User Guide and Examples for Matlab Script beam_3D.m

By Tom Irvine

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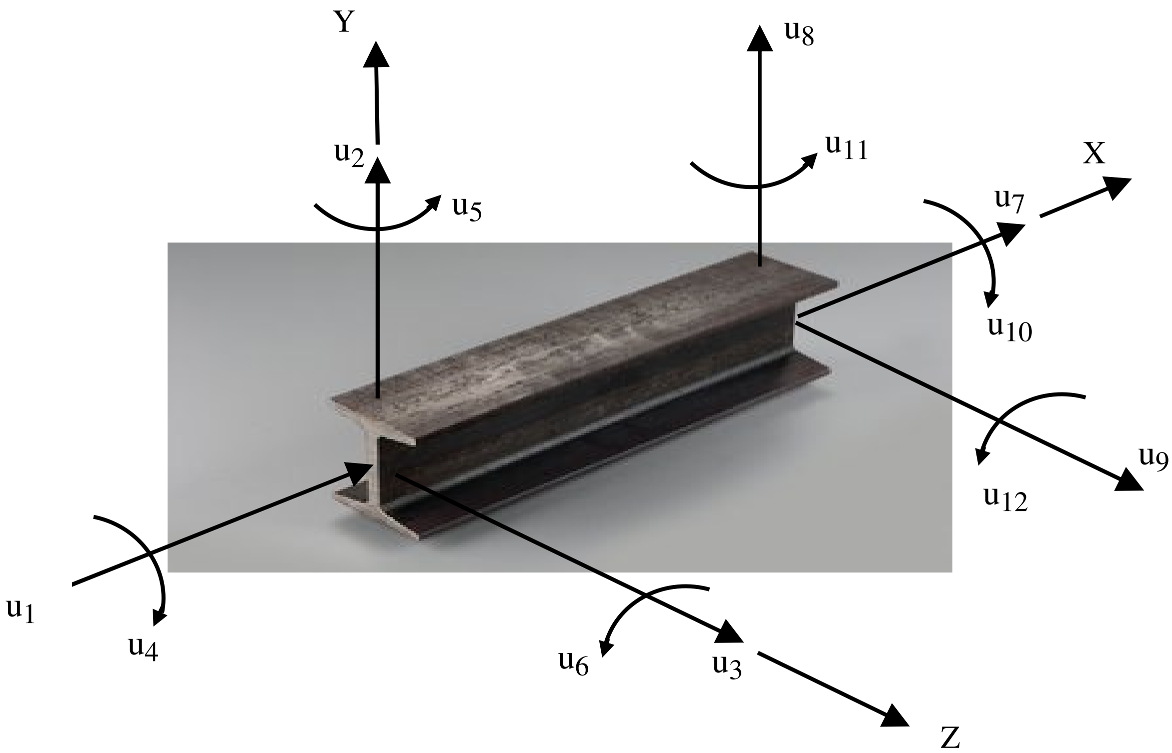

Figure 1. Three-dimension Beam Element, Degrees-of-freedom

Example: Cantilever Beam with End Mass

Consider an aluminum beam with solid circular cross section.

Properties

L	Length	12 inch	
D	Diameter	1 inch	
A	Area	0.7854 in^2	
Iyy	area MOI	0.04909 in^4	
Izz	area MOI	0.04909 in ^4	
E	elastic modulus	1e+07 lbf/in^2	
$\hat{\rho}$	mass/volume	0.1 lbm/in^3	0.000259 lbf sec^2/in^4
ρ	mass/length	0.07854 lbm/in	0.00020338 lbf sec^2/in^2
ρL	Beam mass without end mas	0.942 lbm	0.0024402 lbf sec^2/in
Poisson	Ratio	0.3	
M	end mass	0.5 lbm	0.0012952 lbf sec^2/in

Hand Calculation

The natural frequency from Reference 1 is

$$\mathrm{fn} \approx \frac{1}{2\pi}\sqrt{\frac{3EI}{(0.2235\rho L + m)L^3}} \tag{1}$$

$$\mathrm{fn} \approx \frac{1}{2\pi}\sqrt{\frac{3\,(1e+07\,\mathrm{lbf/in^{\wedge}2})\,(0.04909\,\mathrm{in^{\wedge}4})}{\left(0.2235(0.00244\,\mathrm{lbf\ sec^{\wedge}2/in}) + (0.001295\,\mathrm{lbf\ sec^{\wedge}2/in})\right)(12\,\mathrm{in})^3}} \tag{2}$$

$$\mathrm{fn} \approx 108.4\ \mathrm{Hz} \tag{3}$$

FEA Model via Matlab script beam_3D.m

Use 6 equally spaced elements, with 7 nodes total.

The node array is:

na= [0 0 0; 2 0 0; 4 0 0; 6 0 0; 8 0 0; 10 0 0; 12 0 0]

The element array is

ea = [1 2; 2 3; 3 4; 4 5; 5 6; 6 7]

The finite element output is

```
 beam_3D.m,  ver 1.2, March 4, 2010

 by Tom Irvine

 This script calculates the natural frequencies of a system of beams.
 via the finite element method.

 Select units
 1=English  2=metric
 1

 mass(lbm), stiffness(lbf/in), length(in), mass density(lbm/in^3)

 The nodal coordinates must have three columns: X, Y & Z  (inches)
 Select the nodal coordinate input method

 Select data input method
   1=external ASCII file
   2=file preloaded into Matlab
   3=Excel file
2
 Enter the node matrix name:  na

       Node Table
 Number    X     Y     X
    1      0     0     0
    2      2     0     0
    3      4     0     0
    4      6     0     0
    5      8     0     0
    6      10     0     0
    7      12     0     0

 The element file must have two columns: node1 node2  (integers)
 Select the element input method
```

```
Select data input method
   1=external ASCII file
   2=file preloaded into Matlab
   3=Excel file
2
 Enter the element matrix name:  ea

       Element Table
 Number    N1    N2   Length
    1         1     2           2
    2         2     3           2
    3         3     4           2
    4         4     5           2
    5         5     6           2
    6         6     7           2

 Assume uniform material

 Enter material:
 1=aluminum  2=steel  3=other 1

 Assume uniform cross-section geometry

 Axis convention for a rectangular beam is:
 X is length.   Y=width.   Z=thickness

 Enter the cross-section:
 1=rectangle  2=solid cylinder  3=other 2

 Enter the diameter (inch) 1

 Add point mass?
 i=yes  2=no
 1

 Enter number of point masses
 1
 Enter node number
 7
 Enter mass (lbm)0.5

 Apply Translation Constraints?
 1=yes  2=no
 1

 Enter node number to apply constraint 1

 Select dofs to constrain
 1=TX
 2=TY
 3=TZ
 4=TX & TY
 5=TX & TZ
 6=TY & TZ
 7=TX,TY,TZ
 7
```

```
 Apply another translational constraint?
 1=yes  2=no
 2

 Apply Rotational Constraints?
 1=yes  2=no
 1

 Enter node number to apply constraint 1

 Select dofs to constrain
 1=RX
 2=RY
 3=RZ
 4=RX & RY
 5=RX & RZ
 6=RY & RZ
 7=RX,RY,RZ
 7

 Apply another rotational constraint?
 1=yes  2=no
 2

 Apply mass condensation?
 1=yes  2=no
 1

 Enter number of degrees-of-freedom
 to retain in reduced model
 10

Natural Frequencies
 No.      f(Hz)
1.        107.27
2.        107.27
3.        914.09
4.        917.51
5.        2803.6
6.          2824
7.        3086.8
8.        6105.3
9.        8514.3
10.         12038
```

Recall that the hand calculation fundamental frequency was

$$fn \approx 108.4 \text{ Hz} \tag{4}$$

Thus the two methods produce nearly the same fundamental frequency.

References

1. T. Irvine, Bending Frequencies of Beams, Rods, and Pipes, Revision K, Vibrationdata, 2004.
2. T. Irvine, Table of Spring Stiffness, Rev B, Vibrationdata, 1999.
3. R. Blevins, Formulas for Natural Frequency and Mode Shapes, R, Krieger, Malabar, Florida, 1979.

APPENDIX A

Example: Fixed-Free, Rod

Consider an aluminum rod with solid circular cross section. Determine the torsional fundamental frequency. All translational degrees-of-freedom are fixed. The rotational degrees-of-freedom at the fixed boundary are also fixed, obviously.

Properties

L	Length	12 inch	
D	Diameter	2 inch	
A	Area	3.142 in^2	
J	Polar Area MOI	1.571 in^4	
Jm	Polar Mass MOI	0.004883 lbf sec^2/in	
E	elastic modulus	1e+07 lbf/in^2	
G	shear modulus	3.76e+06 lbf/in^2	
$\hat{\rho}$	mass/volume	0.1 lbm/in^3	0.000259 lbf sec^2/in^4
ρ	mass/length	0.3142 lbm/in	0.000814 lbf sec^2/in^2

Hand Calculation

The natural frequency from References 2 and 3 is

$$k_r = \frac{G\,J}{L} \tag{A-1}$$

$$k_r = (3.76e+06 \ \ \text{lbf/in\^{}2})(1.571\,\text{in\^{}4})/(12\,\text{in}) \tag{A-2}$$

$$k_r = 4.9210e{+}05 \ \ \text{lbf/in} \tag{A-3}$$

$$fn = \frac{1}{2\pi}\left(\frac{\pi}{2}\right)\sqrt{\frac{k_r}{J_m}} \tag{A-4}$$

$$fn = \frac{1}{2\pi}\left(\frac{\pi}{2}\right)\sqrt{\frac{4.9210e+05 \ \text{lbf/in}}{0.004883\,\text{lbf sec\^{}2/in}}} \tag{A-5}$$

$$fn = 2510 \text{ Hz} \tag{A-6}$$

Matlab Analysis

Use 4 equally spaced elements, with 5 nodes total.

The node array is:

```
nne =

    [0      0      0;
     3      0      0;
     6      0      0;
     9      0      0;
    12      0      0]
```

The element array is

```
elem =

     [1      2;
     2      3;
     3      4;
     4      5]
```

Matlab output excerpts:

```
beam_3D.m,  ver 2.1, March 5, 2010

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 This script calculates the natural frequencies of a system of beams.



 Natural Frequencies
 No.       f(Hz)
1.         2552.9
2.         8053.4
3.          14629
4.          21155
```

The Matlab fundamental frequency is 2% higher than the hand calculation value.

APPENDIX B

Tuning Fork, 440 Hz, A Note

A tuning fork is constructed from 0.125-inch diameter steel. There are no constraints.

Grids

```
[0	0	0;
0	0.25	0;
0	0.5	0;
0	0.75	0;
-0.23097	1.15433	0;
-0.17678	1.07322	0;
-0.0957	1.01903	0;
0	1	0;
0.095671	1.01903	0;
0.17678	1.07322	0;
0.23097	1.15433	0;
-0.25	1.25	0;
-0.25	1.45417	0;
-0.25	1.65833	0;
-0.25	1.8625	0;
-0.25	2.06667	0;
-0.25	2.27083	0;
-0.25	2.475	0;
-0.25	2.67917	0;
-0.25	2.88333	0;
-0.25	3.0875	0;
-0.25	3.29167	0;
-0.25	3.49583	0;
-0.25	3.6914	0;
0.25	1.25	0;
0.25	1.45417	0;
0.25	1.65833	0;
0.25	1.8625	0;
0.25	2.06667	0;
0.25	2.27083	0;
0.25	2.475	0;
0.25	2.67917	0;
0.25	2.88333	0;
0.25	3.0875	0;
0.25	3.29167	0;
0.25	3.49583	0;
0.25	3.6914	0]
```

Elements

```
[1     2;
2      3;
3      4;
4      8;
12     5;
5      6;
6      7;
7      8;
8      9;
9      10;
10     11;
11     25;
12     13;
13     14;
14     15;
15     16;
16     17;
17     18;
18     19;
19     20;
20     21;
21     22;
22     23;
23     24;
25     26;
26     27;
27     28;
28     29;
29     30;
30     31;
31     32;
32     33;
33     34;
34     35;
35     36;
36     37]
```

```
>> beam_3D

 beam_3D.m,  ver 2.2, March 8, 2010

 by Tom Irvine

 This script calculates the natural frequencies of a system of beams.


Natural Frequencies
 No.       f(Hz)
1.            0
2.            0
3.            0
4.      0.001158
5.     0.0017086
6.     0.0032686
7.          440
8.        694.42
9.        1598.7
10.        1735.4
11.        2804.5
12.          3821
13.        4163.3
14.        4220.5
15.          5778
16.        7859.9
```

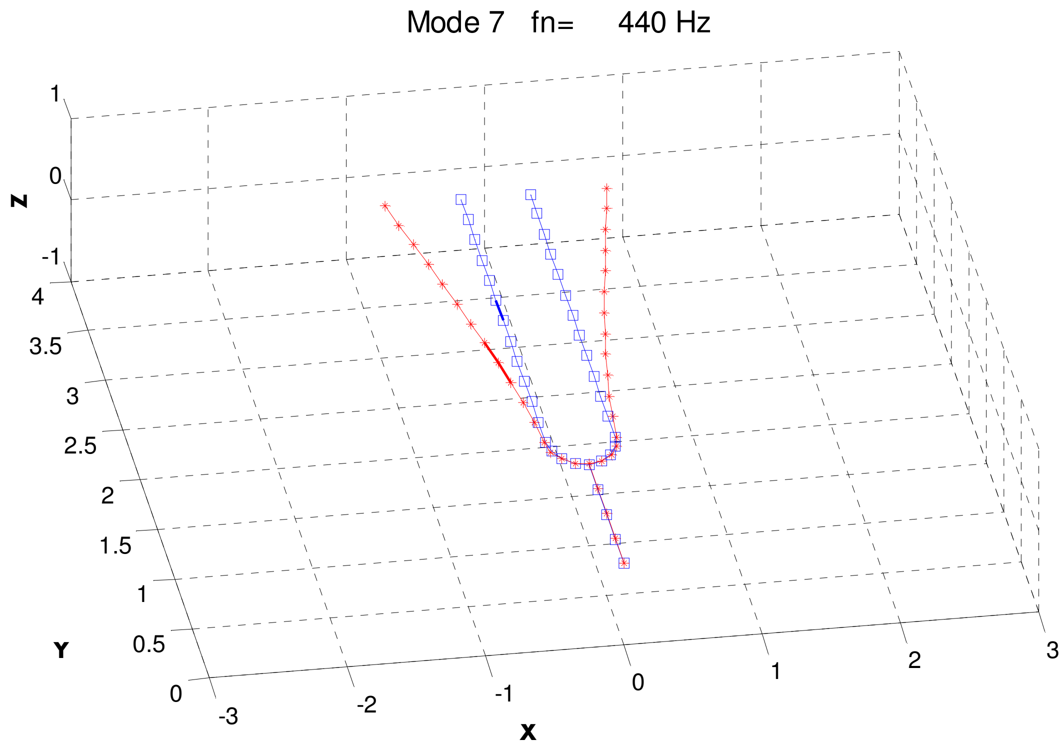

Figure B-1.

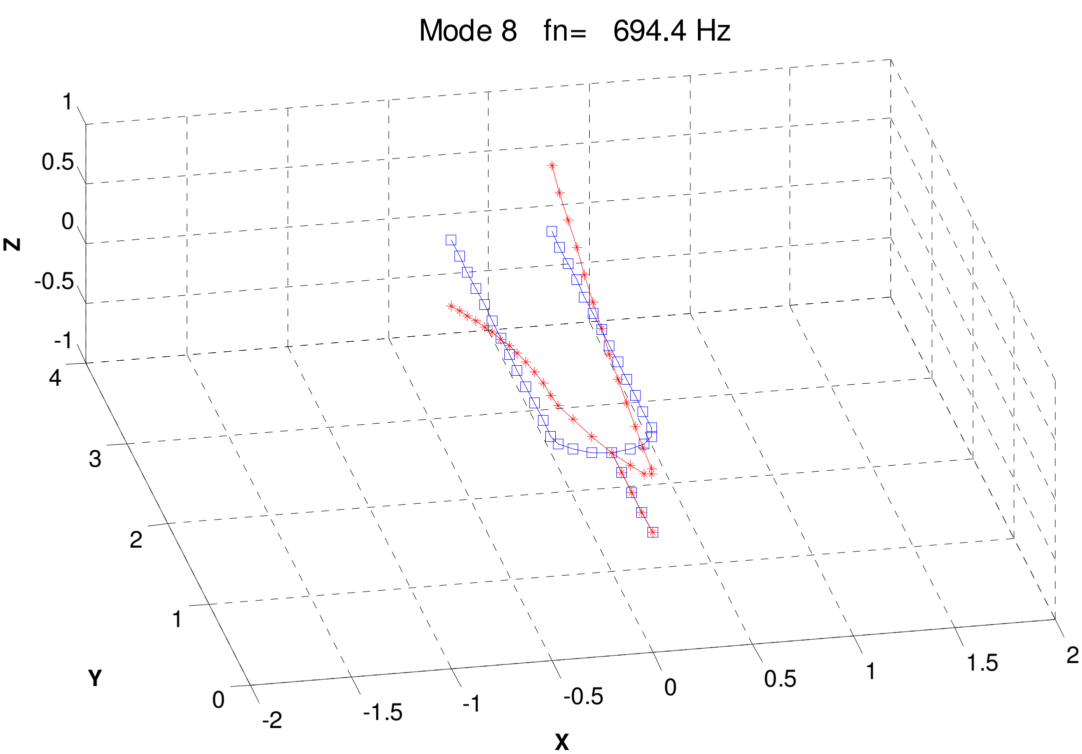

Figure B-2.

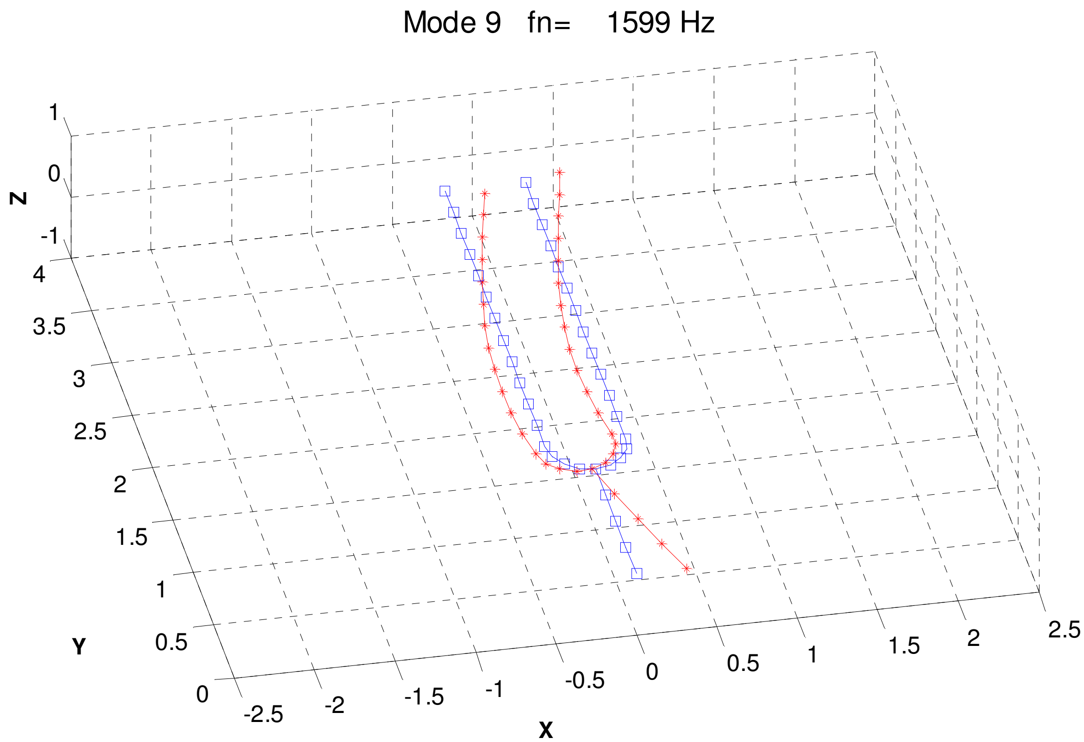

Figure B-3.