ACOUSTIC FATIGUE OF A SMALL RECTANGULAR PANEL Rev B

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Variables

m	is the mass
c	is the damping coefficient
k	is the stiffness
x	is the displacement
ξ	is the viscous damping ratio
f(t)	is the applied force
F	is the applied force amplitude
F_t	is the force transmitted through the spring
P	is the applied pressure
P_t	is the pressure transmitted through the spring
P_{PSD}	Is the pressure power spectral density
ω	is the forcing frequency (rad/sec)
ω_n	is the natural frequency (rad/sec)
f	is the forcing frequency (Hz)
f_n	is the natural frequency (Hz)
E	is the modulus of elasticity
Q	is the amplification factor
t	is time

Variables (Continued)

D	is the plate stiffness factor
ν	is the Poisson ratio
h	is the plate thickness
a	is the plate length
b	is the plate width
μ	is the mass per area
Rn	Miner's cumulative fatigue index
q	is the uniform static pressure
σ	is stress
1σ	is one standard deviation

Introduction

This tutorial follows the method in Reference 1 but adds some clarification.

The length and width of the panel must be sufficiently small that the panel can be regarded as a single-degree-of-freedom system, as shown in Figure 1.

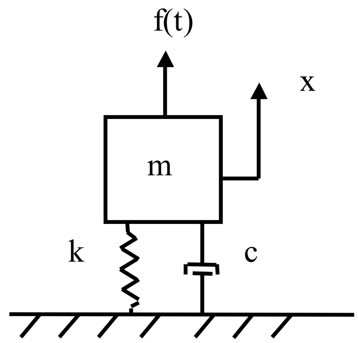

Figure 1.

The force transmitted through the spring to the ground is

$$\left|\frac{F_t}{F}\right| = \frac{\sqrt{1+(2\xi\rho)^2}}{\sqrt{\left[1-\rho^2\right]^2+[2\xi\rho]^2}}, \quad \rho = \frac{\omega}{\omega_n} \tag{1}$$

Equation (1) can be restated for a single-degree-of-freedom panel subjected to an applied pressure.

$$\left|\frac{P_t}{P}\right| = \frac{\sqrt{1+(2\xi\rho)^2}}{\sqrt{\left[1-\rho^2\right]^2+[2\xi\rho]^2}}, \quad \rho = \frac{\omega}{\omega_n} \tag{2}$$

Equation (2) has the same form as that for the acceleration response of a mass subjected to base excitation. Thus the transmitted pressure in terms of RMS can be calculated for an applied pressure power spectral density via Miles equation.

$$P_{t,\,RMS}\left(f_n, Q\right) = \sqrt{\left(\frac{\pi}{2}\right) f_n \;Q\; P_{PSD}(f_n)} \tag{3}$$

where

$$Q = \frac{1}{2\xi} \tag{4}$$

The next step is to calculate the resulting static displacement due to the transmitted force. Both the bending and membrane responses must be considered.

The resulting bending and membrane stresses are then calculated. Miner's cumulative damage equation is used for the fatigue life calculation. These steps are shown in the example in Appendix A.

Alternate Method

An alternate method for calculating the stress is given in Appendix B.

References

1. D. Steinberg, Vibration Analysis for Electronic Equipment, Third Edition, Wiley, New York, 2000.

2. W. Young, Roark's Formulas for Stress & Strain, 6th edition, McGraw-Hill, New York, 1989.

3. R. Blevins, Flow-Induced Vibration, Krieger, Malabar, Florida, 2001.

APPENDIX A

Example 1

Consider a panel with dimensions 12 x 8 x 0.040 inches. The material is aluminum 6061-T4. The panel has a fixed boundary condition on all four edges. It is subjected to the sound pressure level in MIL-STD-1540C, as shown in Figure A-1. The duration, however, is undefined in this example.

The amplification factor is assumed as Q=10.

The plate stiffness factor is

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{A-1}$$

$$D = \frac{1(10^7)\ \mathrm{lbf/in^2}\ (0.040\ \mathrm{in})^3}{12(1-0.33^2)} \tag{A-2}$$

$$D = \quad 59.85 \quad \mathrm{lbf\ in} \tag{A-3}$$

The mass per area is

$$\mu = (0.1\ \mathrm{lbm/in^3})\ (\ 0.040\ \mathrm{in}\ ) \tag{A-4}$$

$$\mu = (0.00025904\ \mathrm{lbf\ sec^2/in^4}\ )\ (\ 0.040\ \mathrm{in}\ ) \tag{A-5}$$

$$\mu = (\ 1.036e-05\ \mathrm{lbf\ sec^2/in^3}\ ) \tag{A-6}$$

$$\frac{D}{\mu} = \frac{59.85 \quad \mathrm{lbf\ in}}{1.036e-05\ \mathrm{lbf\ sec^2/in^3}} \tag{A-7}$$

$$\frac{D}{\mu} = 5.776e+06 \frac{in^4}{sec^2} \tag{A-8}$$

The fundamental frequency for a plate clamped on all edges is

$$f_n = \left(\frac{\pi}{1.5}\right)\sqrt{\frac{D}{\mu}\left(\frac{3}{a^4} + \frac{2}{a^2 b^2} + \frac{3}{b^4}\right)} \tag{A-9}$$

$$f_n = \left(\frac{\pi}{1.5}\right)\sqrt{5.776e+06 \frac{in^4}{sec^2}\left(\frac{3}{12^4} + \frac{2}{12^2 8^2} + \frac{3}{8^4}\right)\frac{1}{in^4}} \tag{A-10}$$

$$f_n = 166.5 \text{ Hz} \tag{A-11}$$

The transmitted pressure is

$$P_t = \sqrt{\left(\frac{\pi}{2}\right) f_n \;\; Q \;\; P_{PSD}(f_n)} \tag{A-12}$$

$$P_t = \sqrt{\left(\frac{\pi}{2}\right)(166.5\,\text{Hz})(10)\;(1.06e-06 \text{ psi\^{}2/Hz})} \tag{A-13}$$

$$P_t = 0.053 \text{ psi RMS} \tag{A-14}$$

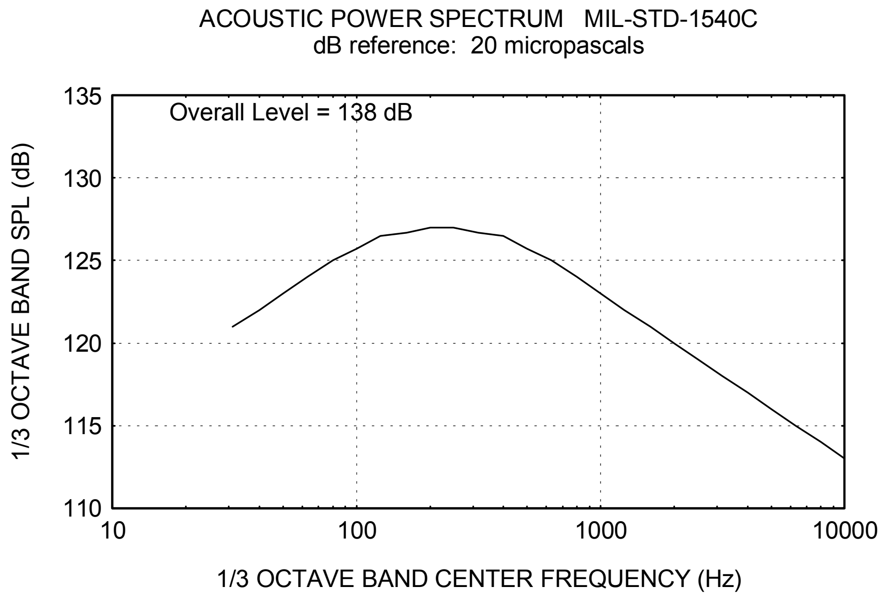

Figure A-1.

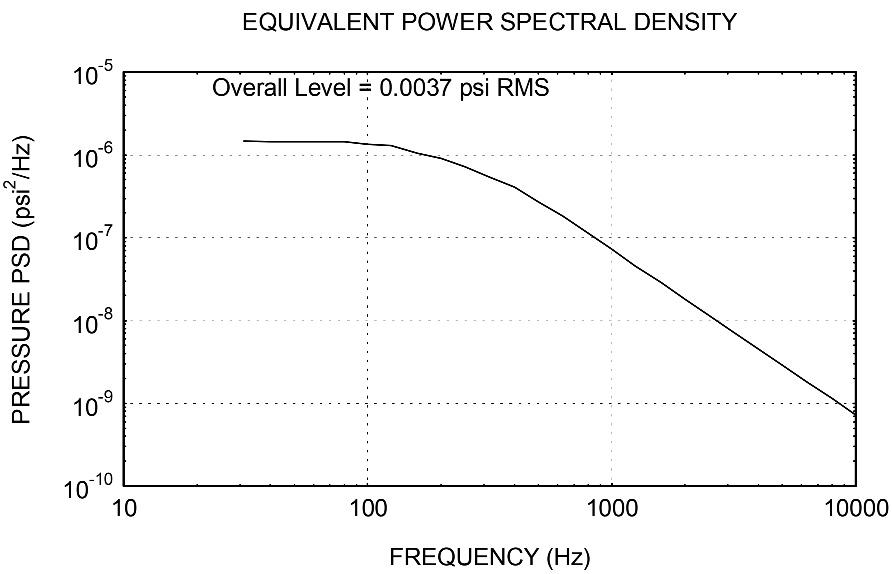

Figure A-2.

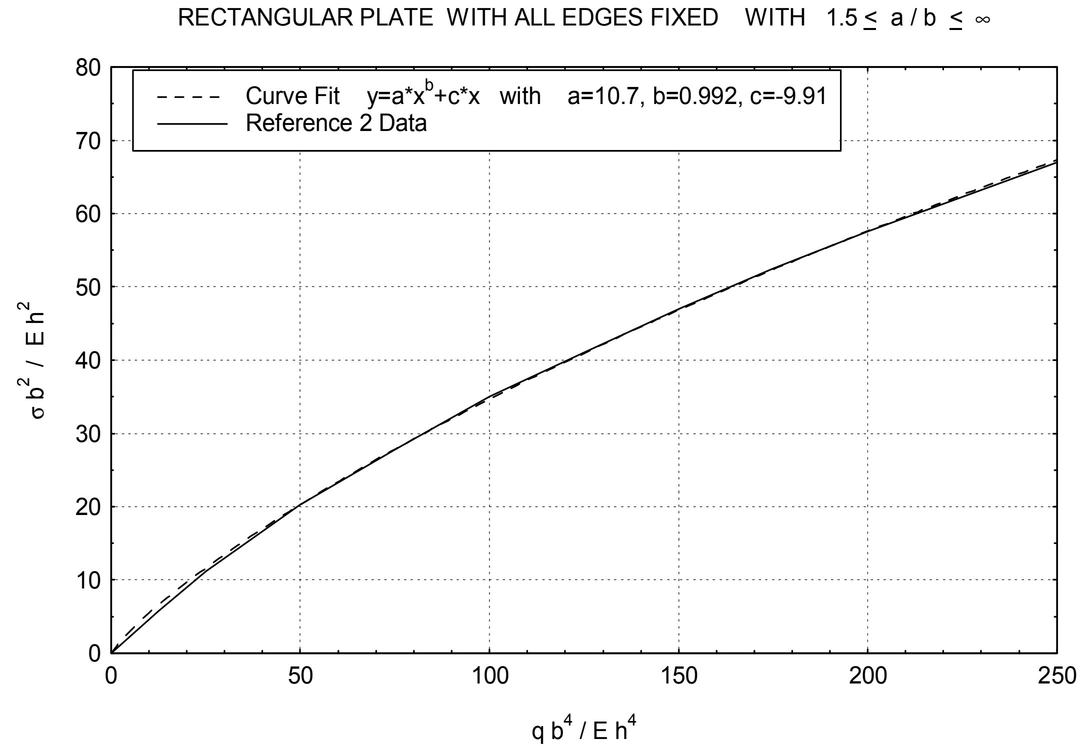

Figure A-3. Large Deflection Theory Curve

The next step is to calculate the displacement for pure bending. As a rule-of-thumb, large deflection theory must be used when the maximum panel displacement is greater than one-third the panel thickness.

The large deflection theory accounts for both membrane and bending stress. The panel will carry some of the load in direct bending and some of the load as membrane tension for large deflections. Large loads can be carried with a relatively small displacement when the panel is loaded as a membrane.

Note that the membrane stress is also referred to as the diaphragm stress.

The bending stress can be easily calculated from textbook formulas.[1] Membrane stress formulas, however, are not readily available except for the special case of a square plate with fixed boundary conditions.

As alternative, Reference 2 gives tabular values of dimensionless coefficients for the load, deflection, and stresses of a rectangular plate undergoing a large deflection. These values are plotted in Figure A-3 for a panel with fixed edges.

[1] For example, see Reference 2, chapter 10, table 26.

The maximum deflection of the rectangular plate with fixed edges for pure bending is

$$Y = \frac{\alpha q b^4}{E h^3} \tag{A-15}$$

Note that α = 0.024 for a/b = 1.5. Recall that b = width.

The deflection equation is taken from Roark and Young, chapter 10, table 26.

$$Y = \frac{0.024\,(0.053 \text{ psi RMS})\,(8\,\text{inch})^4}{(1.0e+07 \text{ psi})(0.040 \text{ inch})^3} \tag{A-16}$$

$$Y = 0.0081 \text{ inch RMS} \tag{A-17}$$

This displacement is about 20% of the panel thickness. This is less than the one-third threshold. Nevertheless, the deflection will have 3-sigma peaks of 0.024 inches, which is more than one-half the thickness. Thus, this is a somewhat borderline case. Large deflection theory will be used anyway to solve for the maximum stress.

The curve in Figure A-3 can be approximated as

$$\frac{\sigma b^2}{E h^2} = 10.7\left(\frac{q b^4}{E h^4}\right)^{0.992} - 9.91\left(\frac{q b^4}{E h^4}\right) \tag{A-18}$$

$$\sigma = \left[\frac{E h^2}{b^2}\right]\left[10.7\left(\frac{q b^4}{E h^4}\right)^{0.992} - 9.91\left(\frac{q b^4}{E h^4}\right)\right] \tag{A-19}$$

Note that $q = P_t$.

Equation (A-19) is solved by a computer program. The resulting intermediate values are

$$\left(\frac{q\,b^4}{E\,h^4}\right) = 8.48 \tag{A-20}$$

$$\frac{\sigma\,b^2}{E\,h^2} = 5.161 \tag{A-21}$$

$$\left[\frac{E\,h^2}{b^2}\right] = 250 \quad \text{psi} \tag{A-22}$$

The maximum combined stress from the computer program is thus

$$\sigma_T = 1290 \text{ psi RMS} \tag{A-23}$$

Note that the maximum stress occurs at the center of each long edge.

Also note that the maximum stress would have been 949 psi RMS if large deflection theory had not been used.

Now assume a stress concentration factor K=2, which could be due to a hole for example.

The total stress including the stress factor is

$$\sigma_{T,K} = 2580 \text{ psi RMS} \tag{A-24}$$

The 3-sigma total stress is

$$\sigma_{T,K} = 7740 \text{ psi 3-sigma} \tag{A-25}$$

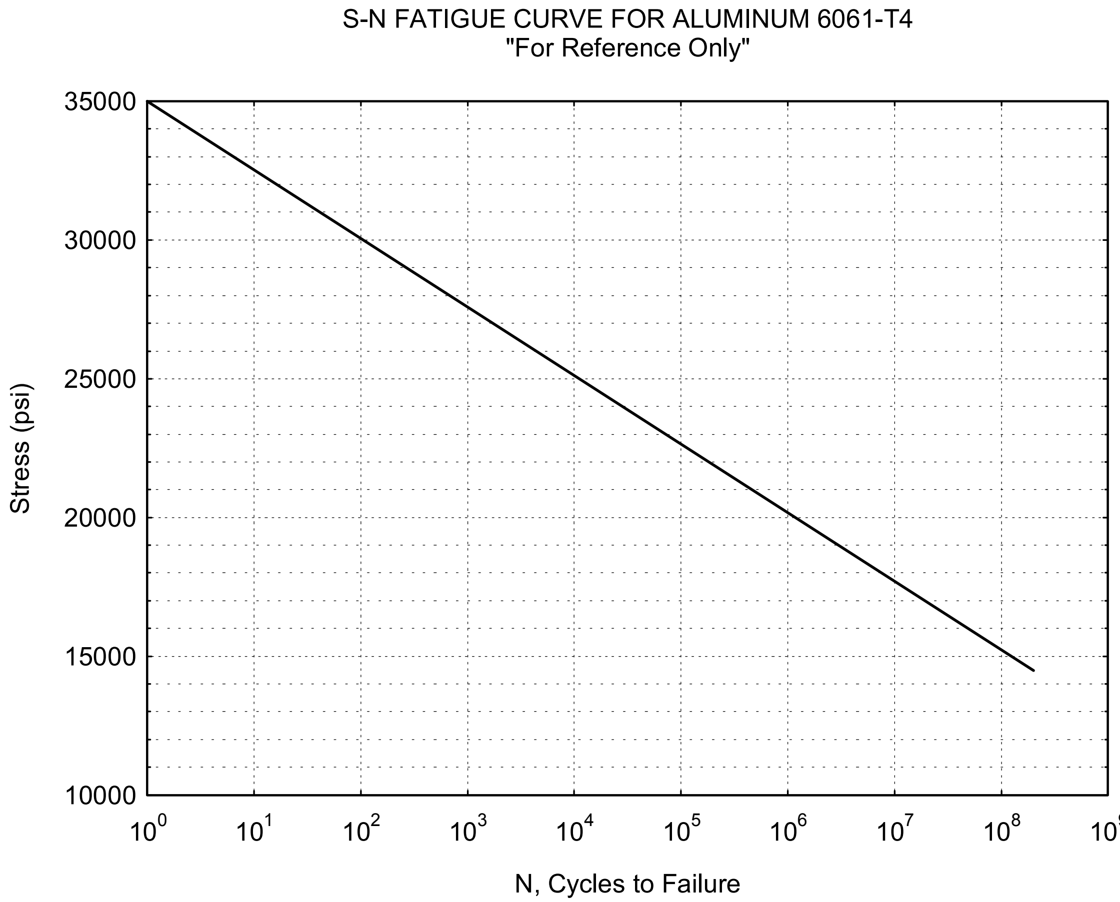

Figure A-4.

The 3-sigma stress level of 7740 psi is well below the stress limit for 10^8 cycles. Thus the panel will endure at least 100 million 3-sigma stress reversal cycles, based on the input sound pressure level in Figure A-1. No further calculations are necessary.

Example 2

The panel is to be subject to the level in Figure A-1 plus 12 dB. Determine the fatigue life. The overall sound pressure level is now 150 dB. The duration is 1000 seconds.

The transmitted pressure can be easily scaled from the previous example.

$$P_t = 0.21 \text{ psi RMS} \tag{A-26}$$

The large deflection calculation, however, must be repeated in detail. Equation (A-19) is again solved by a computer program. The resulting intermediate values are

$$\left(\frac{q b^4}{E h^4}\right) = 33.6 \tag{A-27}$$

$$\frac{\sigma b^2}{E h^2} = 16.8 \tag{A-28}$$

$$\left[\frac{E h^2}{b^2}\right] = 250 \text{ psi} \tag{A-29}$$

The maximum combined stress from the computer program is thus

$$\sigma_T = 4144 \text{ psi RMS} \tag{A-30}$$

Note that the maximum stress occurs at the center of each long edge.

Now assume a stress concentration factor K=2, which could be due to a hole for example.

The total stress including the stress factor is

$$\sigma_{T,K} = 8288 \text{ psi RMS} \tag{A-31}$$

The 3-sigma total stress is

$$\sigma_{T,K} = 24{,}864 \text{ psi 3-sigma} \tag{A-32}$$

The number of stress-reversal cycles required to produce a failure at the 1σ, 2σ and 3σ stresses are determined from the S-N fatigue curve for the 6061-T4 aluminum panel.

Miner's cumulative damage indeed Rn is given by

$$R_n = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \cdots \tag{A-33}$$

In theory, the panel should fail when

$$R_{n\ (\text{theory})} = 1.0 \tag{A-34}$$

For aerospace structures, however, a more conservative value is used

$$R_{n\ (\text{aero})} = 0.7 \tag{A-35}$$

The actual time at each stress level for the example problem is calculated in Table 1 through 5, for various durations.

Note that

Number of cycles = [time ratio] [natural frequency (Hz)] [duration(sec)]

Table A-1. Stress versus Time, 150 dB input

Duration = 1000 sec
Natural Frequency = 166.5 Hz
Material = Aluminum 6061-T4

Stress Level (psi)	Time Ratio	Test Cycles	Limit Cycles from S-N Curve	$\frac{n_i}{N_i}$
1σ = 8288	0.6827	n1 = 1.14e+05	N1 > 1.0e+08	-
2σ = 16,576	0.2718	n2 = 4.53e+04	N2 = 2.9e+07	0.0016
3σ = 24,864	0.0428	n3 = 7.13e+03	N3 =1.50e+04	0.475

Note that the 1σ stress level is not a factor since the number of limits cycles is greater than 100 million.

The results in Table A-1 show that Rn = 0.477 for 1000 seconds. The value is less than the 0.7 threshold. Thus the panel will survive.

Example 3

Repeat example 2 with a duration of 1465 seconds.

Table A-2. Stress versus Time, 150 dB input Duration = 1465 sec Natural Frequency = 166.5 Hz Material = Aluminum 6061-T4				
Stress Level (psi)	Time Ratio	Test Cycles	Limit Cycles from S-N Curve	$\frac{n_i}{N_i}$
1σ = 8288	0.6827	n1 = 1.67E+05	N1 > 1.0e+08	-
2σ = 16,576	0.2718	n2 = 6.63E+04	N2 = 2.9e+07	0.0023
3σ = 24,864	0.0428	n3 = 1.04E+04	N3 =1.50e+04	0.696

The results in Table A-2 show that Rn = 0.698 for 1465 seconds. This is very near the 0.7 limit. Thus the panel is on the verge of a fatigue failure.

APPENDIX B

Alternate Miles Equation for Stress Calculation

The following method is taken from Reference 3, section 7.2.4.

$$\sigma_{RMS}(f_n, Q) = \sqrt{\left(\frac{\pi}{2}\right) f_n \;\; Q \;\; P_{PSD}(f_n)} \;\; \sigma_o \tag{B-1}$$

where σ_o is the dimensionless stress induced by a uniform unit surface pressure.

Recall Example 1 from Appendix A.

Again, the panel has dimensions 12 x 8 x 0.040 inches. The material is aluminum 6061-T4. The panel has a fixed boundary condition on all four edges.

The following equation is taken from Roark and Young.

$$\frac{\sigma_o}{q} = \frac{\beta b^2}{h^2} \tag{B-2}$$

Reference 2 gives $\beta = 0.45$ for $a/b = 1.5$

$$\frac{\sigma_o}{q} = \frac{0.45(8\,\text{in})^2}{(0.040\,\text{in})^2} = 18{,}000 \tag{B-3}$$

Recall that $f_n = 166.5$ Hz and Q=10.

The sound pressure level at 166.5 Hz is $1.06E-06\,\text{psi}^2\,/\,\text{Hz}$.

By substitution,

$$\sigma_{RMS}\left(f_n, Q\right) = 18{,}000\sqrt{\left(\frac{\pi}{2}\right)(166.5\,\mathrm{Hz})(10)\left(1.06\mathrm{E}-06\,\mathrm{psi}^2\,/\,\mathrm{Hz}\right)} \tag{B-4}$$

$$\sigma_{RMS} = 948 \text{ psi RMS} \tag{B-5}$$

The value shown in equation (B-5) is less than that in (A-23).

The value in (B-5) was calculating without accounting for large deflection theory, however. It thus does not account for membrane stress.