex conjugates. The resulting amplitude spectra have an amplitude dimension of (peak pressure^2). Furthermore, the spectra are real. The phase information is thus discarded.

2. The next step is the two-sided to one-sided conversion. The spectral components represent "half-amplitude-squared" spectra which are symmetric about the Nyquist frequency. The Nyquist frequency is equal to one-half the sampling rate. The spectral components above the Nyquist frequency should be discarded. The spectral components below the Nyquist frequency should be multiplied by a factor of 4. An exception is the component at zero Hz which should be taken "as is." The resulting amplitude spectra still have an amplitude dimension of (peak pressure^2), but the power spectrum is now one-sided.

3. The next step is to convert the peak values to rms values. Multiply the spectral amplitudes by a factor of 0.5. Again, the component at zero Hz is exempted. The resulting amplitude spectra have an amplitude dimension of (pressure rms^2).

The steps are interchangeable in some sense. For example, the two-sided to one-sided conversion could be performed prior to the complex conjugate multiplication, but the multiplication factor would decrease from 4 to 2.

Frequency

The Fourier transform gives the acoustic pressure in terms of a constant frequency bandwidth. On the other had, the acoustic power spectrum is typically given in terms of "proportional bandwidths." This report will give an example using 1/3 octave bands.

The three parameters which defined each 1/3 octave band are the center frequency f_c, the lower frequency f_l, and the upper frequency f_u.

The formulas relating these parameters are

$$f_c = \sqrt{f_u f_l} \tag{1}$$

$$f_u = 2^{1/3} f_l \tag{2}$$

Furthermore, consider the respective center frequencies of two adjacent bands.

$$f_{c\,(\text{band}\,i+1)} = 2^{1/3} f_{c\,(\text{band}\,i)} \tag{3}$$

In practice, these formulas are applied in an approximate manner. An example of a 1/3 octave band spectrum is given in Table 1.

Table 1. Example of 1/3 Octave Band

Band	Lower Freq (Hz)	Center Freq (Hz)	Upper Freq (Hz)
1	89	100	112
2	112	125	140
3	140	160	180
4	180	200	224

Note that the upper frequency of one band is the lower frequency of the next higher band.

Power Spectrum Calculation

The next step is to determine which pressure points fall in which bands. The square root of the sum of the squares of the pressure points is then taken to determine the root-mean-square (rms) pressure in each band. An equivalent decibel value is then calculated.

Example

Consider some sample data which has a bandwidth of 5 Hz, as shown in the table below. The data can be converted to a 1/3 octave spectrum as shown in Table 2.

Table 2. Sample Conversion to Acoustic Power Spectra

Freq (Hz)	Pressure^2 (psi rms)^2	1/3 Octave Band No.	Running Sum of Pressure^2 (psi rms)^2 in Band	Square Root of Sum of Squares (psi rms)	Sound Pressure Level in Air (dB)
90	0.00001	1	0.00001	-	-
95	0.00002	1	0.00003	-	-
100	0.00001	1	0.00004	-	-
105	0.00003	1	0.00007	-	-
110	0.00004	1	0.00011 (final)	0.0104	131.
115	0.00003	2	0.00003	-	-
120	0.00005	2	0.00008	-	-
125	0.00001	2	0.00009	-	-
130	0.00002	2	0.00011	-	-
135	0.00003	2	0.00014	-	-
140	0.00004	2	0.00018 (final)	0.0134	133.

The sound pressure level SPL is calculated for each band as

$$SPL(f_c) = 20 \log \left[\frac{P_{rms}(f_c)}{P_{ref}} \right]$$

where

$$P_{ref} = \begin{cases} 20\ \mu\text{Pa rms for air} \\ \\ 1\ \mu\text{Pa rms for water} \end{cases} \tag{4}$$

P rms is the root-mean-square sound pressure in equation (4).

The equivalent reference for air in terms of English units is $P_{ref} \approx 2.9(10^{-9})$ psi rms.

The 1/3 octave band at 100 Hz thus has an amplitude of 131 dB.

The 1/3 octave band at 125 Hz thus has an amplitude of 133 dB.

Additional Methods

A second method is to calculate the acoustic power spectrum from a one-third octave pressure power spectral density.

A third method is to use successive bandpass filtering of the pressure time history in the time domain. The RMS value can be then calculated from the filtered data for each band.

References

1. T. Irvine, The Fast Fourier Transform, Vibrationdata, 1998.