SIGNAL IDENTIFICATION FROM ALIASED DATA

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Introduction

The purpose of this tutorial is to demonstrate a method for identifying the frequency and amplitude of a sine function from aliased data. Some prior knowledge of the expected signal in its analog form is useful for this process.

Shannon's Sampling Theorem

Shannon's sampling theorem states that a sampled time signal must not contain components at frequencies above the Nyquist frequency. Otherwise, an aliasing error will occur.

Note that the Nyquist frequency is equal to one-half the sampling rate.

Numerical Experiment

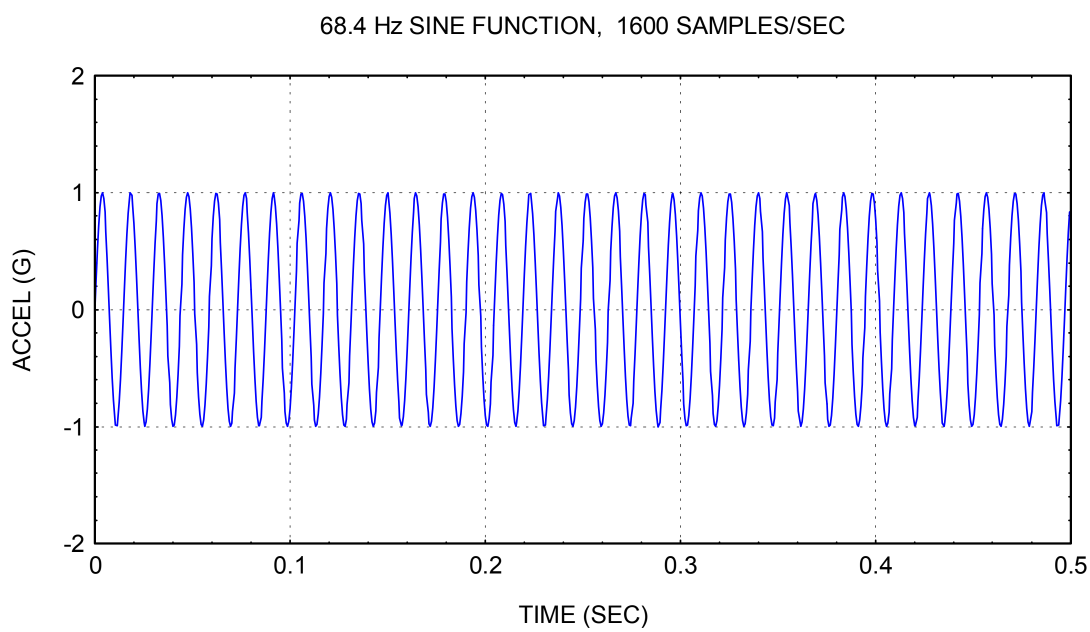

Figure 1.

A sample sine function is shown in Figure 1. The frequency is 68.4 Hz. The digitization rate is 1600 samples per seconds, which provides 23.4 points per each cycle.

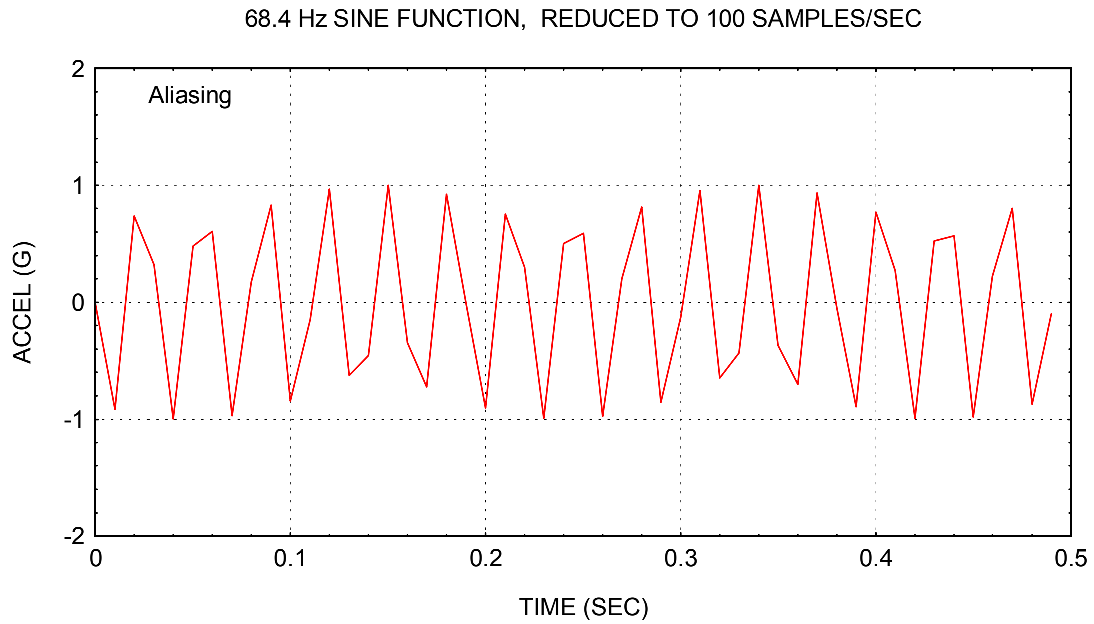

Figure 2.

The sample time history is reduced to 100 samples per second in Figure 2. The Nyquist frequency is 50 Hz, which is below the 68.4 signal frequency. Aliasing has thus occurred. The apparent frequency is 31.6 Hz.

See Reference 1 for aliasing formulas.

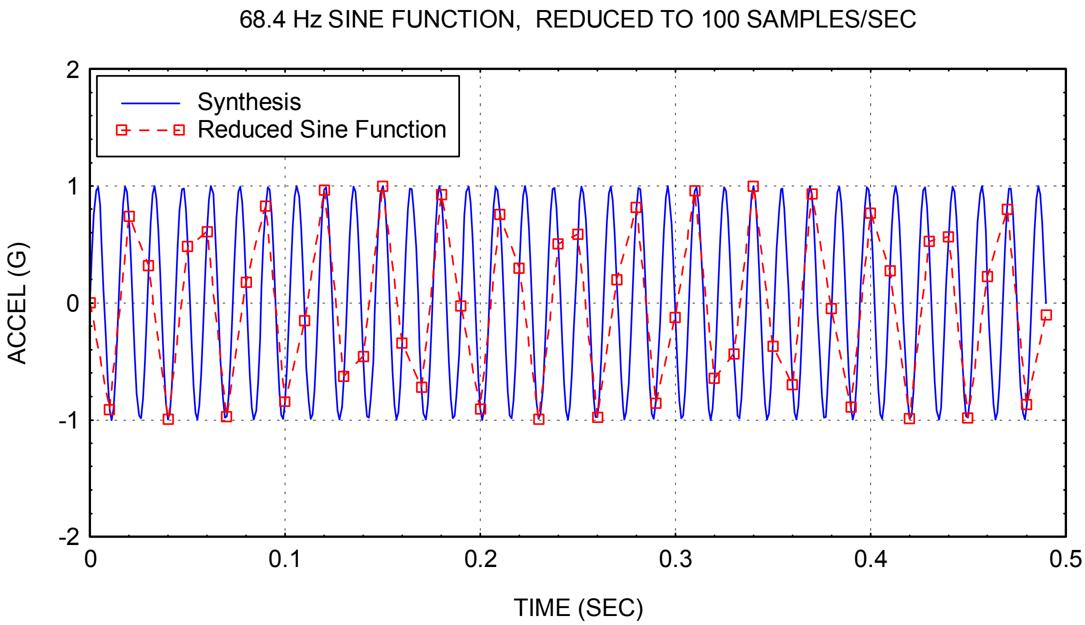

Figure 3.

The method in Reference 2 can be used to perform a curve-fit to estimate the frequency and amplitude of the original signal. The curve-fit was carried out using the Matlab script: sinefind_alias.m, version 1.9. The script uses a brute-force method which relies heavily on random number generation. It also has some convergence steps.

The Matlab script was instructed to perform the curve-fit using a sine frequency between 60 and 80 Hz. These limits were necessary to prevent the curve-fit from converging to 31.6 Hz.

The curve-fit yielded the following results:

Case	Amplitude	fn(Hz)	Phase(rad)
1	1.0003	68.4005	0.0024

The curve-fit thus successfully estimated the frequency and amplitude of the original signal with negligibly low error.

The synthesized signal in Figure 3 is derived from these values, with a sample rate of 1000 samples per second.

Flight Data

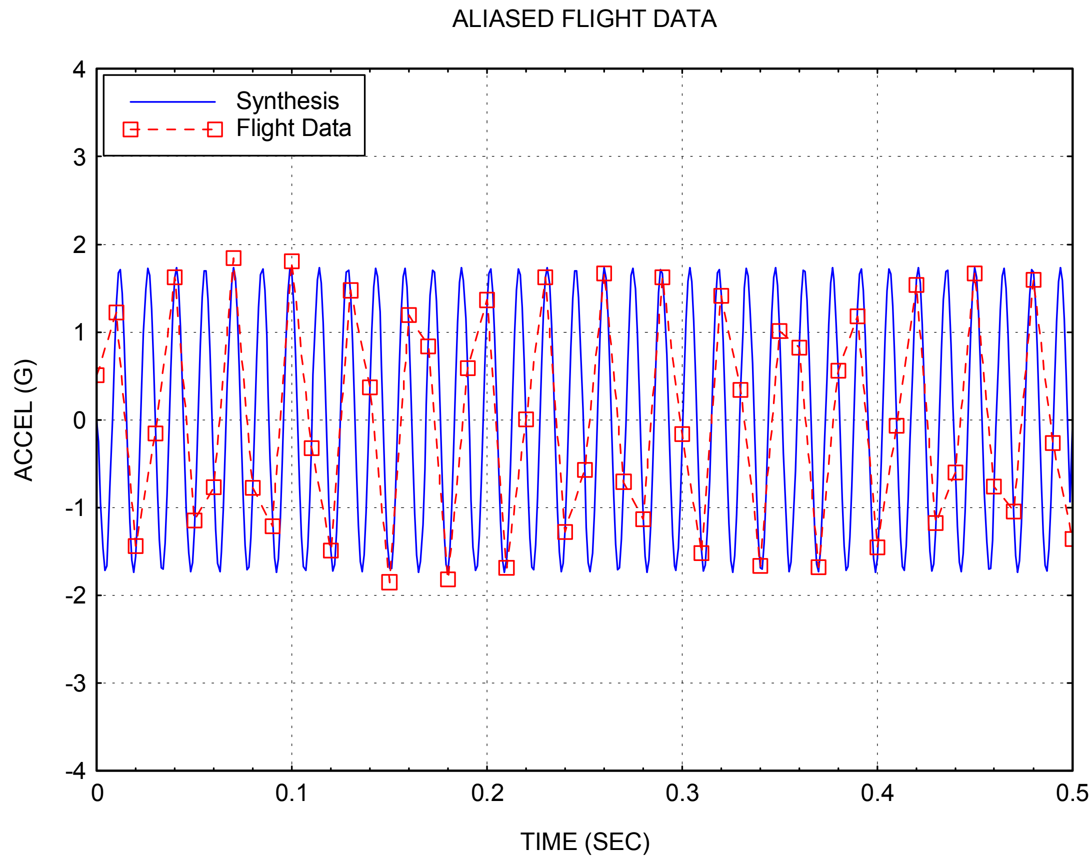

Figure 4.

Flight data from an Inertial Navigation System translational accelerometer is shown in Figure 4. The data was sampled at 100 samples per second, with a Nyquist frequency of 50 Hz. The data was then highpass filtered at 20 Hz to remove rigid-body acceleration effects, as a post-processing step. The resulting flight data has an oscillation with an apparent frequency of approximately 32 Hz. This data is aliased, however, due to a known excitation source with a corresponding frequency somewhere between 60 to 80 Hz.

A curve-fit was performed within the 60 to 80 Hz frequency domain. The following parameters were obtained:

Case	Amplitude	fn(Hz)	Phase(rad)
1	1.7361	68.4085	3.4256

The synthesized signal in Figure 4 is derived from these values, with a sample rate of 1000 samples per second.

The exact parameters of the original signal are unknown, but the results seem very reasonable based on past flight data for similar vehicles which had high-sample rate accelerometers.

Reference

1. T. Irvine, Notes on sample rate and aliasing, Vibrationdata, 2007.
2. T. Irvine, A Time Domain, Curve-Fitting Method for Accelerometer Data Analysis, AIAA-2003-1972, 44th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Norfolk, Virginia, Apr. 7-10, 2003.