NOTES ON SAMPLE RATE AND ALIASING Revision B

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The Nyquist frequency is equal to one-half the sampling rate.

INTRODUCTION

Engineers collect accelerometer data in a variety of settings. Examples from the launch vehicle industry include:

1. Launch vehicle flight data
2. Stage separation and other ground tests where pyrotechnic devices are initiated.
3. Component shock and vibration tests performed in the lab

The accelerometers measure the data in analog form. The accelerometer may have an integral mechanical lowpass filter. Furthermore, the signal conditioning unit may have an analog lowpass filter.

Lowpass filtering of the *analog* signal is necessary to prevent aliasing.

Eventually, the data is passed through an analog-to-digital converter.

The proper lowpass frequency and sampling rate must be selected to ensure that the digitized data is accurate. This report establishes guidelines for selection of these parameters.

FIRST REQUIREMENT FOR SAMPLING RATE

Rule-of-Thumb

The first requirement is that the sampling rate must be greater than the maximum analysis frequency. Industry has established guidelines for this requirement, as discussed in References 1 and 2.

These guidelines are summarized in Table 1, where

$$(\text{minimum sampling rate}) \geq (N)(\text{maximum analysis frequency})$$

Table 1. First Requirement	
Analysis Type	N
Frequency Domain	2
Time Domain	10

Frequency Domain

The frequency domain requirement is based on the fact that at least two time-domain coordinates per cycle are required to resolve a sine wave.

The frequency domain analysis thus extends up to the Nyquist frequency which is one-half the sample rate.

Note that some conservative references specify an N of 2.5 for frequency domain calculation.

Time Domain

A sampling rate of 100 KHz is thus required for a shock response spectrum analysis extending to 10 KHz. Recall that the shock response spectrum is calculated in the time domain.

The IES Handbook for Dynamic Data Acquisition and Analysis gives the following guidelines:

> Unlike other spectral quantities evolving from the discrete Fourier transform computations, the SRS is essentially a time domain quantity.
>
> Hence, the digital sampling rate given by Rs=1/(delta t), introduces errors beyond those associated with aliasing about the Nyquist frequency.
>
> Thus, Rs must be high enough to accurately describe the response of the SRS oscillators.
>
> To minimize potential error, it is recommended that the SRS computations be performed with a sampling rate of Rs $\geq$ 10 fh, where fh is the highest natural frequency of the SRS computation.

SECOND REQUIREMENT FOR SAMPLING RATE

Shannon's Sample Theorem

Shannon's sampling theorem states that a sampled time signal must not contain components at frequencies above the Nyquist frequency.

Again, the Nyquist frequency is equal to one-half the sampling rate. Shannon's theorem applies to frequency domain analysis.

Rule-of-Thumb

Thus, the second requirement is that the sampling rate must be greater than the maximum frequency present in the source energy at the measurement location.

This requirement is independent of the maximum analysis frequency.

The guidelines are summarized in Table 2, where

$$(\text{minimum sampling rate}) \geq (\text{ M })(\text{ maximum frequency in source energy })$$

Table 2. Second Requirement	
Analysis Type	M
Frequency Domain	2
Time Domain	10

Note the similarity between Tables 1 and 2.

Aliasing Examples

The following examples show the consequences of failure to comply with the guidelines in Table 2. An aliasing error results in two of the cases.

Consider sine waves sampled at 2000 samples per second. The Nyquist frequency is thus 1000 Hz. The Nyquist frequency is also the upper limit for a frequency domain calculation, per Table 1.

The power spectral density function of a 200 Hz sine wave sampled at this rate is given in Figure 1. As expected, a spectral line appears at 200 Hz.

The power spectral density of an 1800 Hz sine wave is given in Figure 2. Note that aliasing occurs. The 1800 Hz signal is folded about the Nyquist frequency such that a spectral line appears at 200 Hz. The Nyquist frequency thus forms a line of symmetry.

The power spectral density of a 200 Hz sine wave appears to equal that of a 1800 Hz sine. Again, this error occurs due to inadequate sampling rate.

The time histories for each of these sine waves are given in Figure 3. Note that the 1800 Hz sine wave appears to equal a 200 Hz sine wave with a 180 degree phase difference.

Now consider a 3600 Hz sine wave sampled at 2000 samples per seconds. The power spectral density is shown in Figure 4. A spectral line appears at 400 Hz. Again, this error occurs due to inadequate sampling rate.

The time histories of the 1800 Hz sine wave and the 3600 Hz sine wave are shown together in Figure 5.

The alias frequency is summarized in equation (1).

$$\text{Alias frequency} = \begin{cases} S_f - E_f, \text{ if } \frac{1}{2} S_f < E_f < S_f \\ E_f - m S_f, \text{ if } m S_f < E_f < \left(m + \frac{1}{2} \right) S_f, m = 1,2,3,4,... \\ (m+1) S_f - E_f, \text{ if } \left(m + \frac{1}{2} \right) S_f < E_f < (m+1) S_f, \end{cases} \tag{1}$$

where

S_f is the sample rate

E_f is the energy frequency

(1)

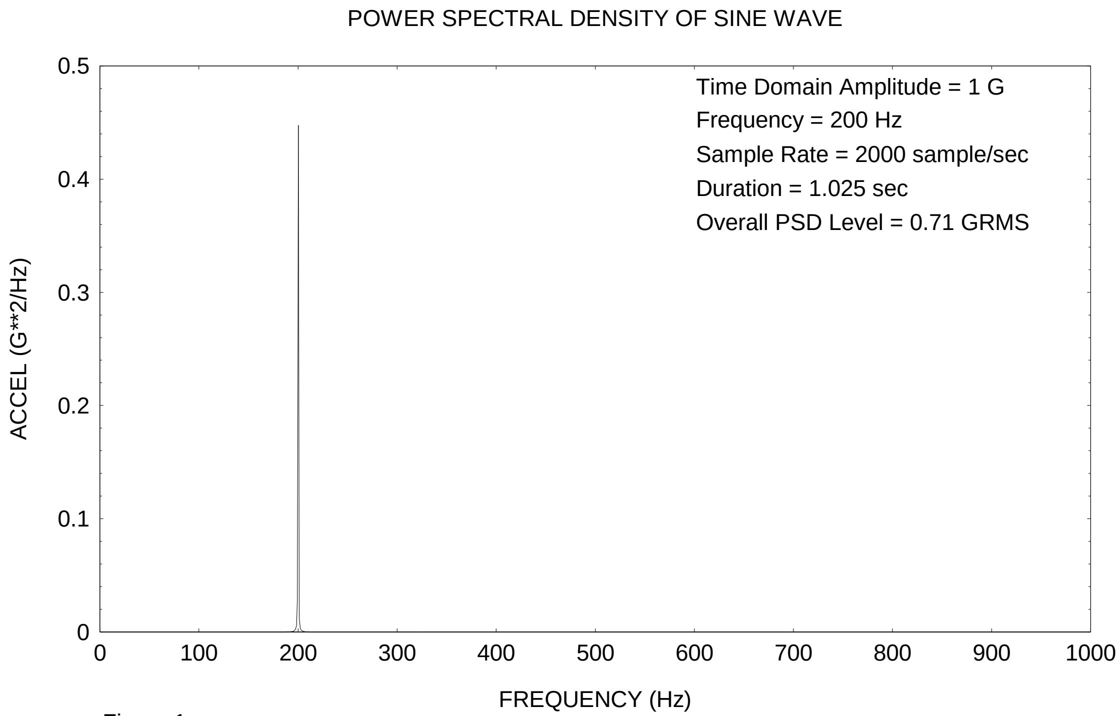


Figure 1.

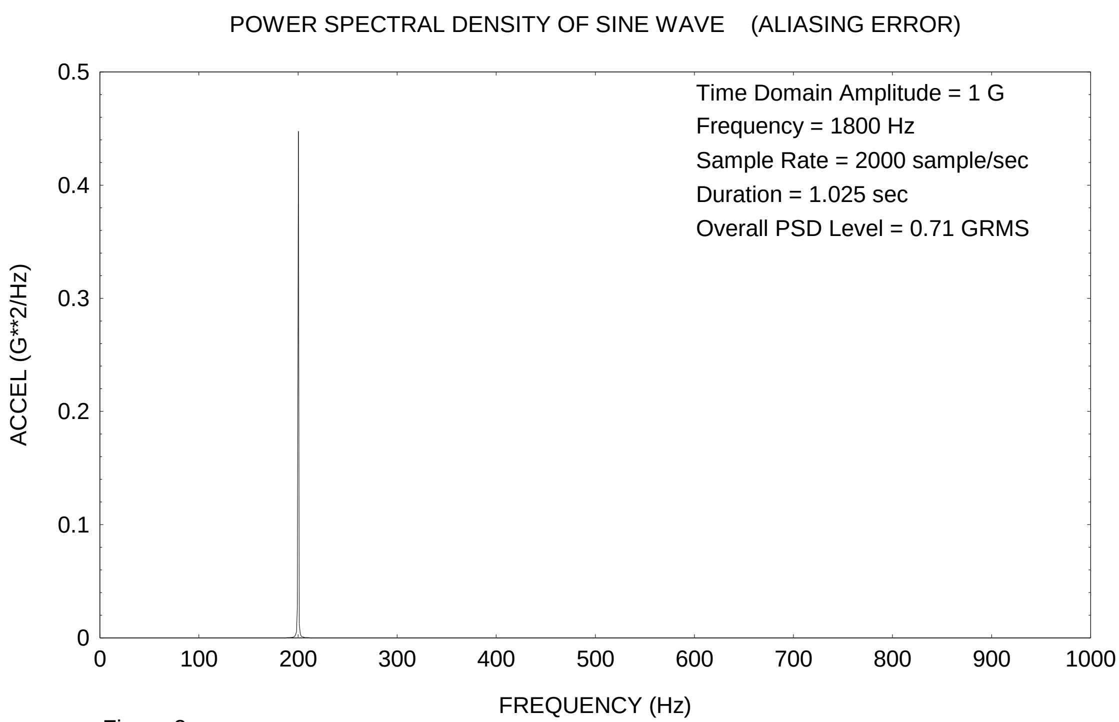


Figure 2.

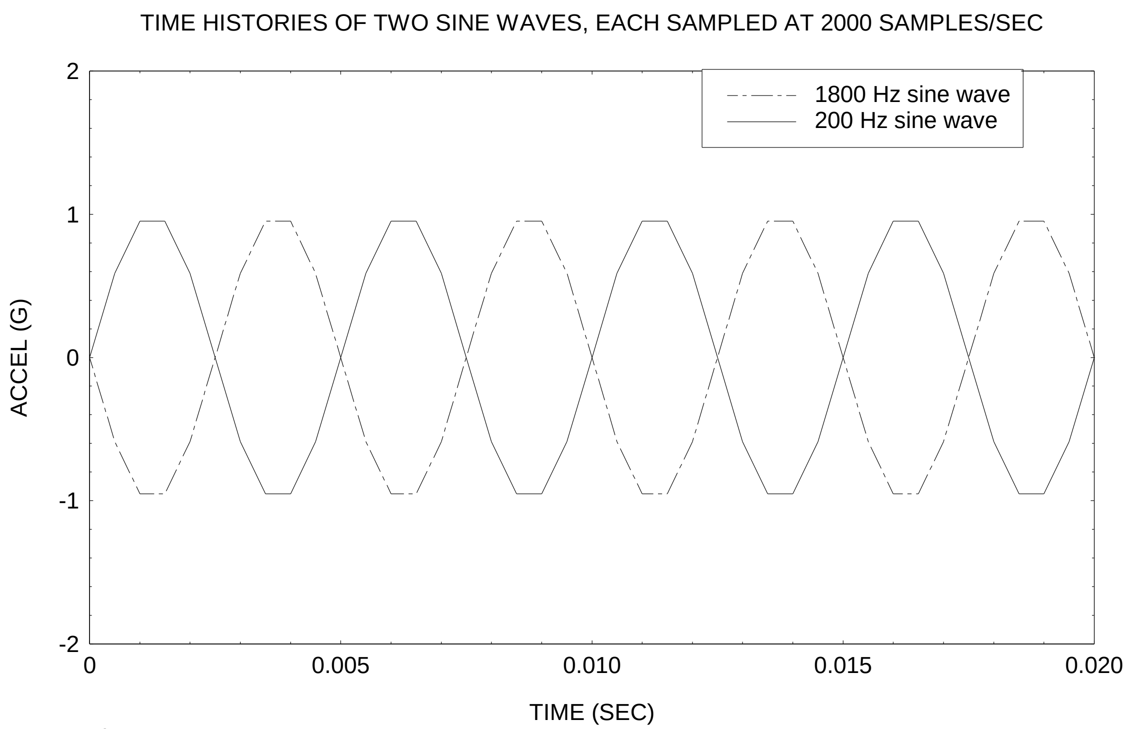


Figure 3.

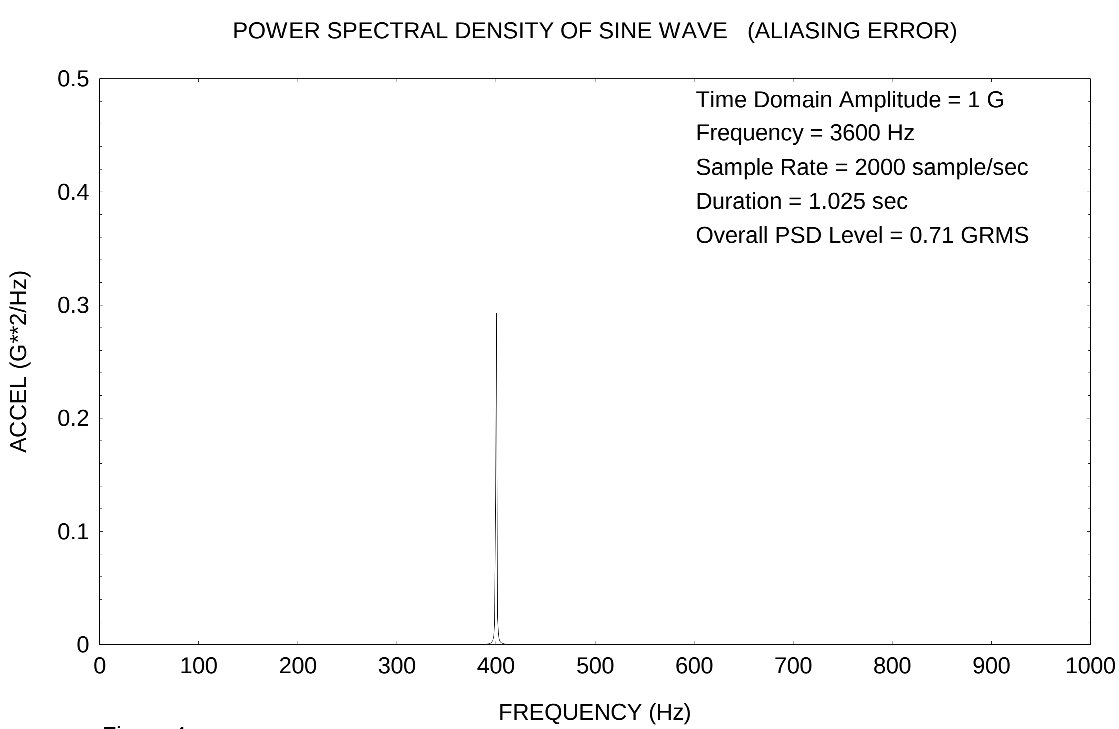


Figure 4.

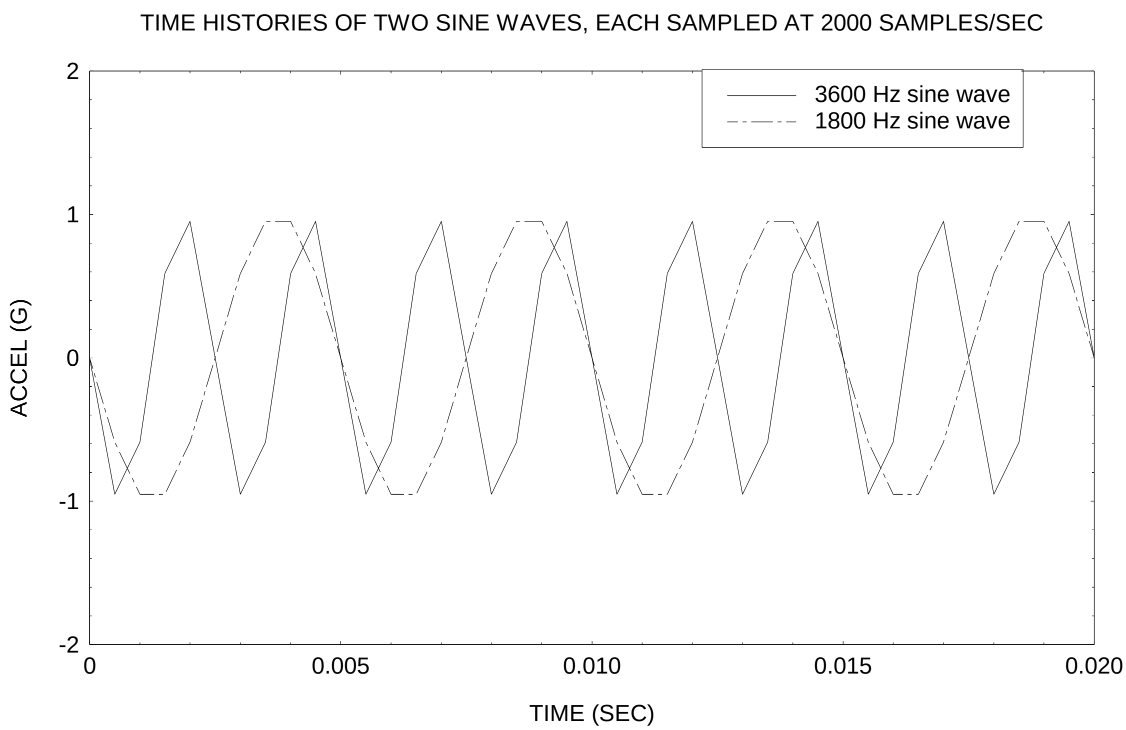


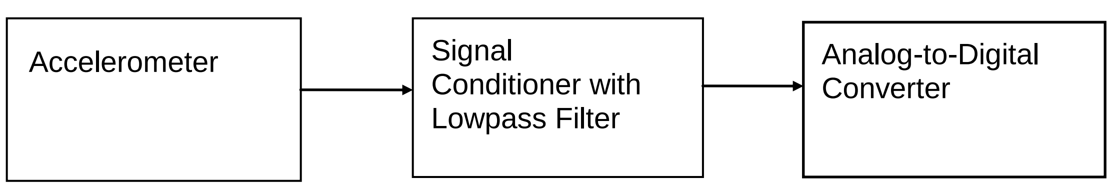
Figure 5.

LOWPASS FILTERING

Aliasing can be prevented by lowpass filtering the analog data.

Consider a stage separation test or a launch vehicle flight. The maximum expected frequency in the source energy is essentially unknown. Thus, there is no proper means to set the sampling rate, other than setting it at some exceedingly high value.

The simple solution is to pass the analog data through a lowpass filter as shown in the flowchart.



The filter can be part of the signal conditioning system. Typically, a Butterworth filter is used. The Butterworth filter has a roll-off which attenuates the signal by 3 dB at the cut-off frequency. Further details are given in Reference 3.

The cut-off frequency is typically set at, or slightly above, the maximum analysis frequency.

The IES Handbook for Dynamic Data Acquisition and Analysis gives the following guidelines:

Let

f_C be the cutoff frequency

f_N be the Nyquist frequency

1. A lowpass anti-aliasing filter with a cutoff rate of at least 60 dB/octave should be used for the analog-to-digital conversion of all dynamic data.

2. With a 60 dB/octave cutoff rate, the half-power point cutoff frequency of the filter should be set at $f_C \leq 0.6 f_N$.

If the anti-aliasing filter has a more rapid cutoff rate, a higher cutoff frequency can be used, but the bound $f_C \leq 0.8 f_N$ should never be exceeded.

SAMPLE RATE EXAMPLE

Ideally, the sampling rate could be chosen after the maximum frequencies were identified. Practical considerations often require a reverse approach.

Consider a telemetry system for a launch vehicle. Several accelerometers will be mounted in the vehicle. The data will be digitized on-board the vehicle. The digitized signal will be sent via a radio link to a ground station.

The flight dynamic environments are unknown. The maximum sampling rate, however, is 4000 samples per second for each accelerometer channel. This sampling rate is constrained by the available radio link bandwidth and other considerations.

Given this constraint, choose an analog lowpass filter with a cut-off frequency at 2000 Hz. This frequency is somewhat higher than that of the IES recommendation, which would be 1200 to 1600 Hz depending on the filter roll-off. The choice of 2000 Hz as the cut-off frequency is a compromise because the maximum frequency of interest is 2000 Hz.

Place the filter between the accelerometer and the vehicle's analog-to-digital converter.

Now consider that the vehicle has flown and the digital data has been received at the ground station.

Power spectral density functions of the flight data can be calculated up to 2000 Hz, per Table 1. Some roll-off may appear starting at about 1600 Hz depending on the filter characteristics, but this is a practical trade-off.

Shock response spectra of the flight data can be calculated accurately up to 200 Hz, per Table 1. This frequency can be extended somewhat if greater error margins are allowed. Interpolations of the data can reduce this error somewhat as mentioned in Reference 1.

This telemetry system will thus yield usable vibration data.

On the other hand, the telemetry system will yield marginal shock data. The resulting shock data may be adequate to characterize motor ignition and launch shock. Unfortunately, the data will be inadequate to characterize near-field and mid-field shock from stage separation events.

Stage separation shock must thus be measured during ground development tests prior to flight. Data acquisition systems with high sampling rates can be used during ground tests.

ADDITIONAL PARAMETERS

Amplitude Resolution

Analog-to-digital conversion systems have an amplitude resolution which is measured in bits.

The amplitude resolution is one part in 2^(number of bits). Thus, a 12-bit system has a resolution of one part in 4096.

Consider a 12-bit system set up to measure a full scale amplitude of 10 V peak-to-peak. The resolution is (10 V / 4096) = 2.4 mV. This example is shown in Table 3, along with two other bit cases.

Table 3. Example for 10 V peak-to-peak	
Bit Resolution	Voltage Resolution (mV)
8	39.1
12	2.4
16	0.15

<u>References</u>

1. OSC ME File: MISC 030-074, Shock Response Spectra Sampling Rate Criteria, 1997.
2. IES Handbook for Dynamic Data Acquisition and Analysis, Institute of Environmental Sciences, Illinois. *Particularly, paragraphs 3.7.2 and 5.5.3.5.*
3. Stearns and David, Signal Processing Algorithms in Fortran and C, Prentice-Hall, New Jersey, 1993.

APPENDIX A

ALIASING CASE HISTORY

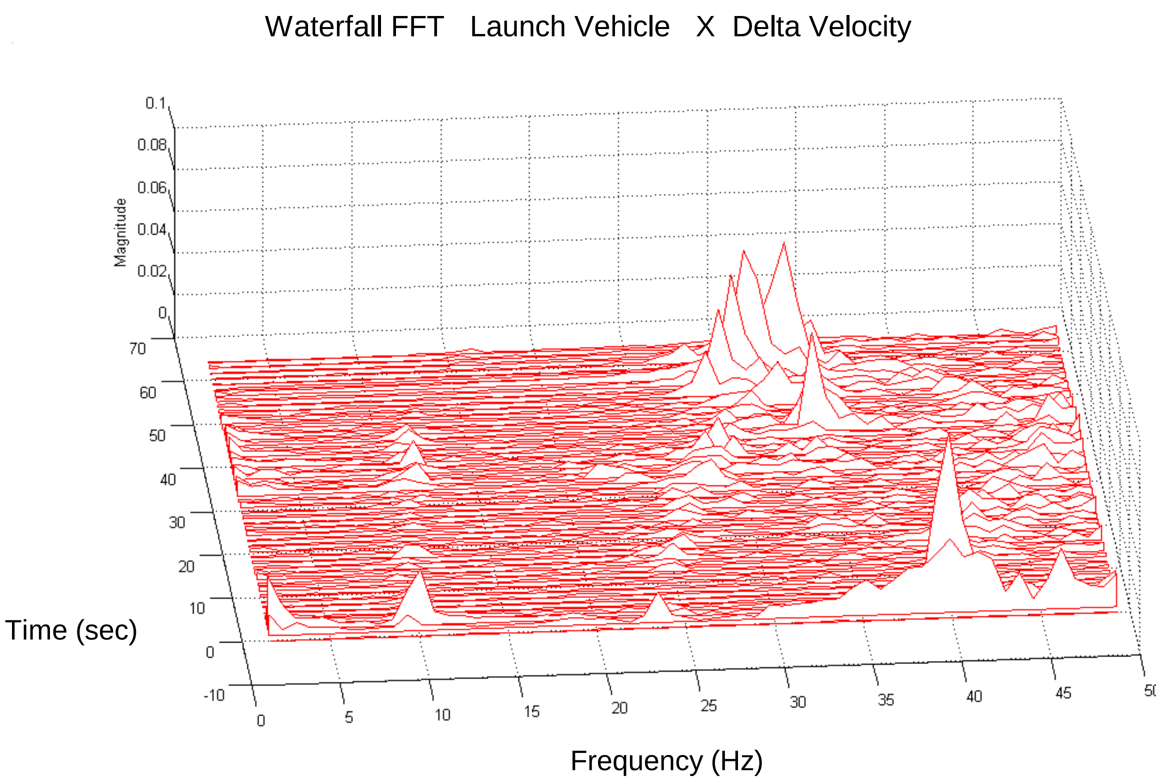


Figure A-1.

The sensor was from an Inertial Navigation System (INS).

The data was sampled at 100 samples per second with no anti-aliasing filter. The waterfall FFT is given up to 50 Hz, which is the Nyquist frequency.

The spectral peaks from 25 to 35 Hz and from 50 to 60 seconds are due to aliasing about the Nyquist frequency. The source energy was a motor pressure oscillation that swept downward from 75 to 65 Hz.

As an aside, the spectral peaks near 10 Hz were due to the fundamental body bending frequency.