# DAMPING COEFFICIENT MATRIX FROM MODAL DAMPING RATIOS FOR ALL MODES

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The equation of motion for a multi-degree-of-freedom system is

$$M\ddot{x} + C\dot{x} + Kx = 0 \tag{1}$$

where

| | |
|---|---|
| M | is the mass matrix |
| C | is the damping coefficient matrix |
| K | is the stiffness matrix |
| x | is the displacement |

The mass-normalized eigenvectors in column format are $\Phi$

$$\Phi^T M \Phi = I \tag{2}$$

Note that

$$\Phi^T M \Phi \Phi^{-1} = \Phi^{-1} \tag{3}$$

$$\Phi^T M = \Phi^{-1} \tag{4}$$

And

$$\left(\Phi^T\right)^{-1} \Phi^T M \Phi = \left(\Phi^T\right)^{-1} \tag{5}$$

$$M \Phi = \left(\Phi^T\right)^{-1} \tag{6}$$

$$\left(\Phi^{T}\right)^{-1} = M\,\Phi \tag{7}$$

Now consider a three-degree-of freedom system.

Derive a damping coefficient matrix as follows.

$$\Phi^{T}\begin{bmatrix} c11 & c12 & c13 \\ c21 & c22 & c23 \\ c13 & c32 & c33 \end{bmatrix}\Phi = \begin{bmatrix} 2\xi_1\omega_1 & 0 & 0 \\ 0 & 2\xi_2\omega_2 & 0 \\ 0 & 0 & 2\xi_3\omega_3 \end{bmatrix} \tag{8}$$

where

$\xi_i$ is the damping ratio for mode i

$\omega_i$ is the natural frequency for mode i

$$\Phi^{T}\begin{bmatrix} c11 & c12 & c13 \\ c21 & c22 & c23 \\ c13 & c32 & c33 \end{bmatrix}\Phi\,\Phi^{-1} = \begin{bmatrix} 2\xi_1\omega_1 & 0 & 0 \\ 0 & 2\xi_2\omega_2 & 0 \\ 0 & 0 & 2\xi_3\omega_3 \end{bmatrix}\Phi^{-1} \tag{9}$$

$$\Phi^{T}\begin{bmatrix} c11 & c12 & c13 \\ c21 & c22 & c23 \\ c13 & c32 & c33 \end{bmatrix} = \begin{bmatrix} 2\xi_1\omega_1 & 0 & 0 \\ 0 & 2\xi_2\omega_2 & 0 \\ 0 & 0 & 2\xi_3\omega_3 \end{bmatrix}\Phi^{T}M \tag{10}$$

$$\left(\Phi^{T}\right)^{-1}\Phi^{T}\begin{bmatrix} c11 & c12 & c13 \\ c21 & c22 & c23 \\ c13 & c32 & c33 \end{bmatrix} = \left(\Phi^{T}\right)^{-1}\begin{bmatrix} 2\xi_1\omega_1 & 0 & 0 \\ 0 & 2\xi_2\omega_2 & 0 \\ 0 & 0 & 2\xi_3\omega_3 \end{bmatrix}\Phi^{T}M \tag{11}$$

$$\begin{bmatrix} c11 & c12 & c13 \\ c21 & c22 & c23 \\ c13 & c32 & c33 \end{bmatrix} = M\,\Phi\begin{bmatrix} 2\xi_1\omega_1 & 0 & 0 \\ 0 & 2\xi_2\omega_2 & 0 \\ 0 & 0 & 2\xi_3\omega_3 \end{bmatrix}\Phi^{T}M \tag{12}$$

The method can be readily extended to systems with any number of degrees-of-freedom.

The resulting damping coefficient is symmetric.