VIBROACOUSTIC CRITICAL AND COINCIDENCE FREQUENCIES OF STRUCTURES   Revision G

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This is a work in progress.

Variables

a	=	radius
C	=	speed of sound in air
C_B	=	bending wave speed
C_L	=	longitudinal wave speed
d	=	diameter
D	=	plate flexure rigidity
E	=	elastic modulus
G	=	shear modulus
f	=	frequency
f_{co}	=	coincidence frequency
f_{cr}	=	critical frequency
f_r	=	ring frequency
h	=	plate thickness or core thickness
N	=	plate shear rigidity
n	=	circumferential mode number

t	=	total face sheet thickness
θ	=	angle of incidence
ρ	=	mass per area
ω	=	frequency (rad/sec)
ν	=	Poisson ratio
k	=	acoustic wavenumber
k_b	=	free structural wavenumber in a plate
k_{cs}	=	free structural wavenumber in a cylindrical shell
$\hat{k}$	=	shear correction coefficient
k_s	=	circumferential wavenumber in a cylinder
k_x	=	wavenumber in the x-axis of a plate
k_z	=	wavenumber in the axial direction of a cylinder

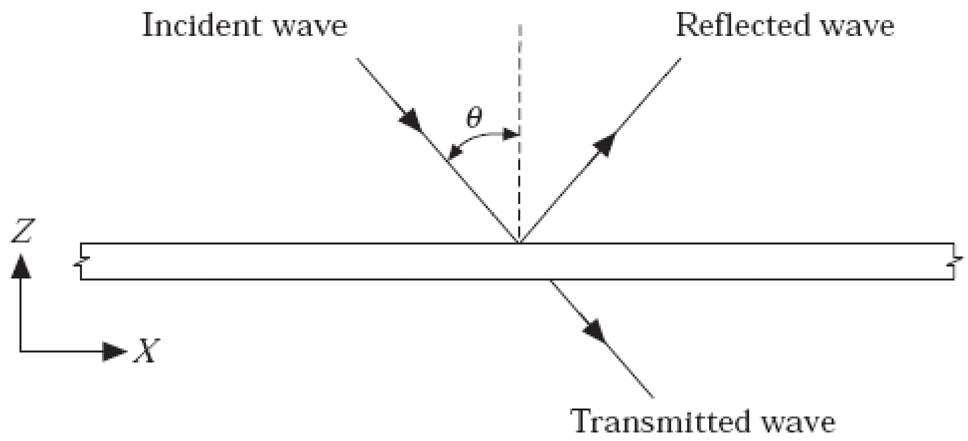

Figure 1. Waveform Diagram (from Reference 3)

Critical Frequency

The critical frequency is the frequency at which the speed of the free bending wave in a structure becomes equal to the speed of the airborne acoustic wave. The sound radiation efficiency is highest at or near the critical frequency. Furthermore, the vibration response of a panel to a reverberant field is highest near the critical frequency.

The critical frequency may be considered as corresponding to "grazing incidence," with $\theta = 90^\circ$ per the coordinate system Figure 1. The angle of incidence is not an explicit variable in the critical frequency formulas for various structures, however.

Note that the bending wavelengths are smaller than the airborne acoustic wavelength at a given frequency below the critical frequency.

Furthermore, the critical frequency is sometimes referred to as the critical coincidence frequency.

It is also referred to as the coincidence cut-off frequency. Below this frequency, coincidence cannot be achieved at any angle.

Coincidence Frequency

The coincidence frequency is the frequency at which the forced bending wave speed equals the free bending wave speed.

The coincidence frequency depends on the angle of incidence θ. The coincidence frequency can be calculated from the critical frequency by applying the appropriate trigonometric term with the angle of incidence as the argument, as shown in Reference 3.

The sound transmission through the structure is highest when the acoustic pressure frequency is at or near the coincidence frequency for the given angle of incidence.

The relationship between the coincidence frequency and the critical frequency for a thin plate is

$$f_{co}^2 = \frac{f_{cr}^2}{\sin^4 \theta} \tag{1}$$

Ring Frequency in a Cylinder

The ring frequency corresponds to the mode in which all points move radially outward together and then radially inward together. This is the first extension mode. It is analogous to a longitudinal mode in a rod.

The ring frequency is the frequency at which the longitudinal wavelength in the structure is equal to the circumference.

$$f_r = \frac{C_L}{\pi d} \tag{2}$$

The ring frequency is also referred to as the cutoff frequency. It is the lowest frequency at which an n=0 axisymmetric-mode resonance can occur, per Reference 6.

Breathing modes cannot propagate below the cutoff frequency, although axial and tangential n = 0 modes can. Note that the n value is the circumferential mode number.

Dispersion

Note that mechanical bending waves are dispersive. The wave speed varies with frequency.

The following waveforms are essentially non-dispersive:

1. Mechanical longitudinal waves
2. Mechanical shear and torsional waves
3. Airborne acoustical waves

Plate Stiffness Factor

Note that the plate stiffness factor D is given by

$$D = \frac{Eh^3}{12\left(1-\nu^2\right)} \tag{3}$$

Thin, Homogeneous Plate

Note that the small-amplitude bending and in-plane longitudinal and shear waves are all uncoupled and can propagate independently in a thin plate.

The bending wave speed from Reference 1, chapter 3 is

$$C_B \approx \sqrt{1.8 C_L h f} \tag{4}$$

The critical frequency f_{cr} for a thin, homogeneous plate is

$$f_{cr} = \frac{c^2}{2\pi h}\sqrt{\frac{12(1-\nu^2)\rho}{E}} \tag{5a}$$

$$f_{cr} \approx \frac{c^2}{1.8\, c_L\, h} \tag{5b}$$

Also, note that the wavenumber relationship from Reference 6 is

$$k_x^2 + k_y^2 = k_b^2 = \sqrt{\frac{\omega^2 \rho}{D}} \tag{6}$$

Also,

$$k_b^2 = \sqrt{\frac{k^2 c^2 \rho}{\omega^2 D}} \tag{7}$$

Isotropic Thick Plate

The bending wave speed from Reference 3 is

$$C_B{}^2 = \frac{2N}{\rho + \sqrt{\rho^2 + \left[\dfrac{4\rho N^2}{\omega^2 D}\right]}} \tag{8}$$

The shear rigidity N for a homogenous thick plate is

$$N = \hat{k}\ G\, h \tag{9}$$

Reference 3 assumes that the shear factor is $\hat{k} = 1$. Reference 5 uses $\hat{k} = \sqrt{5/6}$ for a homogenous plate.

The critical frequency for an isotropic thick plate is

$$f_{cr}{}^2 = \frac{1}{(2\pi)^2}\left[\frac{c^4\rho}{D}\right]\left[\frac{1}{1-\left(c^2\rho / N\right)}\right] \tag{10}$$

Note that the critical frequency does not exist if $\left(c^2\rho / N\right) \geq 1$.

In this case, the free bending wave speed would always be less than airborne wave speed.

Honeycomb Sandwich Panel

The shear rigidity N for a honeycomb sandwich panel is

$$N = \hat{k}\, G\, h\left[1 + (t/h)\right]^2 \tag{11}$$

where t is the total face sheet thickness and h is the core thickness.

The bending wave speed for a honeycomb panel is found by entering the shear rigidity from equation (11) into equation (8).

Likewise, the critical for a honeycomb panel is found by entering the shear rigidity from equation (11) into equation (10).

Cylindrical Shells

The authors of Reference 9 wrote that:

> For cylindrical shells (thick or thin), it is impossible to define a "unique critical frequency" for describing the acoustic properties as for flat plates.

Cylinder, Thin-Walled

The curvature of the walls couples the radial, axial and tangential motions within a cylinder.

Note that air, or a non-viscous fluid, can only exchange energy with a shell via the shell's radial motion.

An example is shown in Appendix A which shows that the critical frequency formula in equation (3) appears to be still valid for the case of a thin-wall cylinder.

Furthermore, the acoustic radiation of a cylinder depends on two major frequency parameters. The first is f / f_c.

The second is

$$f_c / f_r = \left[\frac{c^2}{1.8\, h\, C_L} \right] \Big/ \left[\frac{C_L}{\pi d} \right] \tag{12}$$

Equation (11) is taken from Reference 6, equation (2.115).

Note that large-diameter, thin-wall shells have a ratio of $f_c / f_r > 1$. The shells are thus "acoustically thin."

The radiation efficiency has a peak at the ring frequency in acoustically thin shell.

In this case, there is a frequency band between f_r and f_c in which shell curvature effects on bending wave speed disappear, and the cylinder radiates as a flat plate.

A more general observation is that a cylinder tends to behave as a flat plate above its ring frequency.

Furthermore, the wavenumbers are related by

$$k_z^2 = k_{cs}^2 - k_s^2 = k_{cs}^2 - (n/a)^2 \tag{13}$$

Cylinder, Thick-Walled

The critical frequency for a thick-walled cylinder tends to be less than the ring frequency, such that $f_c / f_r < 1$. The shells are thus "acoustically thick."

The existence of a critical frequency appears to become nebulous or tenuous for a thick-walled cylinder, as shown by example in Appendix B.

The authors in Reference 9 explain:

1. Currently, for acoustically thick cylindrical shells, the physical significance of the critical frequency is unclear because curvature effects would play an important role in determining the flexural wave speed and the acoustic radiation behavior.

2. Analysis of acoustically thick shells has shown that unlike flat plates, for frequencies below the critical frequency, both supersonic and subsonic modes can exist. Consequently, the radiation efficiency is dependent on the geometries and boundary conditions and could reach unity at a frequency much lower than the critical frequency. The behavior of individual modes is important in thick-walled shells.

3. The modal density in thick-walled cylinders is not high enough for statistical analysis.

Acoustically Fast and Slow Modes

Definitions

Note that supersonic modes are also called acoustically fast (AF) modes.

Subsonic modes are also referred to as acoustically slow (AS) modes.

Modes must be categorized as either AF or AS in order to determine their ability to interact with sound waves.

Another distinction between these two classes is that an AF mode has a structural wavenumber smaller than the acoustic trace. In other words, the AF mode has a longer wavelength than the acoustic wave at the corresponding frequency. The converse is true for AS modes.

Flat Plate

All modes above the critical frequency in a flat plate are AF, and all modes below the critical frequency are AS.

Cylindrical Shell

Again, both AF and AS modes can exist below the critical frequency of a cylindrical shell.

Further Notes on Radiation Efficiency

The radiation efficiency for AF modes is

$$\sigma_{rad} = \sqrt{1 - \frac{(k_m{}^2 + k_n{}^2)}{k^2}} \quad , \quad \text{for} \;\; k^2 > k_m{}^2 + k_n{}^2 \tag{14}$$

$$\sigma_{rad} \approx 1 \quad \text{for} \quad k^2 >> k_m{}^2 + k_n{}^2 \tag{15}$$

The original source for equation (14) is Reference 11, equation (2.16).

Furthermore, a separate formula is required for the special case where
$k^2 = k_m{}^2 + k_n{}^2$.

The authors of Reference 9 state that radiation efficiency of all AF modes above the critical frequency in a shell should be unity. This statement appears to be true only if the condition in equation (15) is satisfied.

Improved Definition of the Critical Frequency for a Cylinder

The critical frequency of cylinder should be defined as the lowest frequency above which all modes are AF.

The critical frequency cannot be calculated directly from any of the flat plate critical frequency formulas given previously in his paper.

A search is in progress to derive this formula, perhaps through empirical numerical studies.

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APPENDIX A

Thin-Walled Cylinder Example

Consider a cylinder with the following properties:

Table A-1. Sample Thin-Walled Cylinder, Simply-Supported at Each End	
Diameter	48 inch
Length	96 inch
Thickness	0.125 inch
Material	Aluminum
Mass Density ρ	0.1 lbm/in^3
Elastic Modulus E	10.0e+06 psi
Poisson Ratio	0.3

The ring frequency is 1303 Hz per equation (2).

The critical frequency using the thin plate equation is 3927 Hz per equation (5a).

The natural frequencies and corresponding wave numbers were calculated per Reference 7.

The natural frequencies are plotted as a function of wave number in Figure A-1. The airborne acoustic relation is also plotted in this figure.

The ring frequency mode is a special case of the n=0 modal family. The ring mode radiates sound as a line monopole.

Note that the axial and tangential n=0 modes can propagate below the ring frequency.

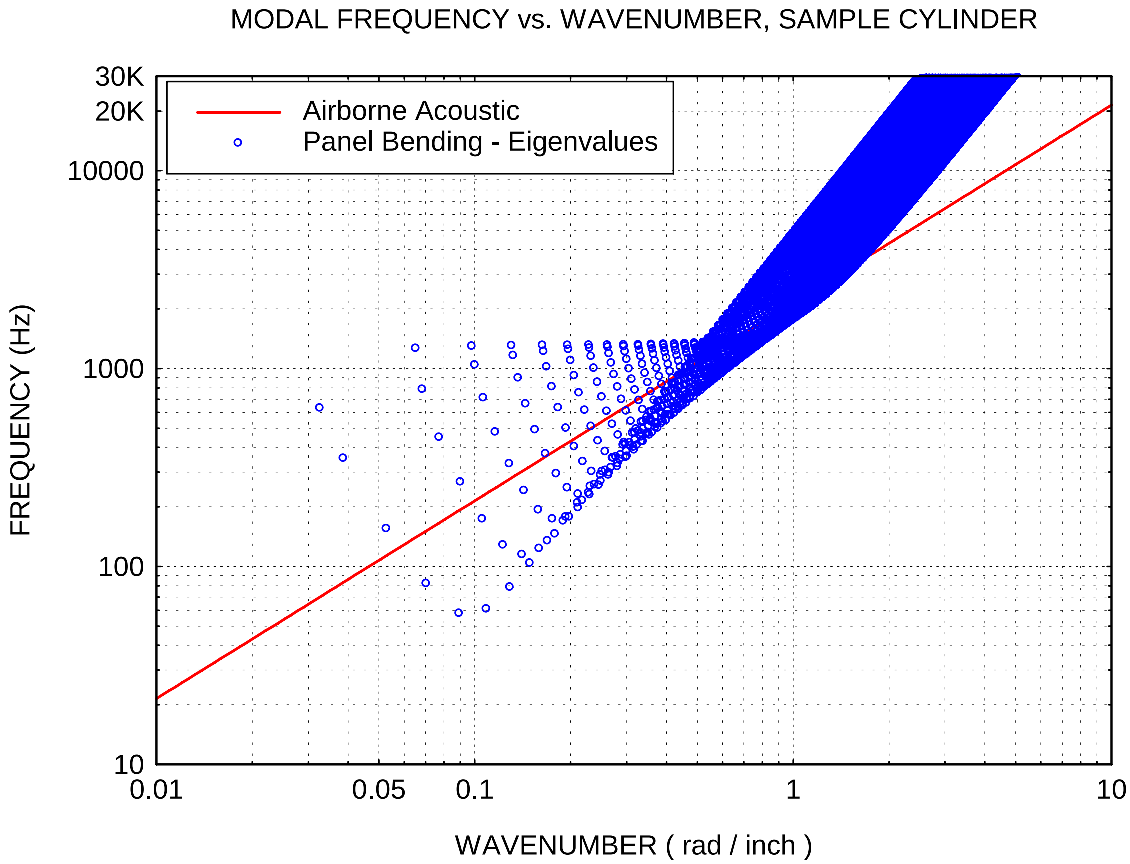
MODAL FREQUENCY vs. WAVENUMBER, SAMPLE CYLINDER
Airborne Acoustic
Panel Bending - Eigenvalues
30K
20K
10000
1000
100
10
FREQUENCY (Hz)
0.01
0.05
0.1
1
10
WAVENUMBER (rad / inch)

Figure A-1.

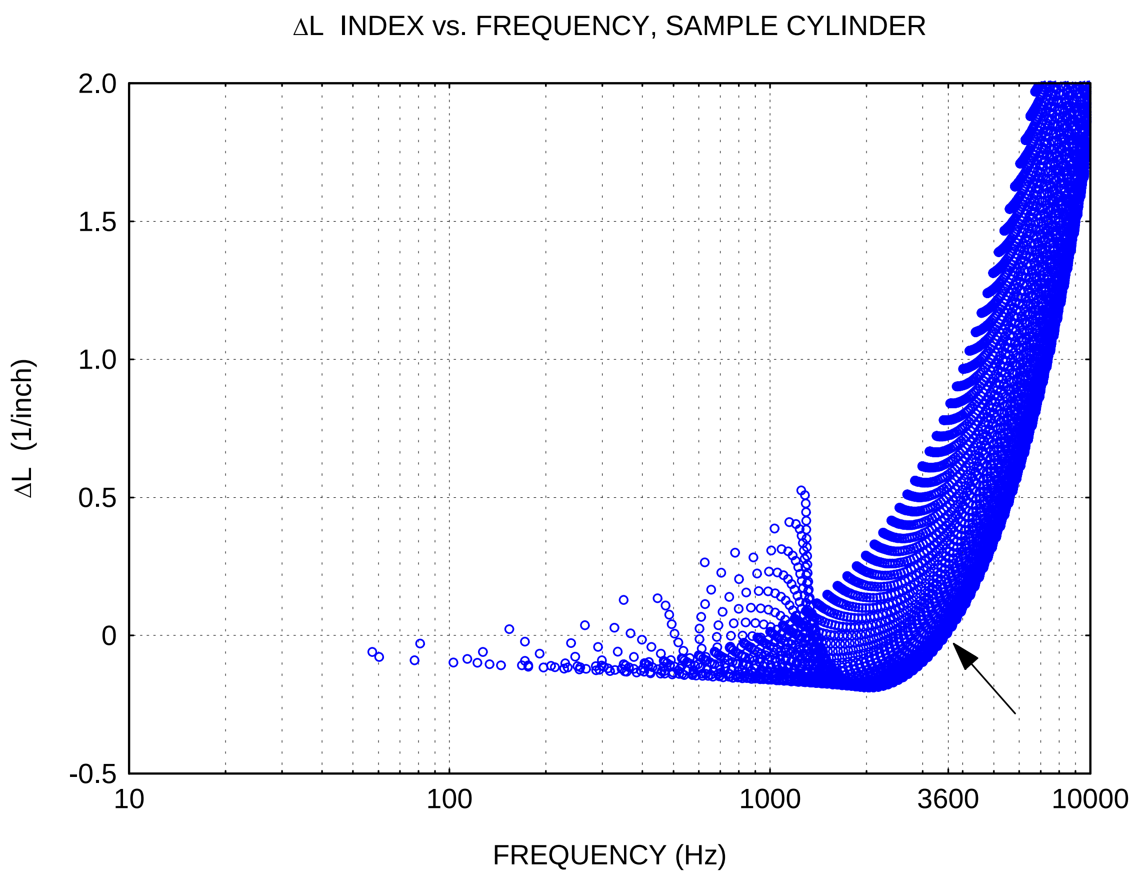

Figure A-2.

The wavenumber difference index ΔL is defined as

$$\Delta L = k - \sqrt{k_m^2 + k_n^2} \qquad \text{(A-1)}$$

Note that

$\Delta L > 0$ for AF modes

$\Delta L < 0$ for AS modes

Thus, the intersection point with the $\Delta L=0$ line at 3600 Hz is the critical frequency, as obtained by numerical experiment. This is the frequency above which all modes are AF.

This experimental frequency is 8% lower than the theoretical flat plate equivalent value.

APPENDIX B

Thick-Walled Cylinder Example 1

Consider a cylinder with the following properties:

Table B-1. Sample Thick-Walled Cylinder, Free at Each End	
Outer Diameter	18 inch
Length	32 inch
Thickness	1 inch
Material	Aluminum
Mass Density	0.1 lbm/in^3 = 0.00026 lbf sec^2/in^4
Elastic Modulus E	10.0e+06 psi
Shear Modulus G	3.85e+06 psi
Poisson Ratio	0.3

The shear rigidity N the thick plate analogy is

$$N = \hat{k}\, G\, h = (1)(3.85e{+}06 \text{ psi})(1 \text{ inch}) = 3.85e{+}06 \text{ lbf/in} \qquad \text{(B-1)}$$

The shear coefficient is assumed to be one in equation (B-1).

Now consider the criteria for a thick, rectangular plate.
Recall that the critical frequency does not exist if $\left(c^2 \rho / N\right) \geq 1$.
For this example,

$$\left(c^2 \rho / N\right) = \frac{[11500 \text{ in/sec}]^2 \, [0.00025904 \text{ lbf sec\^{}2/in\^{}4}][1 \text{inch}]}{3.85e+06 \text{ lbf/in}} \approx 0.009 \qquad \text{(B-2)}$$

Thus, the critical frequency should exist for the sample, thick-walled cylinder, although the flat plate criterion may not be appropriate for a thick-walled cylinder.

The plate stiffness factor D is given by

$$D = \frac{Eh^3}{12\left(1-\nu^2\right)} \tag{B-3}$$

$$D = \frac{\left(10.0e+06\ \text{lbf/in\^{}2}\right)\left(1\text{inch}\right)^3}{12\left(1-0.3^2\right)} \tag{B-4}$$

$$D = 9.16E+05\ \text{lbf in} \tag{B-5}$$

The critical frequency using a flat, thick plate assumption is

$$f_{cr}^2 = \frac{1}{(2\pi)^2}\left[\frac{c^4\rho}{D}\right]\left[\frac{1}{1-\left(c^2\rho / N\right)}\right] \tag{B-6}$$

$$f_{cr}^2 = \frac{1}{(2\pi)^2}\left[\frac{\left(13500\ \text{in/sec}\right)^4\left(0.00026\ \text{lbf sec\^{}2/in\^{}4}\right)\left(1\ \text{in}\right)}{9.16E+05\ \text{lbf in}}\right]\left[\frac{1}{1-0.009}\right] \tag{B-7}$$

$$f_{cr} = 491\ \text{Hz} \tag{B-8}$$

The ring frequency is 3474 Hz per equation (2).

The critical frequency using the flat thick plate equation is 491 Hz. This equation may not be appropriate for thick-walled cylinders, however. (The thin plate equation would have yielded a critical frequency of 506 Hz.)

A finite element analysis was performed. The model name was thick_cylinder_1.nas.

The ring frequency appeared to occur at 2862 Hz, but this mode also had a large axial displacement. This is 18% less than the theoretical value.

There were no bending modes near 491 Hz. The first axial bending mode appeared at 1891 Hz, nearly two octaves higher. This difference would seem to render the critical frequency as irrelevant.

Thick-Walled Cylinder Example 2

Repeat Example 1 except constrain the ends so that no axial motion can occur.

A finite element analysis was performed. The model name was thick_cylinder_2.nas.

The ring frequency occurs at 3927 Hz, which is 13% higher than the theoretical value. The motion was purely radial.

Again, there were no bending modes near 491 Hz.