dB/octave Calculations for Log-Log Plots

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Random Vibration

The section applies to amplitudes with G^2/Hz units.

Let (f_2, y_2) be the known coordinate pair, where f_2 has dimensions of frequency (Hz), and y_2 has dimensions of power spectral density (G^2/Hz).

Let (f_1, y_1) be the other coordinate pair, where f_1 is known but y_1 is unknown.

Frequency f_1 can either be less than or greater than f_2. The formula will work either way.

Let ΔdB be the slope with units of dB/octave from the lower frequency to the higher frequency.

The formula is:

$$y_1 = y_2 \left[\frac{f_1}{f_2}\right]^{\left[\frac{\Delta dB / 10}{\log 2}\right]} \tag{1}$$

Note that if the result for y_1 seems either ridiculously high or low then the polarity of ΔdB may have been entered incorrectly.

Also note that the logarithm is base ten.

The slope N between the coordinate pair is

$$N = \frac{\log[y_2 / y_1]}{\log[f_2 / f_1]} \tag{2}$$

The dB/octave slope is

$$\Delta dB = 10 \text{ N} \log 2 \quad \text{(dB/octave)} \tag{3}$$

Example 1

The known point is (60 Hz, 1.1 G^2/Hz). The unknown amplitude is at 20 Hz. The slope from the lower frequency to the higher frequency is 6 dB/octave. Solve for the unknown amplitude y_1.

$$y_1 = \left(1.1\, G^2 / \mathrm{Hz}\right)\left[\frac{20\ \mathrm{Hz}}{60\,\mathrm{Hz}}\right]^{\left[\frac{6\,\mathrm{dB}/10}{\log 2}\right]} \quad = \quad 0.123\, G^2 / \mathrm{Hz} \tag{4}$$

Example 2

The known point is (1200 Hz, 1.8 G^2/Hz). The unknown amplitude is at 2000 Hz. The slope from the lower frequency to the higher frequency is -18 dB/octave. Solve for the unknown amplitude y_1.

$$y_1 = \left(1.8\, G^2 / \mathrm{Hz}\right)\left[\frac{2000\ \mathrm{Hz}}{1200\,\mathrm{Hz}}\right]^{\left[\frac{-18\,\mathrm{dB}/10}{\log 2}\right]} \quad = \quad 0.085\, G^2 / \mathrm{Hz} \tag{5}$$

<u>Shock and Sinusoidal Vibration</u>

The amplitude for shock and sine vibration is often expressed in units of G. The formula for this unit is:

$$y_1 = y_2 \left[\frac{f_1}{f_2}\right]^{\left[\frac{\Delta \mathrm{dB}/20}{\log 2}\right]} \tag{6}$$

Note that this is the same formula as for the random vibration case except that the ΔdB value is divided by 20 for the shock and sine case.

The corresponding slope equations for sine and shock are:

$$N = \frac{\log\left[y_2 / y_1\right]}{\log\left[f_2 / f_1\right]} \tag{7}$$

The dB/octave slope is:

$$\Delta dB = 20\ N \log 2 \qquad \text{(dB/octave)} \tag{8}$$