ACOUSTIC TRANSMISSION LOSS
Revision F

By Tom Irvine
Email: tomirvine@aol.com

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Introduction

Sound is transmitted through a panel by the forced vibration of the panel. The pressure variations in the sound waves provide the forcing function.

The transmission loss of a panel depends on the surface mass density, dynamic bending stiffness, and internal damping. The contribution of each of these variables depends on the frequency domain.

Forcing Frequency Below Panel Natural Frequencies

Specifically, the panel stiffness governs the transmission loss at low frequencies, below the first series of natural frequencies. At low frequencies, the transmission loss increases about 6 dB per doubling of the panel stiffness. Furthermore, the transmission loss in this domain decreases 6 dB per octave as the forcing frequency increases. Thus, the panel stiffness should be increased as much as possible for low forcing frequencies.

Resonant Excitation of Panel Natural Frequencies

At somewhat higher forcing frequencies, a series of plate-like natural frequencies in the panel cause wide fluctuations in the transmission loss. Sound is radiated at these frequencies. The transmission loss is limited by the panel damping at these frequencies.

Forcing Frequency Above Panel Natural Frequencies

The mass density of the panel controls the transmission loss above the first series of natural frequencies. Theoretically, the transmission loss in this domain increases 6 dB per octave as the forcing frequency increases. Furthermore, the mass controlled region is interrupted by the wave coincidence effect, as explained later in this report.

Transmission Loss Equations

The transmission loss TL in decibels for a panel is defined as

$$\mathrm{TL} = 10 \log\left(\frac{\Pi_s}{\Pi_t}\right), \ \mathrm{dB} \tag{1}$$

where

Π_S is the total power incident on the source of the panel

Π_t is the total power transmitted through the panel

The transmission loss may also be expressed as

$$TL = 10 \log \left(\frac{1}{\tau} \right) \tag{2}$$

where τ is the transmission coefficient.

Note that transmission loss is function of frequency.

<u>Noise Reduction</u>

The noise reduction NR can be calculated by either of the following formulas.

$$NR = 10 \log \left(\frac{I_1}{I_2} \right) \tag{3}$$

$$NR = L_1 - L_2 \tag{4}$$

where I_1 and I_2 are the intensities and L_1 and L_2 are the sound pressure levels in the source and receiver rooms, respectively.

The transmission loss and noise reduction are related by the following formula

$$TL = NR + 10 \log \left(\frac{S}{A} \right) \tag{5}$$

where

S is the surface area

A is the sound absorption of the receiver room

Equation (5) is taken from References 1 and 2.

Mass Law for Single Panel

Consider a single, homogeneous, nonporous panel. The transmission loss for random incidence per Reference 1 is given by the approximate relationship:

$$TL = 20 \log W + 20 \log f - 33 \tag{6}$$

where

W is the surface weight density in pounds per square foot

f is the forcing frequency in Hertz

The transmission loss per equation (6) increases by 6 dB each time either the weight or the frequency is doubled. Equation (6) thus represents the "mass law."

The transmission loss is less than 6 dB in practice, however. The discrepancy between the mass law and actual measurements may be due to:

1. The neglect of stiffness and damping in the mass law equation
2. Panel cracks and porosity
3. Alternate sound transmission paths such as vents

An alternate formula which accounts for the angle of incidence θ is

$$TL(\theta) = 20 \log \left[1 + \frac{\rho_s \ \omega \cos\theta}{2 \rho_o c_o} \right] \tag{7}$$

where

ρ_s = surface mass density

ω = frequency (rad/sec)

$\rho_o c_o$ = acoustic impedance of the surrounding medium

The angle of incidence is defined such that normal incidence occurs at $\theta = 0°$. See Figure 1.

Equation (7) is taken from Reference 7, equation (9.93).

Critical Frequency

The critical frequency is the frequency at which the airborne acoustic wavelength matches the panel bending wavelength. The critical frequency f_c for a homogeneous panel is

$$f_c \approx \frac{C_a^2}{1.8 C_L h} \tag{8}$$

where

C_a = the speed of sound in the surrounding air medium

C_L = the longitudinal wave velocity

h = the panel thickness

Equation (7) is derived in Appendix A.

For example, a 1/4-inch thick steel or aluminum panel has a critical frequency of 2000 Hz.

Wave Coincidence Effect

For all frequencies above the critical frequency, there exists an angle of incidence such that the projection of the incident wave coincides with the bending wave.

$$\lambda_b = \frac{\lambda}{\sin\theta} \tag{9}$$

The geometry of the coincidence effect is shown in Figure 1, as taken from Reference 3.

A relatively high amount of acoustic energy transmission occurs when the bending frequencies coincide with acoustic frequencies above the critical frequency.

Consider a random acoustic field. The random character reduces the significance of the coincidence effect because only a small fraction of the energy is incident at the correct angle.

Honeycomb Panels

Honeycomb Panels are designed to have a high stiffness to mass ratio. The transmission loss is thus controlled by stiffness. The result is poor acoustic attenuation properties, as discussed in Reference 4.

The transmission loss can be improved by filling the core with granular material. This lowers the fundamental frequency and increases the mass density. These changes force the response into the "mass controlled region" of the transfer function magnitude.

Further information is given in Appendix B.

Table 1. Appendix Index	
Appendix	Title
A	Mechanical Bending Waves
B	SDOF Model
C	Transmission through Three Media
D	Transmission Loss Curves from Reference 9

References

1. George Diehl, Machinery Acoustics, Wiley-Interscience, New York, 1973.

2. Lawrence Kinsler et al, Fundamentals of Acoustics, Third Edition, Wiley, New York, 1982.

3. C. Harris, Handbook of Noise Control, McGraw-Hill, New York, 1957.

4. Vibration, Shock, Acoustics; McDonnell Douglas Astronautics Company, Western Division, 1971. (See section 3-1.8)

5. John Miles, On Structural Fatigue Under Random Loading, Journal of the Aeronautical Sciences, November 1954.

6. T. Irvine, Acoustic Fatigue of a Small Rectangular Panel, Rev B, Vibrationdata, 2006.

7. Beranek & Ver, editors; Noise and Vibration Control Engineering Principles and Applications, Wiley, New York, 1992.

8. W. Seto, Acoustics, Schuam's Outline Series in Engineering, McGraw-Hill Book Company, New York, 1971.

9. P. Franken, WADC Technical Report 58-343, Vol II, Methods of Space Vehicle Noise Prediction, Wright-Patterson AFB, Ohio, 1960.

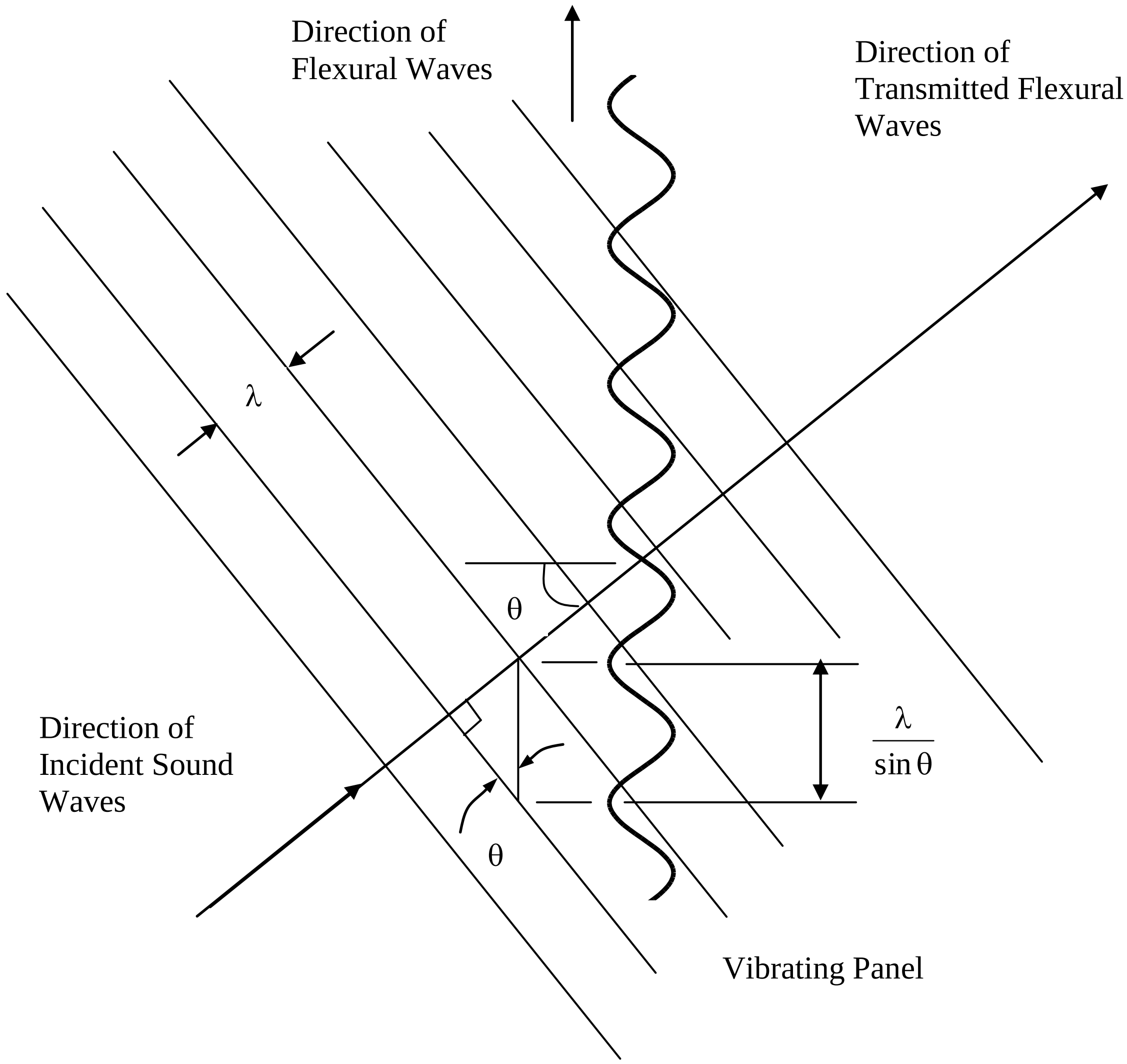

Figure 1. Coincidence Effect Geometry

APPENDIX A

Mechanical Bending Waves

Bending waves are also called flexural waves. They are fundamentally different from all other waveforms.

The classic harmonic wave equation is a second-order equation with respect to both the spatial and time coordinates.

The bending wave equation remains second-order in time, but it is fourth-order in space. The one-dimensional form for the natural response is

$$-B\frac{\partial^4 E}{\partial x^4} = m\frac{\partial^2 E}{\partial t^2} \tag{A-1}$$

where

E	is a field variable
B	is the flexural stiffness
m	is the mass per unit length

The field variable E can represent translation amplitude, rotational amplitude, bending moment, or shear force.

Note that equation (A-1) assumes that the bending wavelength is large compared to the dimensions of the thickness of the beam or plate. Otherwise, correction terms are needed.

A proposed traveling-wave solution is again

$$E(x,t) = A\sin(kx - \omega t - \phi) \tag{A-2}$$

The solution assumes that the bending-wave is propagating along an infinite length. For practical purposes, this assumption is important only for low-frequency bending-modes.

Substitution into equation (A-3) yields the following constraint

$$\frac{B}{m}k^4 = \omega^2 \tag{A-3}$$

The bending wave phase velocity C_B is related to the wave number k and the angular frequency ω by the formula.

$$k = \frac{\omega}{C_B} \tag{A-4}$$

Substitution of equation (A-4) into (A-3) yields the following phase velocity relationship

$$C_B = [\omega]^{1/2} \left[\frac{B}{M}\right]^{1/4} \tag{A-5}$$

The consequence of equation (A-5) is that the phase velocity is no longer a constant. Rather it is proportional to the square root of the frequency.

The phase velocity represents the propagation velocity of only one infinite sinusoidal wave. Waveforms are typically composed of a number of sinusoids, however. A distortion or dispersion effect occurs because the higher frequency waveforms propagate with a higher phase velocity than the lower-frequency components.

The bending wave phase velocity C_B is related to the frequency f and wavelength λ by the familiar formula

$$C_B = f\,\lambda \tag{A-6}$$

Again, phase velocity C_B is no longer a constant.

Consider a plate with thickness h and a longitudinal wave velocity of C_L.

The longitudinal velocity is a constant for a given material. The phase velocity is

$$C_B \approx \sqrt{1.8\ C_L h f} \tag{A-7}$$

By substitution,

$$f\,\lambda \approx \sqrt{1.8\ C_L h f} \tag{A-8}$$

Dividing both sides by $\sqrt{f}$,

$$\lambda\sqrt{f} \approx \sqrt{1.8\ C_L h} \tag{A-9}$$

The bending wavelength is thus

$$\lambda \approx \frac{\sqrt{1.8 C_L h}}{\sqrt{f}} \tag{A-10}$$

The acoustic wavelength at is

$$\lambda = \frac{C_a}{f} \tag{A-11}$$

where C_a is the speed of sound in the surrounding air.

The critical frequency f_c is the frequency at which the bending wavelength equals the acoustic wavelength in the surrounding air. Substitute equation (A-11) into (A-10) and set the frequency equal to the critical frequency.

$$\frac{\sqrt{1.8 C_L h}}{\sqrt{f_c}} \approx \frac{C_a}{f_c} \tag{A-12}$$

$$\frac{f_c}{\sqrt{f_c}} \approx \frac{C_a}{\sqrt{1.8 C_L h}} \tag{A-13}$$

$$\sqrt{f_c} \approx \frac{C_a}{\sqrt{1.8 C_L h}} \tag{A-14}$$

The critical frequency is thus

$$f_c \approx \frac{C_a^{\,2}}{1.8 C_L h} \tag{A-15}$$

APPENDIX B

SDOF Model

The following model is an oversimplification. The purpose is to determine the effects of mass and stiffness on the transmission loss.

Idealize a panel as a single-degree-of-freedom system. Idealize the applied sound pressure as a force.

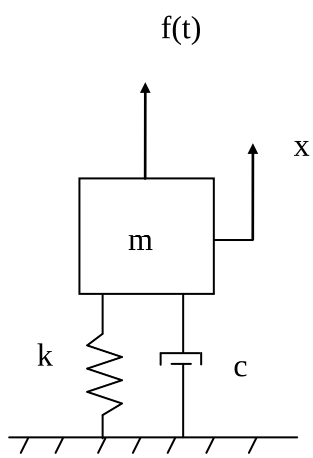

Figure B-1.

Let

k	= the stiffness
m	= the mass
c	= the damping coefficient
x	= the displacement
F	= the applied force
f	= the forcing frequency
fn	= the natural frequency
ξ	= the damping ratio

The transfer function is shown in Figure B-1.

Steady-state Response of a Single-degree-of-freedom System Subjected to an Applied Sinusoidal Force

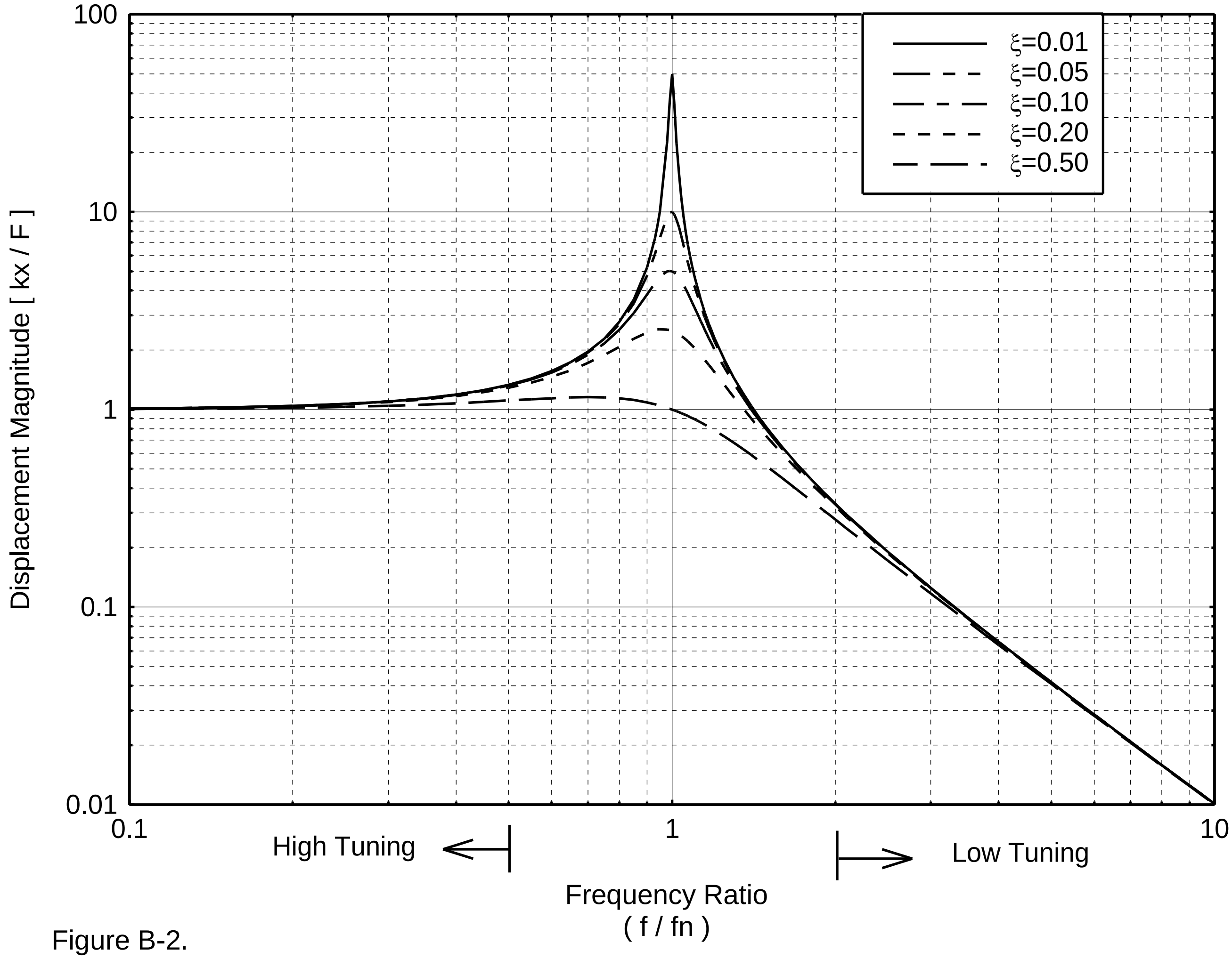

Figure B-2.

The "High Tuning" region of the transfer function is stiffness-controlled.

The middle region is damping-controlled. A greater damping ratio increases the transmission loss.

The "Low Tuning" region of the transfer function is mass-controlled. This is the preferred region for most cases because this is the region over which attenuation occurs.

White Noise Case, Displacement Model

Now consider that the applied force is “white noise,” which is type of broadband, random excitation. The goal is to achieve as much attenuation as possible. Consider each of the dynamic parameters.

The design problem can be reduced to a Miles equation. The displacement response x_{RMS} to a force power spectral density F_{PSD} is

$$x_{RMS} = \frac{1}{2}\left[\frac{\pi}{\xi}\right]^{1/2}\left[\frac{1}{m}\right]^{1/4}\left[\frac{1}{k}\right]^{3/4}\left[F_{PSD}\right]^{1/2} \tag{B-1}$$

Equation (B-1) is derived from Reference 5, equation (3-5b).

The following statements apply if all other variables remain equal:

The displacement response decreases:

1. By 3 dB as the damping is doubled.
2. By 1.5 dB as the mass is doubled.
3. By 4.5 dB as the stiffness is doubled.

These statements may be somewhat misleading in terms of transmission loss, however, because very small displacements at higher frequencies can generate high intensity noise.

White Noise Case, Velocity Model

The velocity RMS can be calculated from the displacement RMS as

$$V_{RMS} \approx 2\pi f_n \left\{X_{RMS}\right\} \tag{B-2}$$

$$V_{RMS} \approx \sqrt{\frac{k}{m}} \left\{X_{RMS}\right\} \tag{B-3}$$

The Miles equation for velocity is thus

$$V_{RMS} \approx \frac{1}{2}\left[\frac{\pi}{\xi}\right]^{1/2}\left[\frac{1}{m}\right]^{3/4}\left[\frac{1}{k}\right]^{1/4}\left[F_{PSD}\right]^{1/2} \tag{B-4}$$

The following statements apply if all other variables remain equal:

The velocity response decreases:

1. By 3 dB as the damping is doubled.
2. By 4.5 dB as the mass is doubled.
3. By 1.5 dB as the stiffness is doubled.

The velocity model is probably a better method than displacement for calculating transmission loss. Reference 7, section 9.7, states that sound transmission through plates can be characterized by the separation impedance Z_s

$$Z_s = \frac{P_s - P_{rec}}{v_n} \tag{B-5}$$

where

P_s = sound pressure amplitude on the source-side face
P_{rec} = sound pressure amplitude on the receiver-side face
v_n = normal velocity of the plate

The plate velocity is assumed to be the same on either side.

White Noise Case, Transmitted Pressure

The amplitude of sound is typically measured in terms of pressure rather than particle displacement or velocity. Furthermore, the receiver side sound pressure is a variable in equation (B-5).

The transmitted pressure in terms of RMS can be calculated for an applied pressure power spectral density via Miles equation. This is the pressure transmitted through the mass and then through the spring into the ground.

$$P_{t,\,RMS}(f_n, \xi) = \sqrt{\left(\frac{\pi}{4\xi}\right) f_n \, P_{PSD}(f_n)} \tag{B-6}$$

where

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{B-7}$$

Equation (B-6) is taken from Reference 6.

The following statements apply if all other variables remain equal:

The transmitted pressure:

1. Decreases by 3 dB as the damping is doubled.
2. Decreases by 1.5 dB as the mass is doubled.
3. Increases by 1.5 dB as the stiffness is doubled.

Note that the “mass law” states that the attenuation should be 6 dB as the mass is doubled. The mass law does not account for damping or stiffness, however.

Summary

Again, the models in this section represent an oversimplification. One reason is that a real panel is a multi-degree-of-freedom system.

The intermediate goal should be to reduce both the plate velocity and the transmitted pressure. Reducing each of these response parameters will increase the transmission loss.

Increasing the mass decreases both the plate velocity and the transmitted pressure.

Increasing the stiffness decreases the plate velocity but increases the transmitted pressure.

Plate velocity and transmitted pressure should <u>not</u> be weighted the same. But assume that they can be as another layer of simplification.

Given this assumption, the net sound power response decreases

1. By 6 dB as the damping is doubled.
2. By 6 dB as the mass is doubled.
3. By 0 dB as the stiffness is doubled.

The previous statements apply if all other variables remain equal.

APPENDIX C

Transmission Through Three Media

The following method is taken from Reference 8. It is another simplified method. It ignores stiffness and damping. It also does not account for the critical frequency.

Consider the transmission coefficient of plane acoustics waves through three homogeneous and isotropic media.

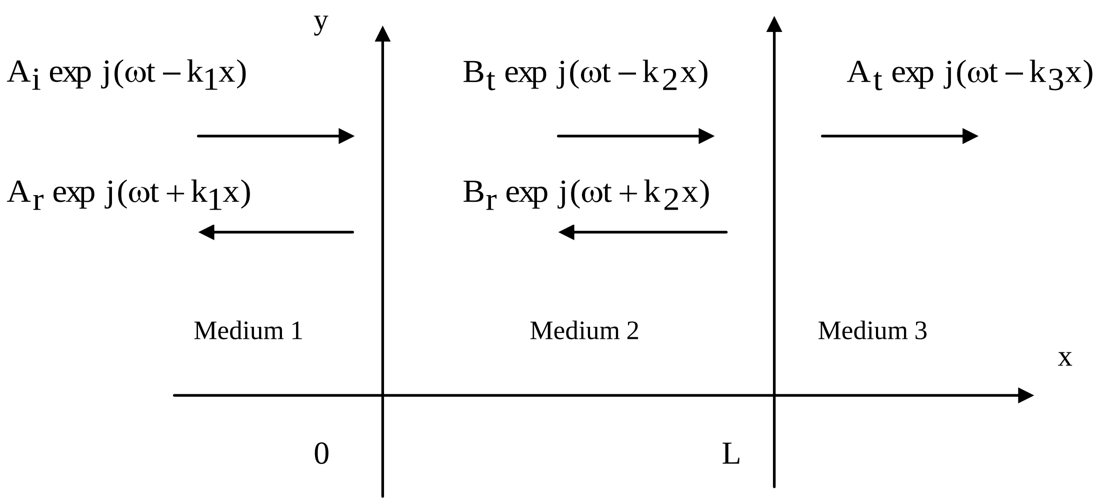

Figure C-1.

The vector functions are velocity components.

For the steady-state condition, two boundary conditions apply at each of the two interfaces:

1. The acoustics pressures at both sides of the boundary are equal
2. The particle velocities normal to the boundary are equal

Consider the case where the first and third media are air. Also the second medium is a dense material such as a wall.

The transmission coefficient τ is

$$\tau = \frac{4}{4\cos^2(k_2 L) + (R_2 / R_1)^2 \sin^2(k_2 L)} \tag{C-1}$$

The transmission coefficient is the faction of incident sound intensity transmitted through the media. Sound intensity is the acoustic power per unit area in the direction of propagation.

k_2 is the wave number in medium 2.

$$k_2 = 2\pi f / c_2 \tag{C-2}$$

where

c_2 is the speed of sound in medium 2
R_1 is the characteristic impedance of air

$$R_1 = \rho_1 c_1 \tag{C-3}$$

where

ρ_1 is the air mass density
c_1 is the speed of sound in air

Typical impedance values for air in equivalent units are:

415	Rayls (Pa sec/m)
415	kg/(m^2 sec)
0.59	lbm/(in^2 sec)
85.0	lbm/(ft^2 sec)
0.01835	slugs/(in^2 sec)
2.6425	slugs/(ft^2 sec)

R_2 is the characteristic impedance of medium 2.

$$R_2 = \rho_2 c_2 \tag{C-4}$$

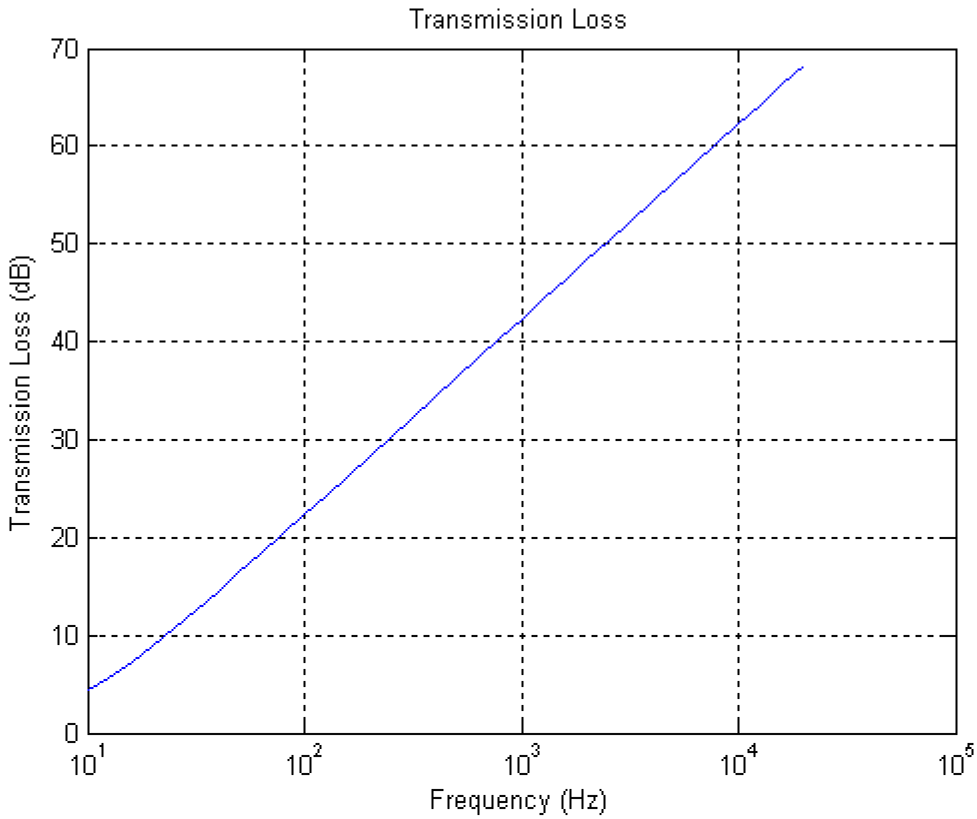

Figure C-2.

As an example, consider the transmission loss through a ¼ inch thick aluminum plate.

Recall that

$$\mathrm{TL} = 10 \log\left(\frac{1}{\tau}\right) \quad , \mathrm{dB} \tag{C-5}$$

The resulting transmission loss is given in Figure C-2.

The transmission loss of a ¼ inch thick aluminum plate is shown in Figure C-2. Again, the curve was calculated using a simplified method.

A more realistic method for calculating the transmission loss through a partition is given in Reference 7, section 9.7.

APPENDIX D

Transmission Loss Curves from Reference 9

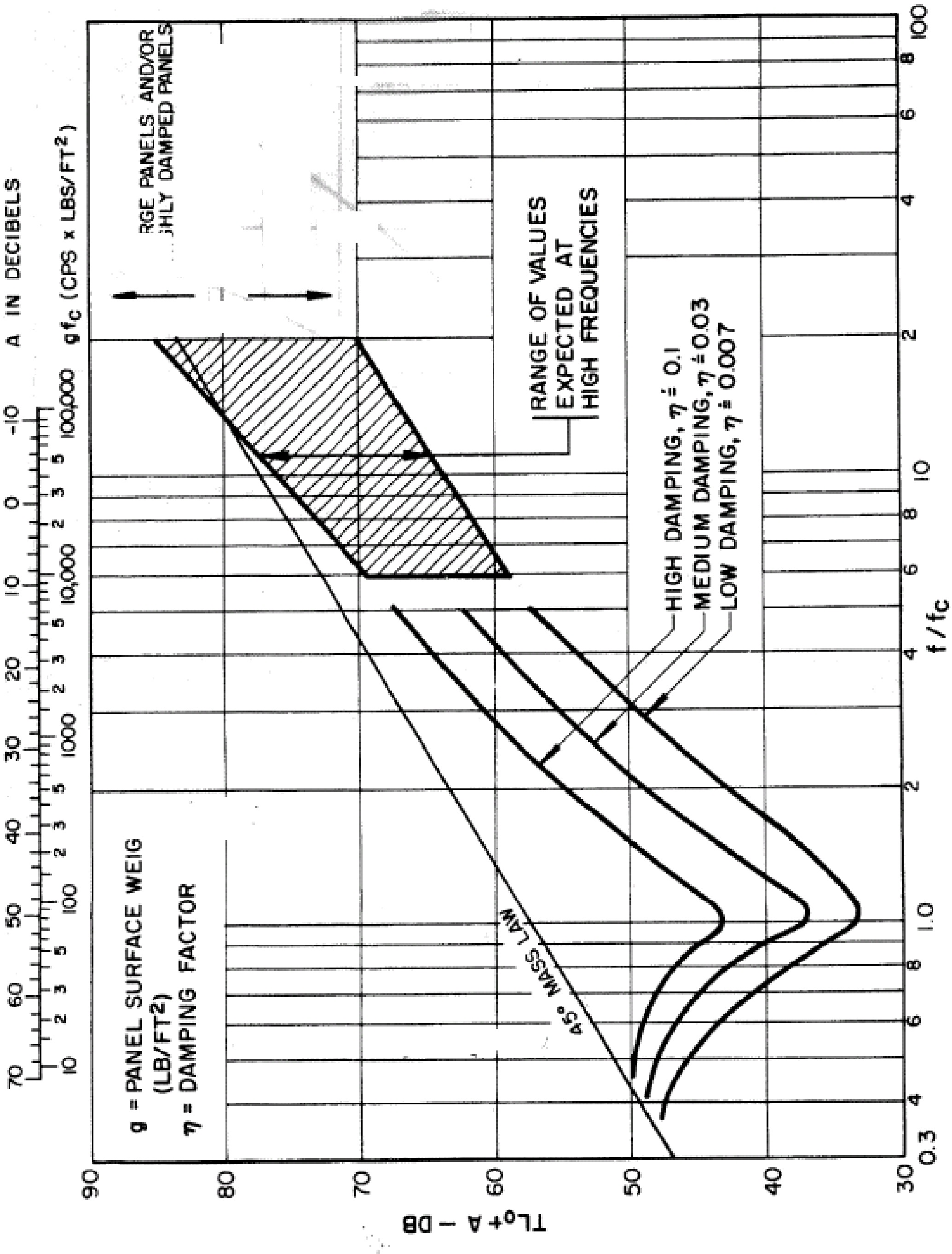

FIG. 109 TL_o FOR PANELS NEAR AND ABOVE THE COINCIDENCE FREQUENCY, f_c.

APPENDIX E

Mass Law Equation

The transmission loss in decibels from Reference 1 is

$$TL = 20 \log W + 20 \log f - 33 \quad \text{(E-1)}$$

where

W is the surface weight density in pounds per square foot
f is the forcing frequency in Hertz

$$TL = 20 \log W + 20 \log f - 28 - 5 \quad \text{(E-2)}$$

The factor of –5 dB in equation (E-2) is needed to account for random incidence.

$$TL = 20 \log W + 20 \log f - 20 \log 25.1 - 5 \quad \text{(E-3)}$$

$$TL = 20 \log \left(\frac{W f}{25.1} \right) - 5 \quad \text{(E-4)}$$

Note that the denominator in the log argument in equation (E-4) is related to the acoustic impedance in air.

$$TL = 20 \log \left(\frac{W f}{R_{air} / \pi} \right) - 5 \quad \text{(E-5)}$$

The impedance in air is

$$R_{air} = \rho c \quad \text{(E-6)}$$

where

ρ	is the air mass density
c	is the speed of sound in air

Note that

$$f = \omega/(2\pi) \tag{E-7}$$

By substitution,

$$TL = 20 \log\left(\frac{W\,\omega/(2\pi)}{R_{air}/\pi}\right) - 5 \tag{E-8}$$

$$TL = 20 \log\left(\frac{W\,\omega}{R_{air}/2}\right) - 5 \tag{E-9}$$