ACCELERATION, VELOCITY, AND DISPLACEMENT POWER SPECTRAL DENSITY FUNCTIONS Revision B

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Introduction

Mechanical vibration is usually characterized in terms of acceleration. The main reason is that acceleration is easier to measure than velocity or displacement, in the context of vibration.

Acceleration time histories may be converted to power spectral density functions for the purpose of deriving test specifications. A typical example is the MIL-STD-1540C acceptance level as shown in Figure 1 and in Table 1.

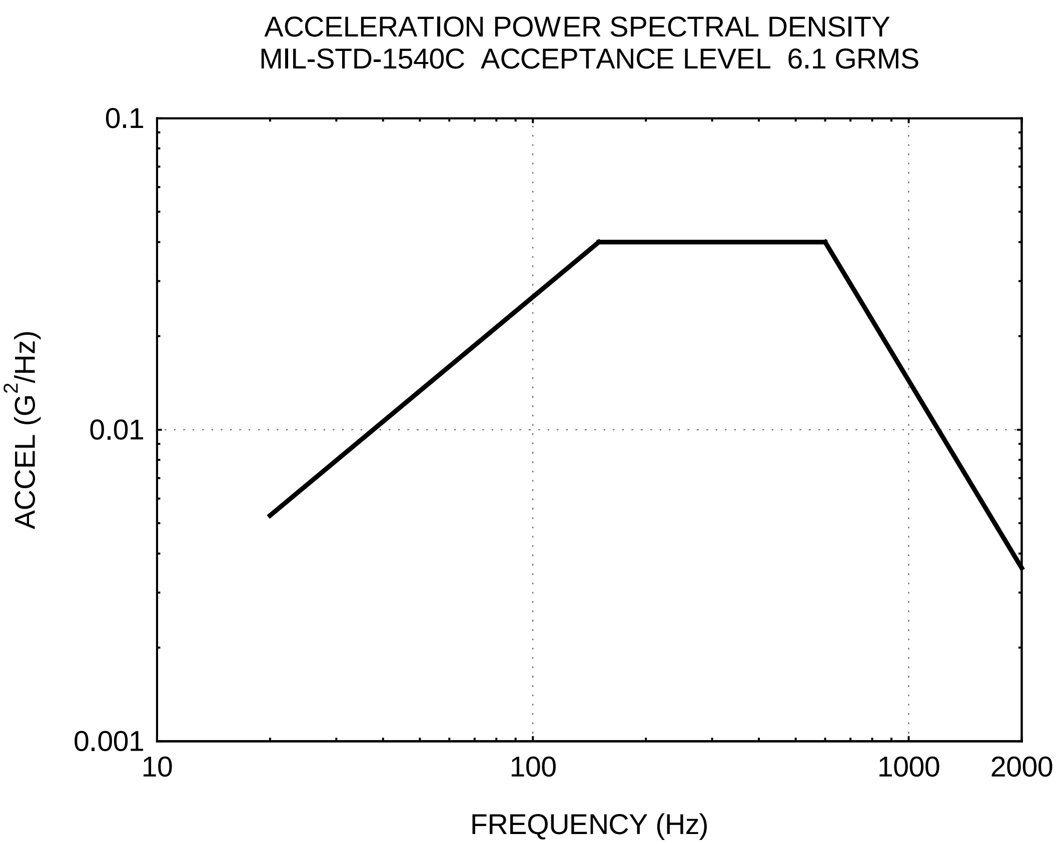

Figure 1.

Table 1. MIL-STD-1540C Acceptance Level, 6.1 GRMS Overall	
Frequency (Hz)	PSD (G^2/Hz)
20	0.0053
150	0.04
600	0.04
2000	0.0036

The curve in Figure 1 is an acceleration power spectral density function. The curve can be integrated to determine the overall GRMS value, as explain in Reference 1.

The same curve may also be integrated, through a separate method, to determine the velocity power spectral density and the displacement power spectral density. The purpose of this report is to describe this method.

Integration Method

Consider a single-degree-of-freedom system undergoing sinusoidal excitation. The displacement amplitude x(t) is

$$x(t) = X \sin(\omega t) \tag{1}$$

where

X is the displacement
ω is the frequency (radians/time)

The velocity $\dot{x}(t)$ is obtained by taking the derivative.

$$\dot{x}(t) = \omega X \cos(\omega t) \tag{2}$$

The acceleration $\ddot{x}(t)$ is obtained by taking the derivative of the velocity.

$$\ddot{x}(t) = -\omega^2 X \sin(\omega t) \tag{3}$$

The relationships are summarized in Tables 2 and 3.

Table 2. Peak Values Referenced to Peak Displacement.	
Parameter	Equation
Displacement	$x_{peak} = X$
Velocity	$\dot{x}_{peak} = \omega X$
Acceleration	$\ddot{x}_{peak} = \omega^2 X$

Note that

$$\ddot{x}_{peak} = \omega^2 x_{peak} \tag{4}$$

Now let A be the peak acceleration. The relationships in Table 3 can be derived via algebra.

Table 3. Peak Values Referenced to Peak Acceleration	
Parameter	Equation
Displacement	$x_{peak} = A / \omega^2$
Velocity	$\dot{x}_{peak} = A / \omega$
Acceleration	$\ddot{x}_{peak} = A$

The relationships in Tables 2 and 3 can be applied to power spectral density functions. Recall that a power spectral density functions have dimension of [(amplitude^2) / Hz]. Thus, the appropriate ω scale factor must be squared.

Let

DPSD = displacement power spectral density
VPSD = velocity power spectral density
APSD = acceleration power spectral density

Note that each PSD function is a function of the frequency f. Furthermore, the angular frequency is $\omega = 2\pi f$.

The resulting relationships for the power spectral density functions are shown in Tables 4 and 5.

Table 4. PSD Functions Referenced to Displacement PSD	
Parameter	Equation
VPSD	$VPSD = \omega^2 \, DPSD$
APSD	$APSD = \omega^4 \, DPSD$

Table 5. PSD Functions Referenced to Acceleration PSD	
Parameter	Equation
DPSD	$DPSD = APSD / \omega^4$
VPSD	$VPSD = APSD / \omega^2$

The steps in Table 4 are actually differentiation steps. Those in Table 5 are integration steps.

Again, each PSD function is a function of frequency. The integration or differentiation must be performed at each breakpoint frequency.

Example 1

The integration method can easily be performed using a computer program or Excel spreadsheet.

Calculate the velocity power spectral density from the acceleration power spectral density in Figure 1. The results are shown in Table 6 and in Figure 2.

Table 6. Integrate APSD to Determine VPSD

Frequency (Hz)	APSD (G^2/Hz)	ω (rad/sec)	ω^2 (rad/sec)^2	VPSD [(G sec)^2]/Hz	VPSD [(in/sec)^2]/Hz
20	0.0053	1.26E+02	1.58E+04	3.36E-07	5.00E-02
150	0.04	9.42E+02	8.88E+05	4.50E-08	6.71E-03
600	0.04	3.77E+03	1.42E+07	2.81E-09	4.19E-04
2000	0.0036	1.26E+04	1.58E+08	2.28E-11	3.40E-06

Note that a unit conversion is necessary to obtain the values in the last column.

Furthermore, the velocity power spectral density can be integrated to determine the overall velocity level from the area under the curve.

Example 2

Calculate the displacement power spectral density from the acceleration power spectral density in Figure 1. The results are shown in Table 7 and in Figure 3.

Table 7. Integrate APSD to Determine DPSD

Frequency (Hz)	APSD (G^2/Hz)	ω (rad/sec)	ω^4 (rad/sec)^4	DPSD [(G sec^2)^2]/Hz	DPSD [in^2]/Hz
20	0.0053	1.26E+02	2.52E+08	2.10E-11	3.13E-06
150	0.04	9.42E+02	7.87E+11	5.08E-14	7.57E-09
600	0.04	3.77E+03	2.02E+14	1.98E-16	2.95E-11
2000	0.0036	1.26E+04	2.52E+16	1.43E-19	2.13E-14

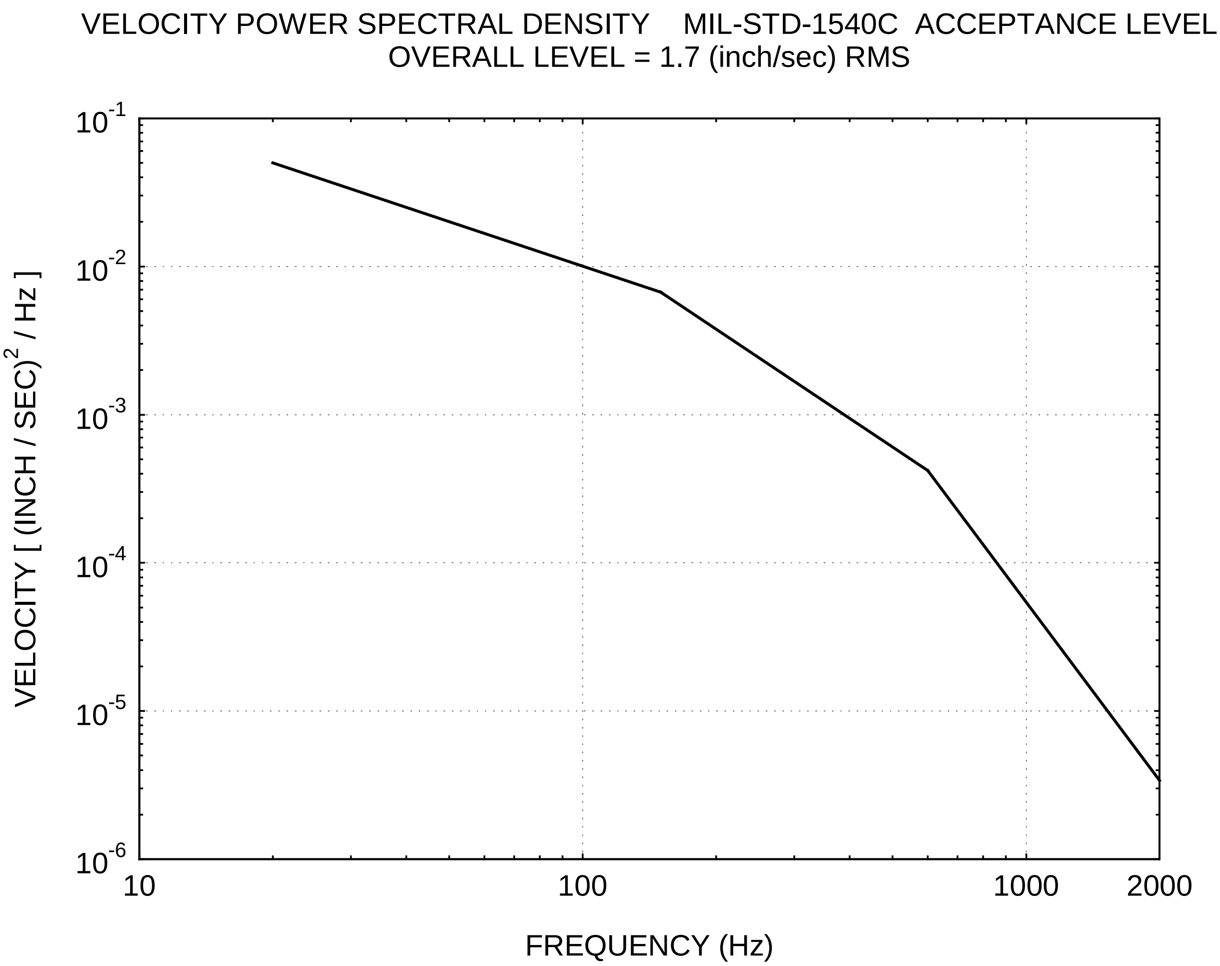
VELOCITY POWER SPECTRAL DENSITY MIL-STD-1540C ACCEPTANCE LEVEL
OVERALL LEVEL = 1.7 (inch/sec) RMS
VELOCITY [(INCH / SEC)^2 / Hz]
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10
100
1000
2000
FREQUENCY (Hz)

Figure 2.

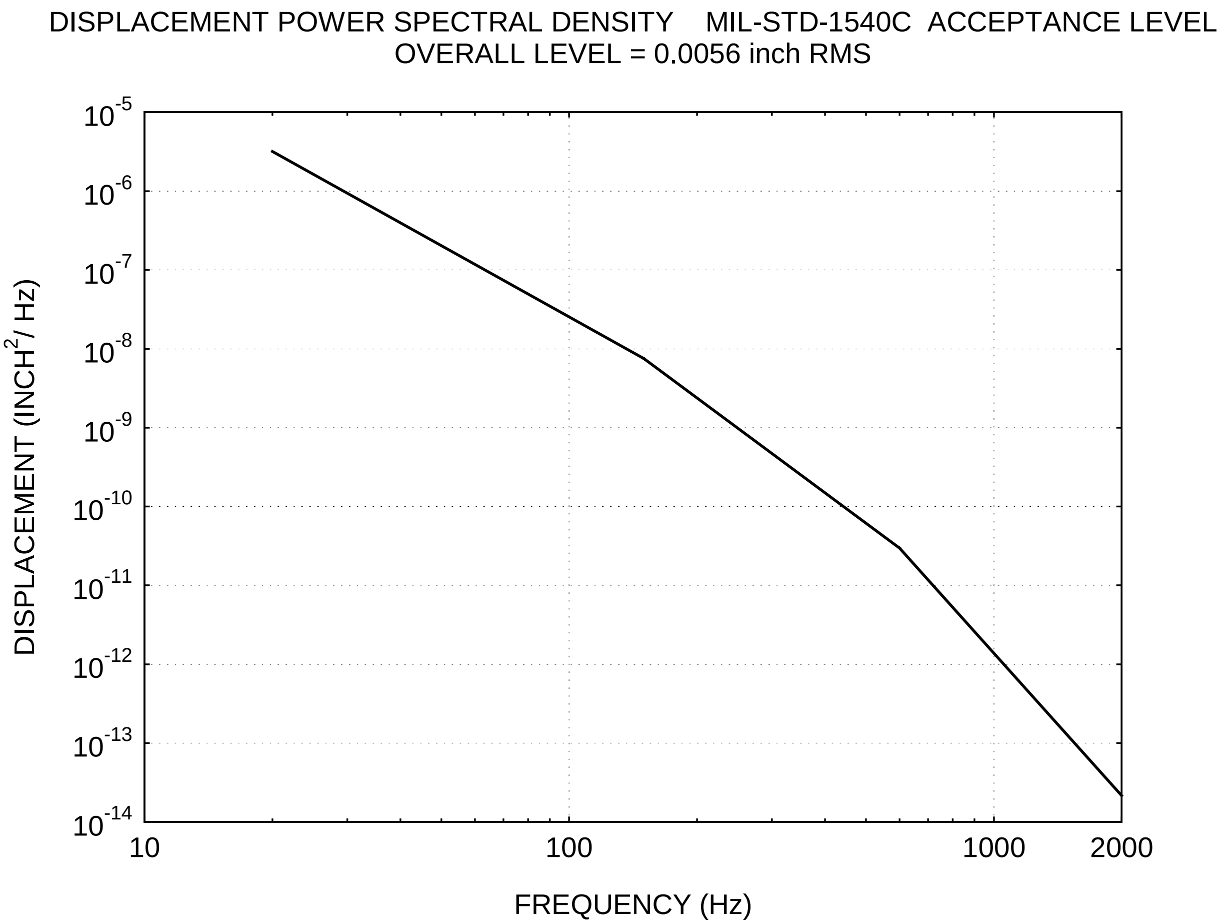

Figure 3.

Note that integration is analogous to a lowpass filtering operation. Thus, the displacement above, say, 100 Hz is negligibly low.

Reference

1. T. Irvine, Integration of the Power Spectral Density Function, Vibrationdata.com Publications, 2000.