THE AUTOCORRELATION FUNCTION
Revision A

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Introduction

Cross-correlation is a measure of similarity of two waveforms as a function of a time-lag applied to one of them.

Autocorrelation is the cross-correlation of a signal with itself. It is a time domain analysis useful for determining the periodicity or repeating patterns of a signal.

Formulas and Properties

The autocorrelation $R(\tau)$ for a continuous function x(t) is

$$R(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\,x(t+\tau)\,dt \tag{1}$$

where

τ is the delay

T is the signal period

The autocorrelation at zero delay is

$$R(0) = \sigma^2 + \mu^2 \tag{2}$$

where

σ is the standard deviation

μ is the mean

Note that the autocorrelation function is symmetric about the x=0 line.

$$R(-\tau) = R(\tau) \tag{3}$$

The autocorrelation for a finite, discrete function is

$$R_n = \frac{1}{M}\sum_{m=1}^{M} x_m \, x_{n+m} \quad , \ n=1, 2, 3, \ldots , M \tag{4}$$

where M is the total number of points.

Examples are given in the appendices.

References

1. W. Thomson, Theory of Vibrations with Applications, Second Edition, Prentice-Hall, New Jersey, 1981.
2. D. Newland, An Introduction to Random Vibrations, Spectral & Wavelet Analysis, Third Edition, Dover, New York, 1993.

APPENDIX A

<u>Sine Function</u>

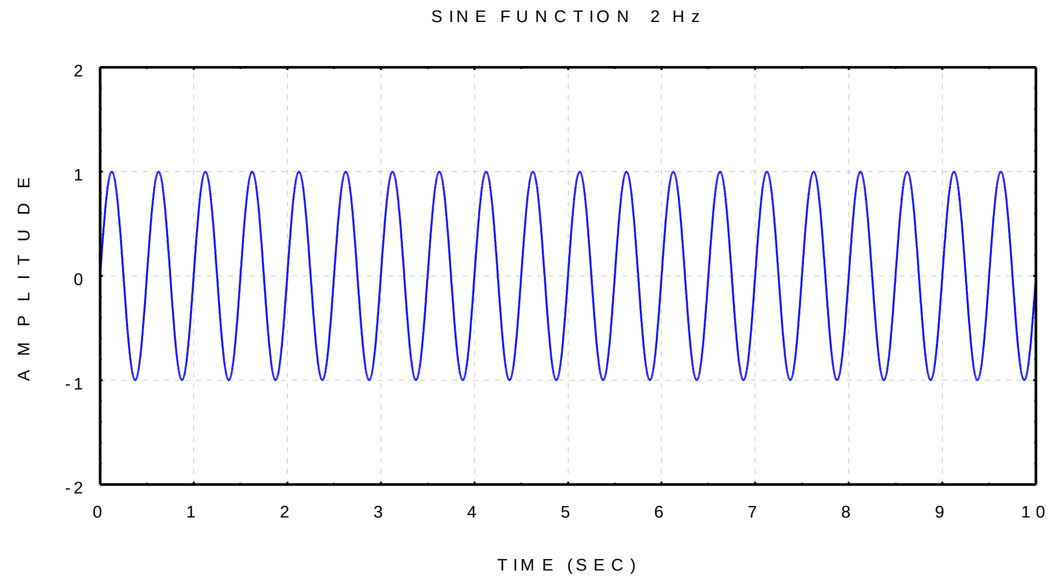


Figure A-1.

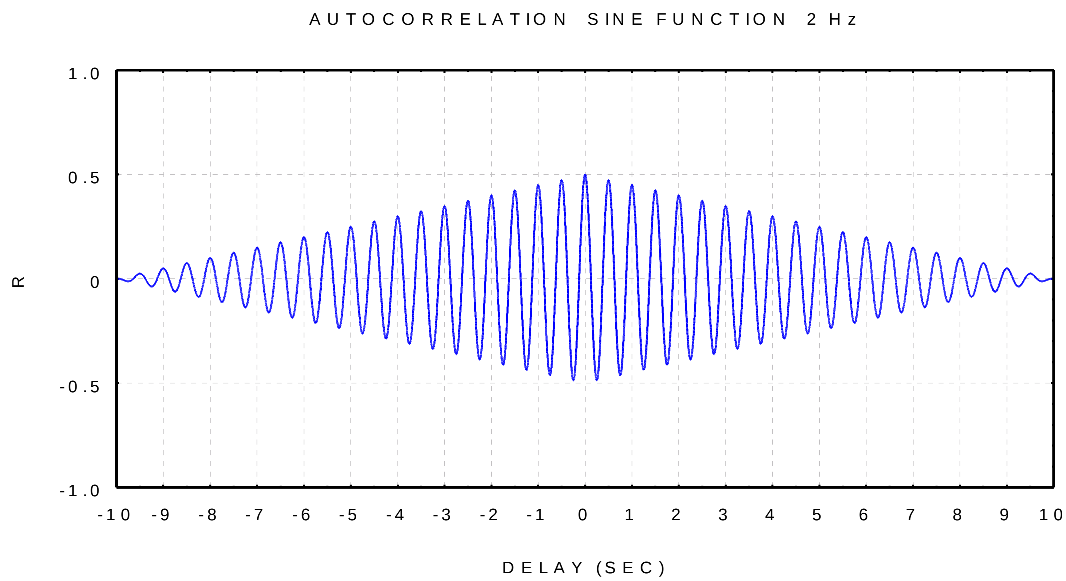


Figure A-2.

The envelope of the autocorrelation function has a piecewise linear variation because the input function has a finite duration. The 2 Hz frequency is otherwise apparent in the autocorrelation function.

The autocorrelation of an infinitely long sine function would itself be a sine function.

APPENDIX B

<u>Broadband Random</u>

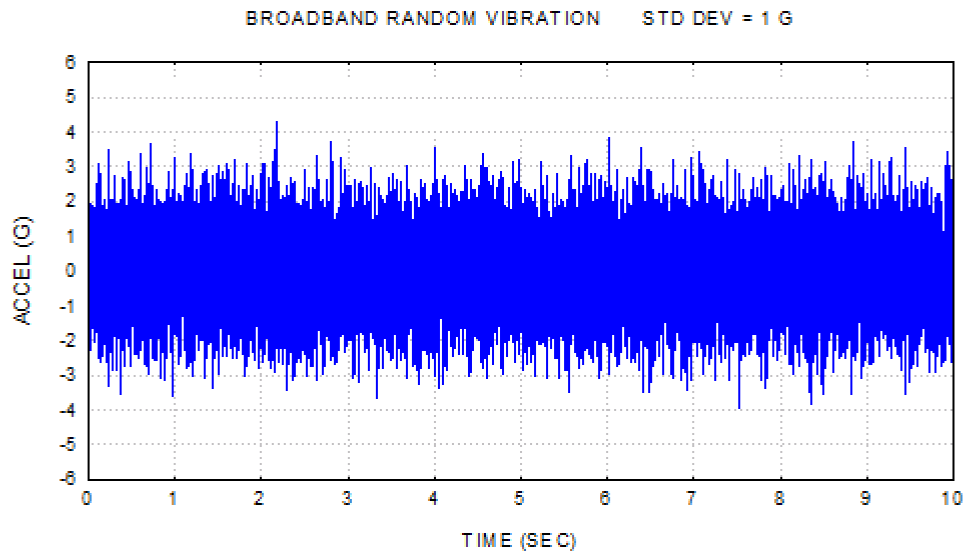


Figure B-1.

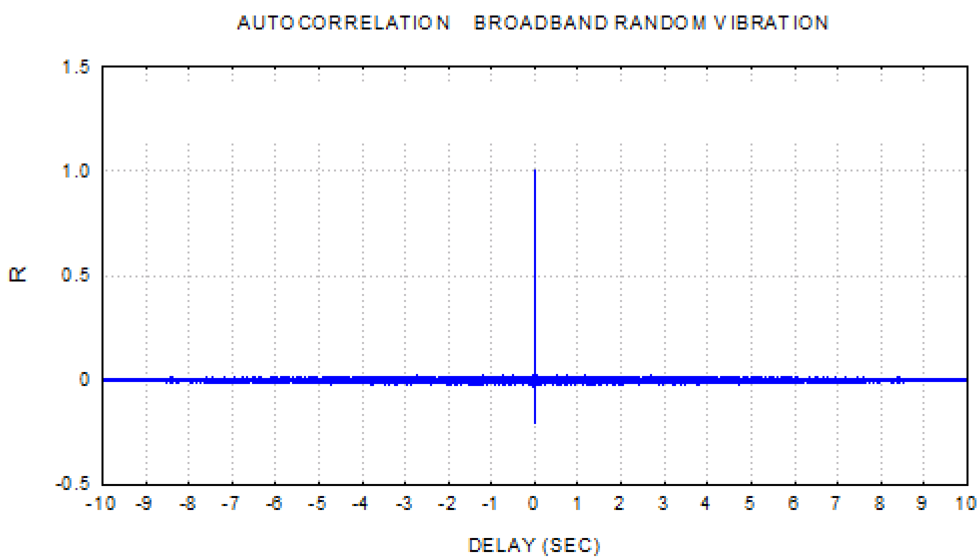


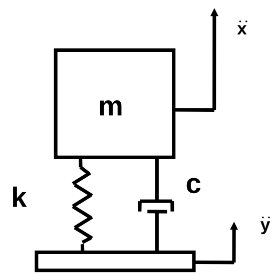
Figure B-2.

A white noise time history was synthesized with a sample rate of 4000 samples/sec. It was then lowpass filtered at 1000 Hz. Then it was scaled to have a standard deviation of 1 G. The time history and autocorrelation plots are shown in Figures B-1 and B-2 respectively.

Idealized, continuous white noise would have an autocorrelation function represented by the Dirac delta function at delay $\tau = 0$.

APPENDIX C

Narrowband Random



m mass

c damping coefficient

k stiffness

$\ddot{x}$ acceleration of mass

$\ddot{y}$ base acceleration

Figure C-1. Single-degree-of-freedom System

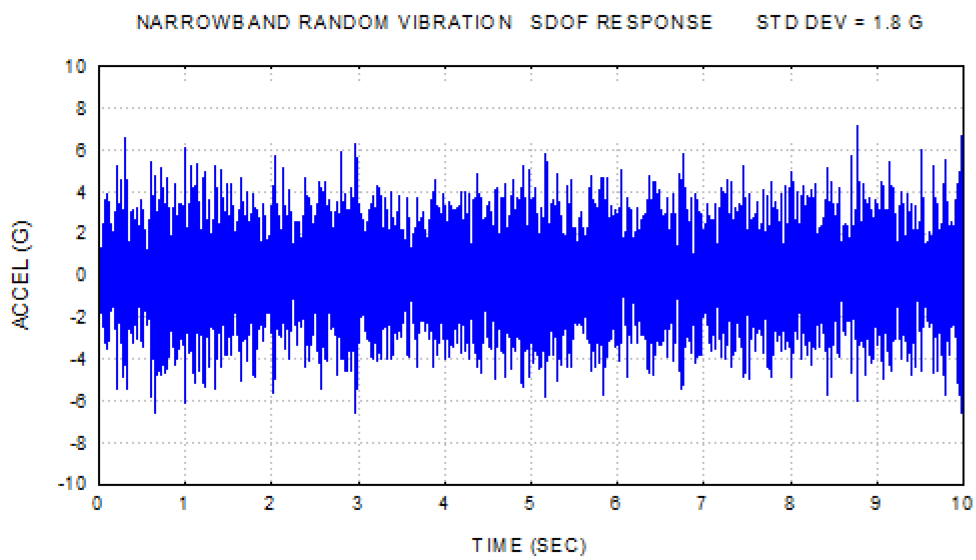


Figure C-2.

The single-degree-of-freedom in Figure C-1 is subjected to the acceleration base input from Figure B-1. The system has a natural frequency of 200 Hz and an amplification factor of Q=10. The response of the system is narrowband random, as shown in Figure C-2.

The autocorrelation function is shown in Figure C-3. A close-up view is shown in Figure C-4.

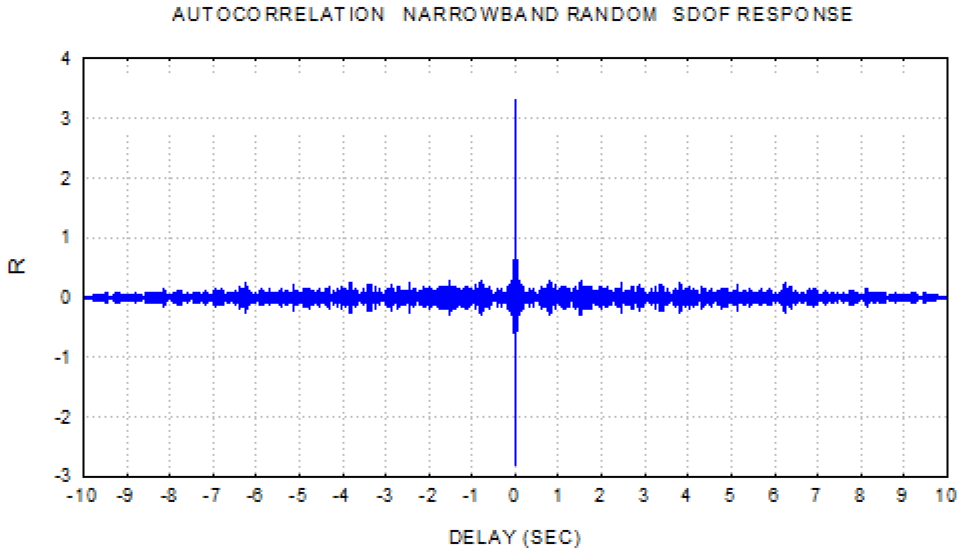


Figure C-3.

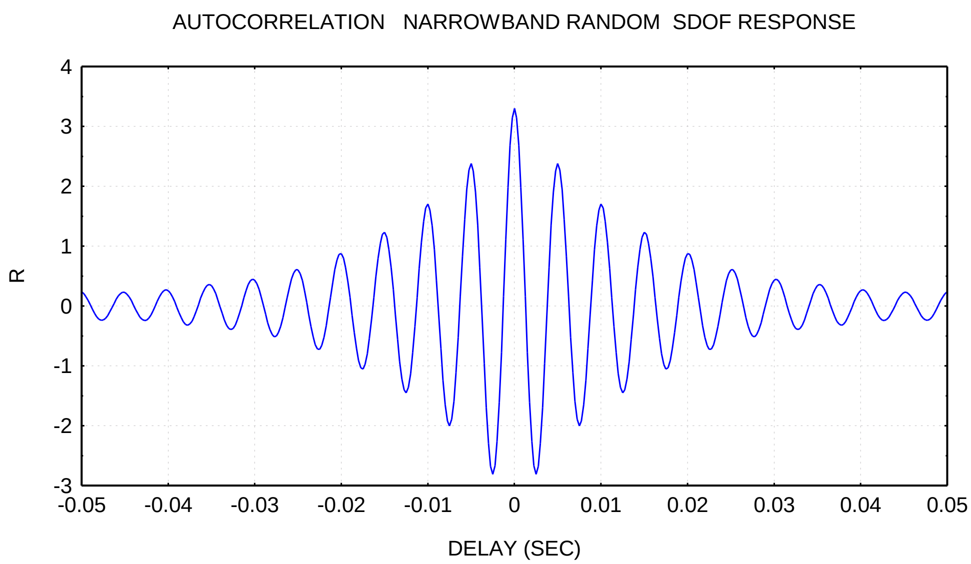


Figure C-4.

The standard deviation of the time history is 1.8 G. The mean is zero.

The autocorrelation peak is: R(0)=3.2

Note that 1.8^2 = 3.2.

The first peak after time zero occurs at 0.005 seconds, which is the period of the 200 Hz system.