ATMOSPHERIC SOUND PROPAGATION FROM A
MONOPOLE SOURCE VIA DIRECT, REFLECTED AND SURFACE WAVES
Revision C

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Consider the system in Figure 1. Assume spherical radiation in a homogeneous atmosphere.

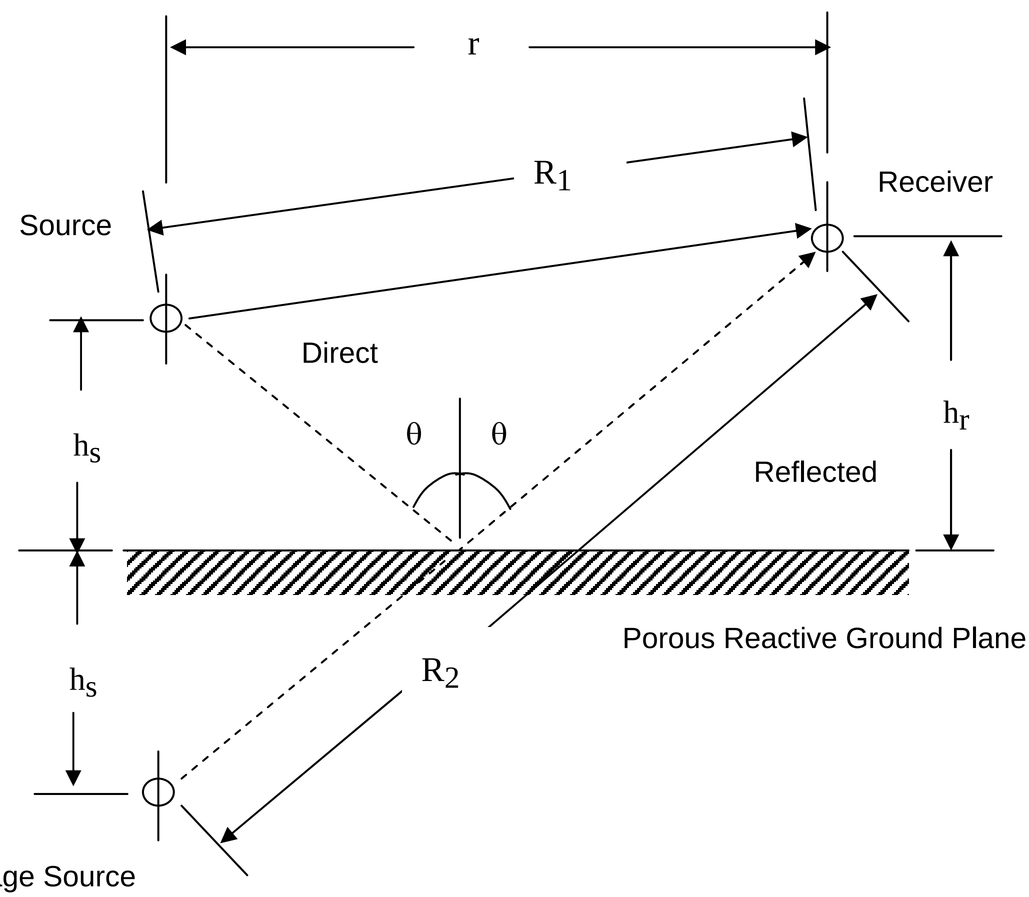


Figure 1.

Variables

c	=	Speed of sound
efrc	=	Complementary error function, complex
f	=	Frequency
F	=	Boundary loss factor, complex
h_r	=	Height of receiver above ground
h_s	=	Height of source above ground
k	=	Wavenumber
p	=	Normalized sound pressure, complex
Q	=	Spherical wave reflection coefficient, complex
r	=	Distance component from source to receiver parallel to the ground
R_p	=	Plane wave reflection coefficient, complex
R_1	=	Direct distance from source to receiver
R_2	=	Distance from image source to receiver, reflected wave
S	=	A constant with dimension (Pressure length)
w	=	Numerical distance, complex
Z	=	Normalized surface impedance, complex
θ	=	Angle of incidence
σ	=	Ground flow resistivity in units of (Pa sec / m^2)
λ	=	Wavelength
ΔL	=	Relative sound pressure level

The following approach is taken from Reference 1, with peripheral information taken from Reference 2.

The normalized sound pressure from the monopole source is

$$p(x,y,z) = S\left\{\frac{1}{4\pi R_1}\exp(jkR_1) + Q\,\frac{1}{4\pi R_2}\exp(jkR_2)\right\} \tag{1}$$

with the following three restrictions:

$$kr >> 1$$

$$r >> h_s + h_r$$

$$\frac{1}{|Z|^2} << 1 \qquad \text{(relatively hard ground surface)}$$

The constant S is determined by applying the known pressure at a given distance to equation (1).

The spherical wave reflection coefficient is

$$Q = \left[Rp + (1 - Rp)\ F(w)\right] \tag{2}$$

The second term on the right-hand-side of equation (2) accounts for the surface wave which propagates close to and parallel to the ground. The surface wave has elliptical motion of the air particles as the result of combining motion parallel to the surface with that normal to the surface in and out of the pores.

Table 1. Typical Q Values	
Absorbing Ground	$\lvert Q\rvert < 1$
Rigid Ground	$Q = 1$
Surface Wave Effect	$\lvert Q\rvert > 1$

The plane wave reflection coefficient is

$$\mathrm{Rp} = \frac{\mathrm{Z}\cos(\theta) - 1}{\mathrm{Z}\cos(\theta) + 1} \tag{3}$$

The normal impedance Z is found from empirical data, as shown for example in Reference 2. Note that Z is complex and varies with frequency. It depends on the "flow resistivity" of the ground.

Note:

1. The real Z component is called the resistance. The imaginary component is the reactance.
2. A surface wave may form if the reactance is greater than the resistance.

An empirical formula for impedance from Reference 2 is

$$\mathrm{Z} = 1 + 0.0511\left(\frac{\sigma}{\mathrm{f}}\right)^{0.75} + \mathrm{j}\,0.0768\left(\frac{\sigma}{\mathrm{f}}\right)^{0.73} \tag{4}$$

Additional empirical formulas are given in both References 1 and 2.

Note that the flow resistivity σ is the ease with which air can move into and out of the ground. A typical value for grasslands is σ = 200 kPa s / m^2.

The boundary loss factor is

$$\mathrm{F(w)} = 1 + \mathrm{j}\sqrt{\pi}\ \mathrm{w}\,\exp(-\mathrm{w}^2)\,\mathrm{erfc}(-\mathrm{jw}) \tag{5}$$

The term $\exp(-\mathrm{w}^2)\,\mathrm{erfc}(-\mathrm{jw})$ is the complex error function also known as the Faddeeva function. Further information regarding this function is given in Appendix A.

The numerical distance is

$$\mathrm{w} = \frac{1}{2}(1+\mathrm{j})\sqrt{\mathrm{kR}_2}\left[\frac{1}{\mathrm{Z}} + \cos\theta\right] \tag{6}$$

The wavenumber is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} \tag{7}$$

Note that

$$\cos\theta = \frac{h_s}{\sqrt{h_s^2 + a^2}} \tag{8}$$

where

$$a = \frac{r}{1 + \left(\frac{h_r}{h_s}\right)} \tag{9}$$

The relative sound pressure level is

$$\Delta L = 20 \log \left| 1 + Q \frac{R_1}{R_2} \exp\left[jk\left(R_2 - R_1\right)\right] \right| \tag{10}$$

Table 2. Typical ΔL Values	
Destructive Interference	$\Delta L < 0$ dB
Rigid Ground with Constructive Interference	$\Delta L = 6$ dB
Surface Wave Effect	$\Delta L > 6$ dB

Sample plots are given in Appendix B and C.

Additional information on the surface wave is given in Appendix D. The effects of temperature and wind are considered in Appendix E.

References

1. Malcolm J. Crocker, Handbook of Noise and Vibration Control, Wiley, 2007.
2. E. M. Salomons, Computational Atmospheric Acoustics, Kluwer Academic Publishers, 2001.
3. G. Daigle & T. Embleton, Air-Ground Interface: Surface Waves, Surface Impedance and Acoustic-to-Seismic Coupling Coefficient, NASA, Langley Research Center, 4th International Symposium on Long-Range Sound Propagation.

APPENDIX A

Error Function

The error function is

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)\, dt \tag{A-1}$$

The complementary error function is

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$$

$$= \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-t^2)\, dt \tag{A-2}$$

The complex error function is

$$\hat{w}(x) = \exp(-w^2)\operatorname{erfc}(-jw) \tag{A-3}$$

For further information, please visit

http://en.wikipedia.org/wiki/Error_function

APPENDIX B

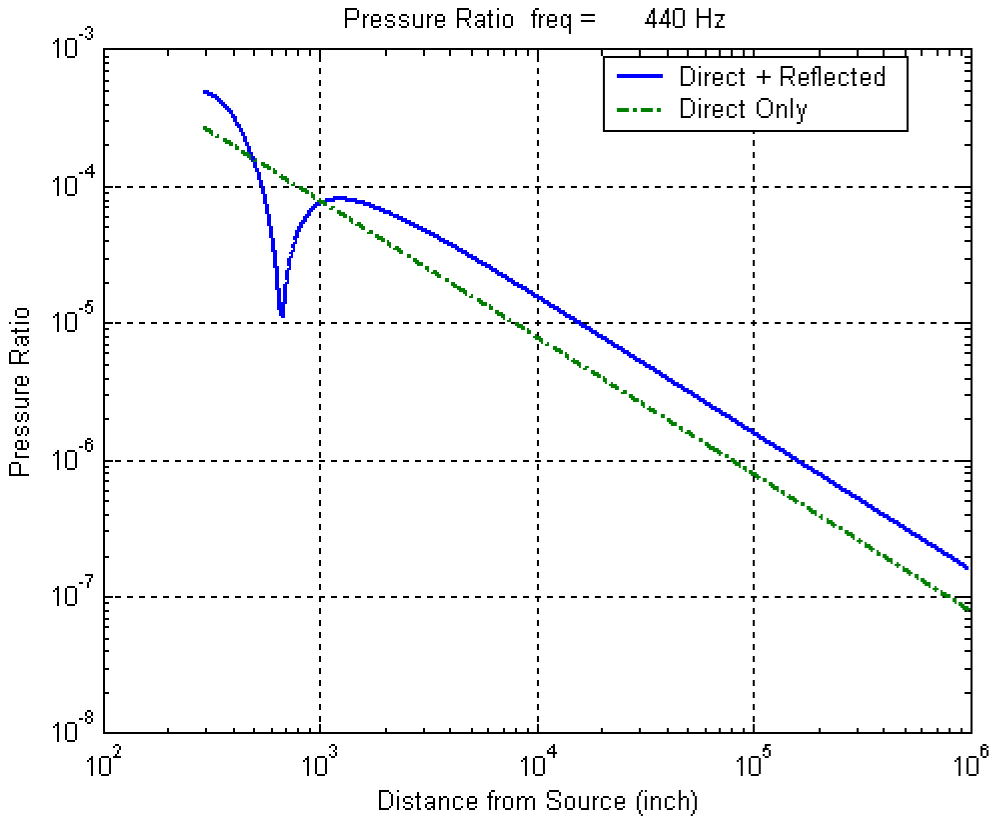
Consider a waveform with the following characteristics:

Frequency	440 Hz
Wavelength	30.68 inch
Ground Flow Resistivity	200 kPa s / m^2
Normalized Impedance	6.0 + j 6.7
Source Height	72 inch
Receiver Height	72 inch

Assume that the constant S is equal to (1 pressure unit inch), given that only relative pressure is of interest.

The imaginary impedance exceeds the real impedance, indicating the possibility of a surface wave.

The results are shown in Figures B-1 through B-7.



Pressure Ratio freq = 440 Hz
Direct + Reflected
Direct Only
Pressure Ratio
10-3
10-4
10-5
10-6
10-7
10-8
10^2
10^3
10^4
10^5
10^6
Distance from Source (inch)

Figure B-1.

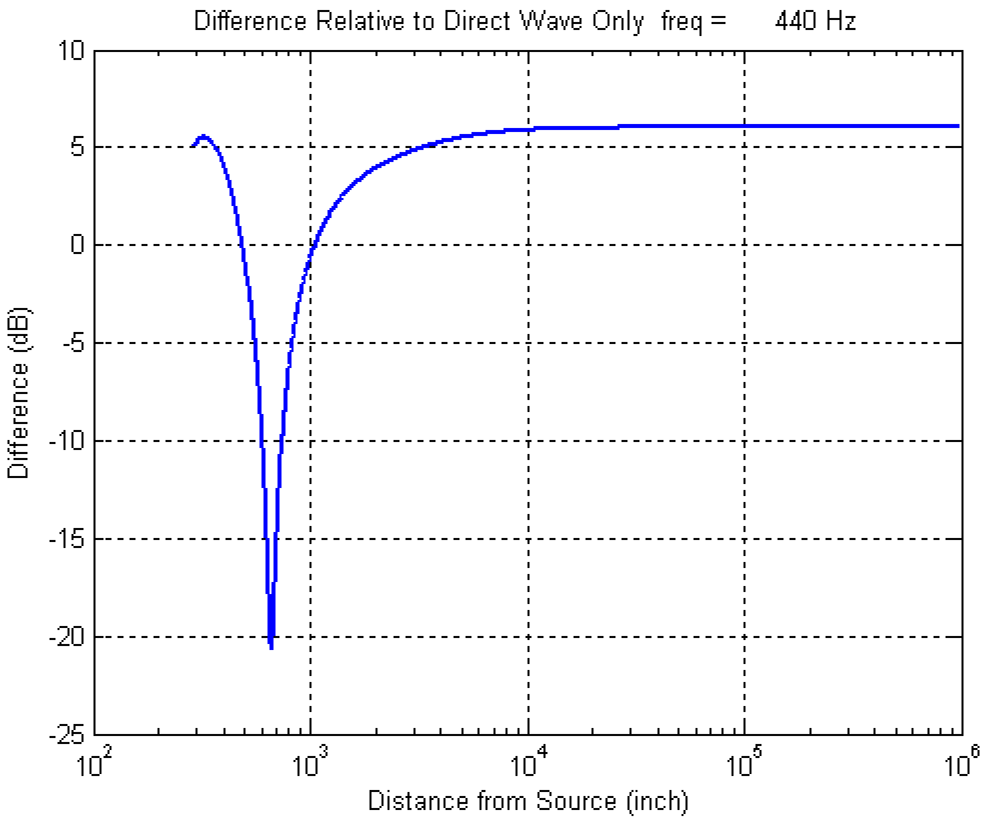


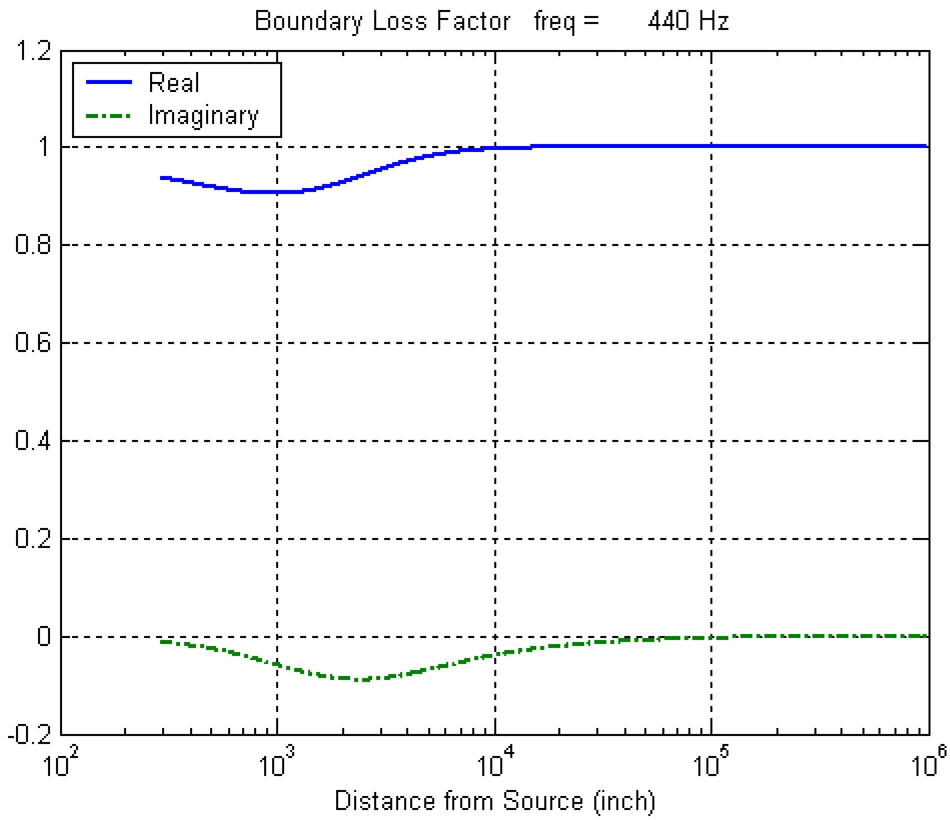
Figure B-2.

The curve in Figure B-2 is the relative sound pressure level ΔL.

The curve dips below 0 dB representing varying degrees of destructive interference. Complete destructive interference may never occur due to the presence of a surface wave.

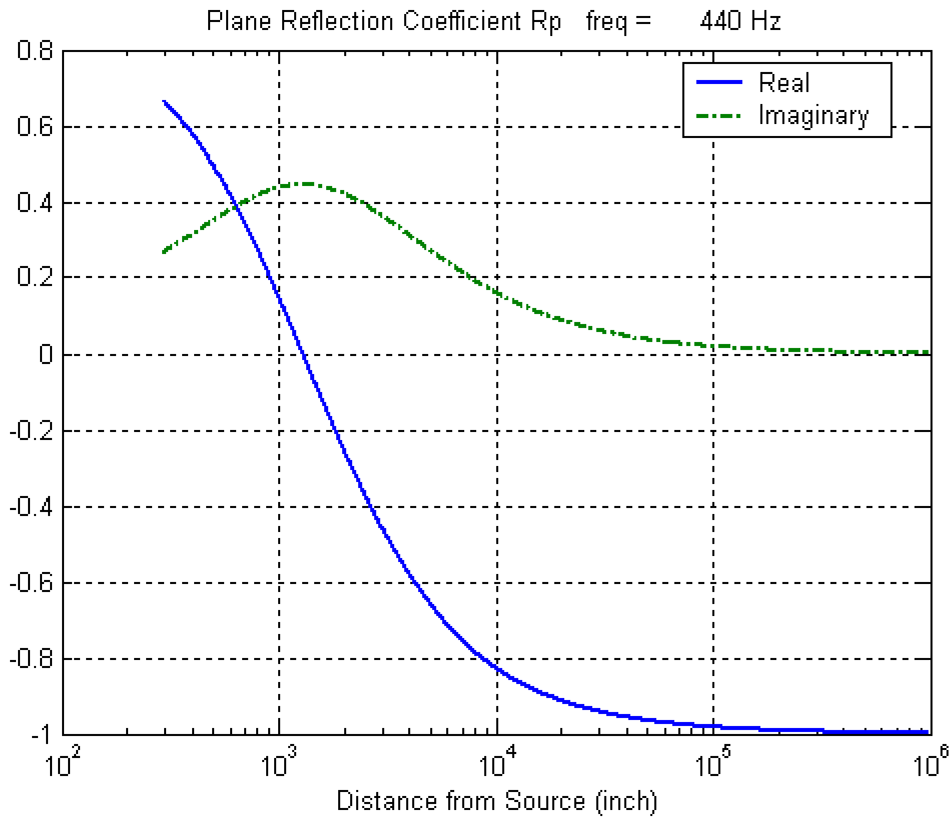
The curve converges to constructive interference with a difference of 6 dB beyond 1.0e+04 inches. Actually, the peak level may exceed 6 dB due to any surface wave contribution.

There is no exact boundary between the near and far field for a pure tone. One convention is to take the distance where the curve permanently crosses the 5 dB line. The corresponding distance is 3200 inches (81 meters).



Boundary Loss Factor freq = 440 Hz
Real
Imaginary
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
10^2
10^3
10^4
10^5
10^6
Distance from Source (inch)

Figure B-3.



Plane Reflection Coefficient Rp freq = 440 Hz
Real
Imaginary
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
10^2
10^3
10^4
10^5
10^6
Distance from Source (inch)

Figure B-4.

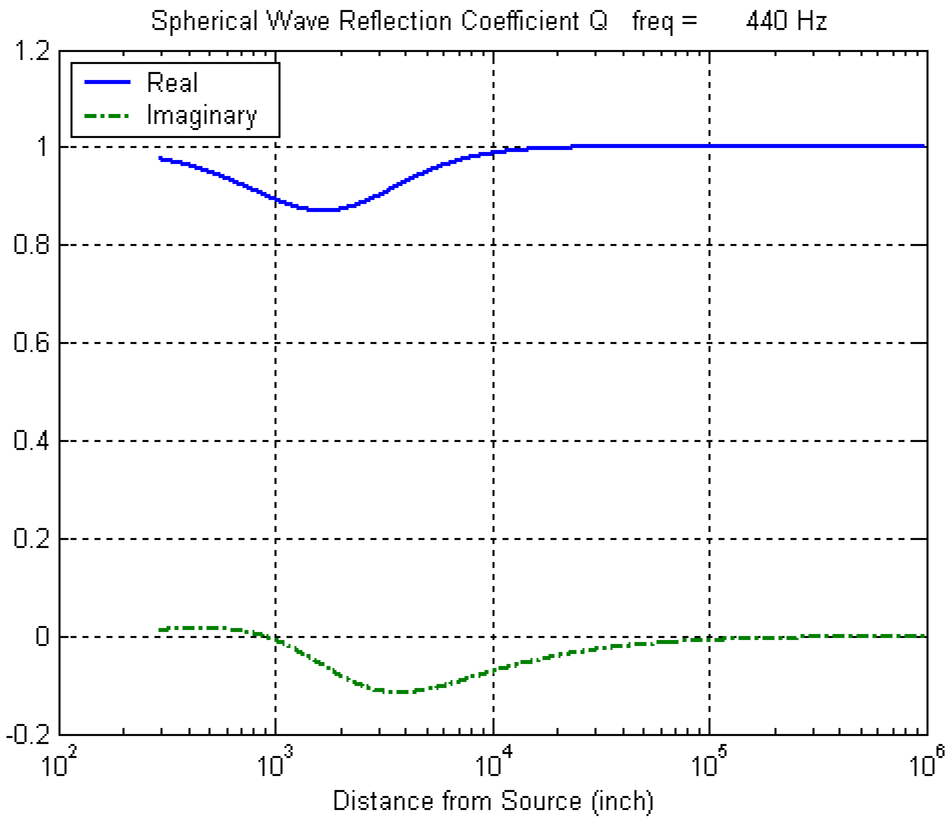


Figure B-5.

The ground is almost a hard surface, since the real Q is nearly equal to unity over much of the distance range. Some absorption occurs in the vicinity of 2000 inches.

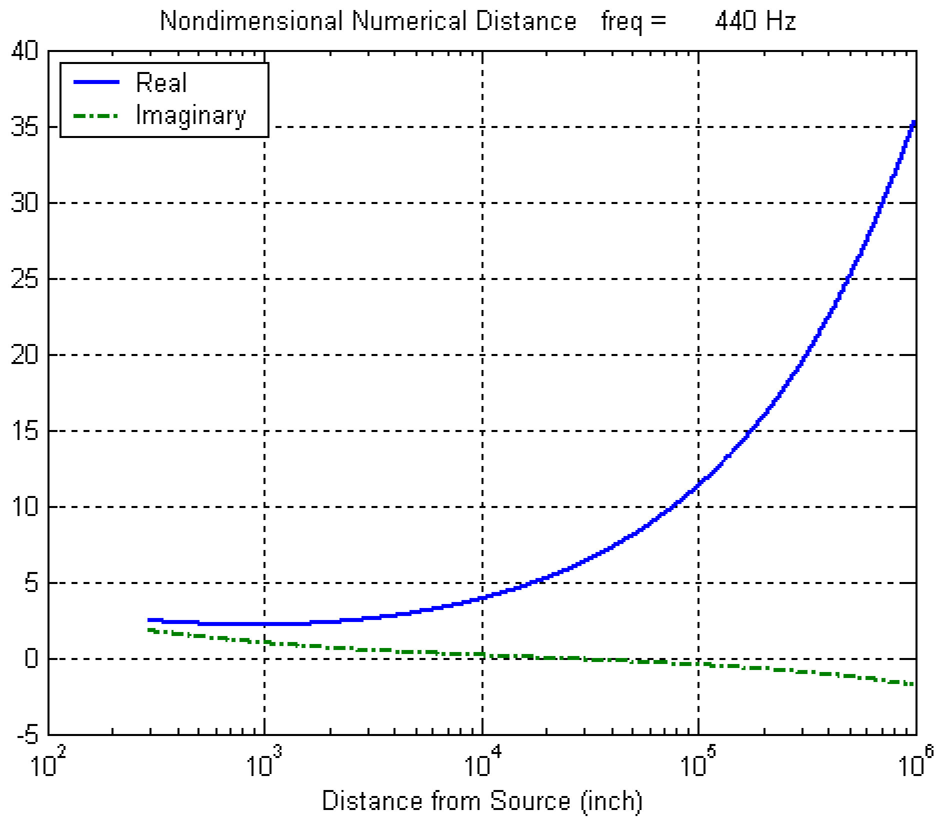

Nondimensional Numerical Distance freq = 440 Hz
Real
Imaginary
40
35
30
25
20
15
10
5
0
-5
10^2
10^3
10^4
10^5
10^6
Distance from Source (inch)

Figure B-6.

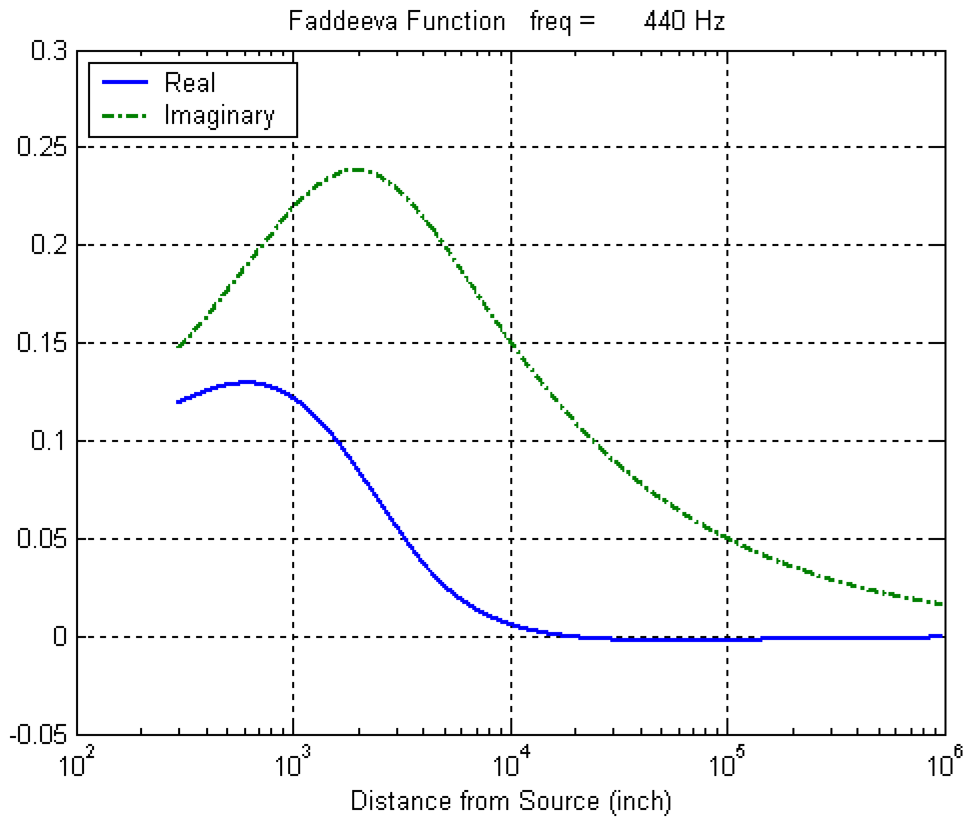

Faddeeva Function freq = 440 Hz
Real
Imaginary
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
10^2
10^3
10^4
10^5
10^6
Distance from Source (inch)

Figure B-7.

APPENDIX C

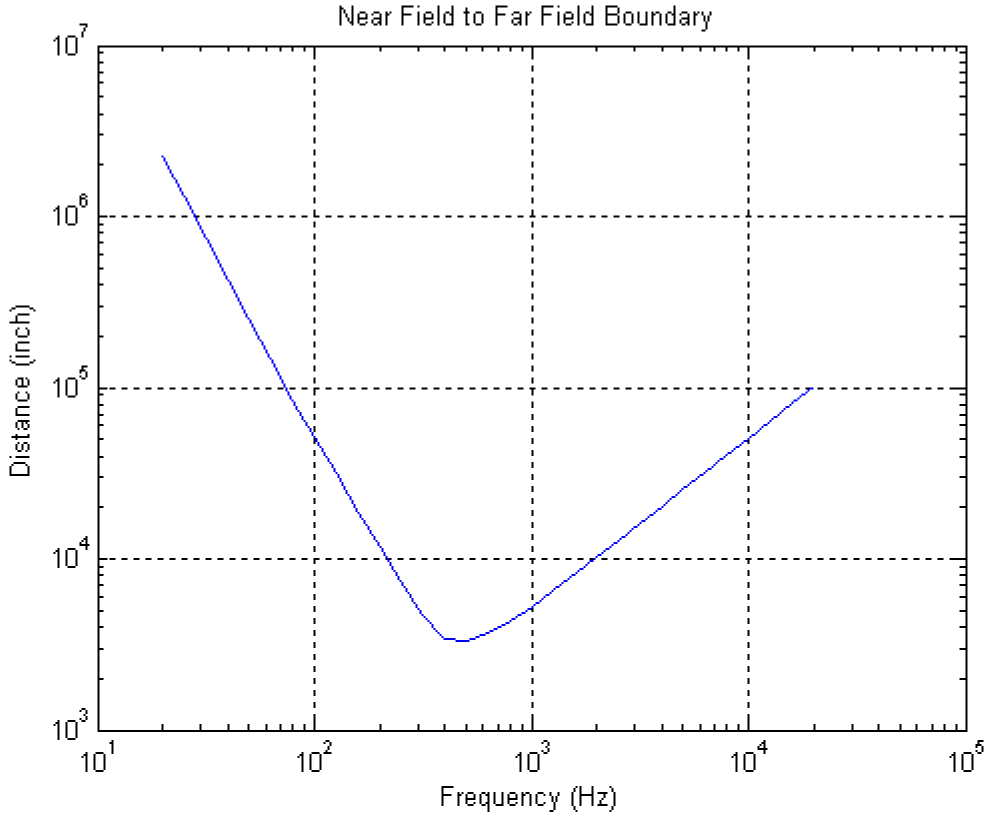
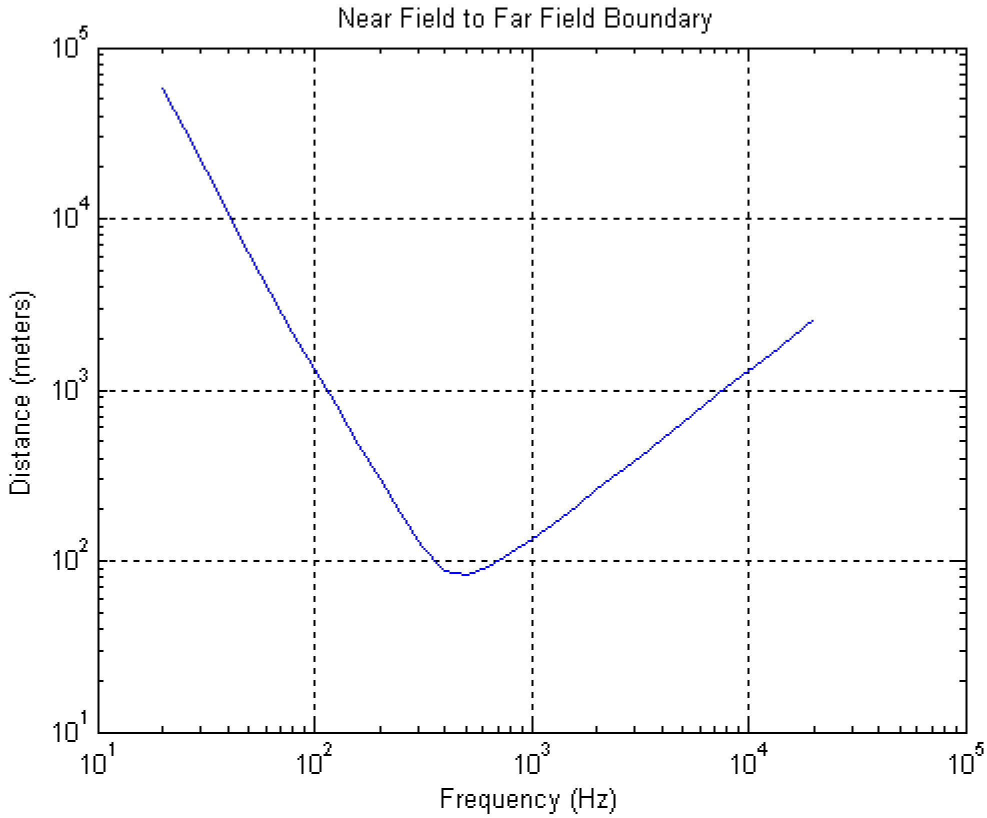


Figure C-1.

The analysis in Appendix B was repeated for one-third octave bands. The details are omitted for brevity.

The resulting transition distances are shown in Figure C-1.



Near Field to Far Field Boundary
Distance (meters)
Frequency (Hz)
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}

Figure C-2.

APPENDIX D

Waveform Particle Motion Diagram

The following diagram is taken from Reference 3.

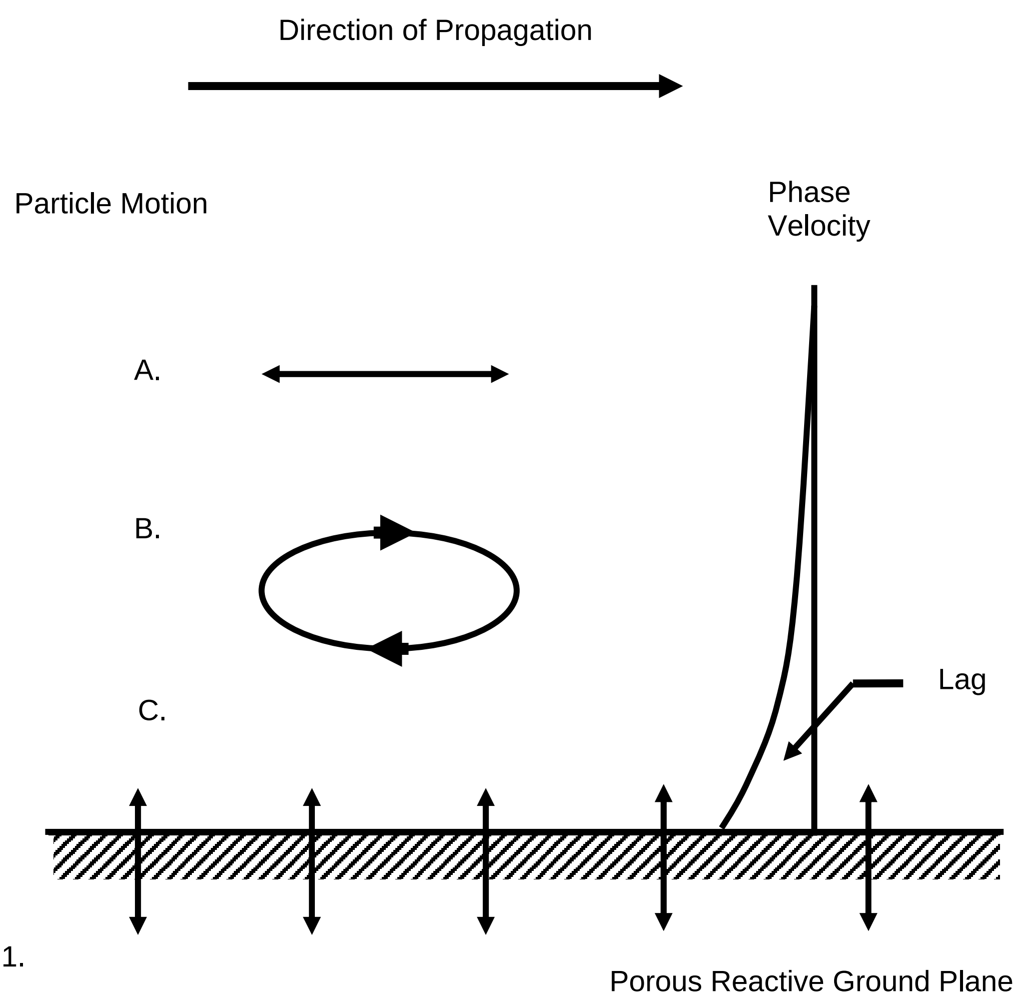


Figure D-1.

The consequence and origin of the reduced phase speed of the surface wave are shown in Figure D-1. The characteristics of the three zones are:

A. The propagating has a horizontal particle motion far from the ground.

B. Zone B represents the surface wave. The particle motion just above the ground surface is elliptical due to the transition between the horizontal motion in zone A and the vertical motion in zone C.

C. The air molecules at the porous ground are entrained in vertical particle motion due to alternating compressing and rarefaction cycles.

The elliptical particle motion results in a reduced phase speed. The resulting lag caused the wavefronts to bend toward the ground, increasing the sound pressure close to the ground and decreasing the sound energy above the surface wave thickness.

APPENDIX E

Effects of Temperature Gradients and Wind

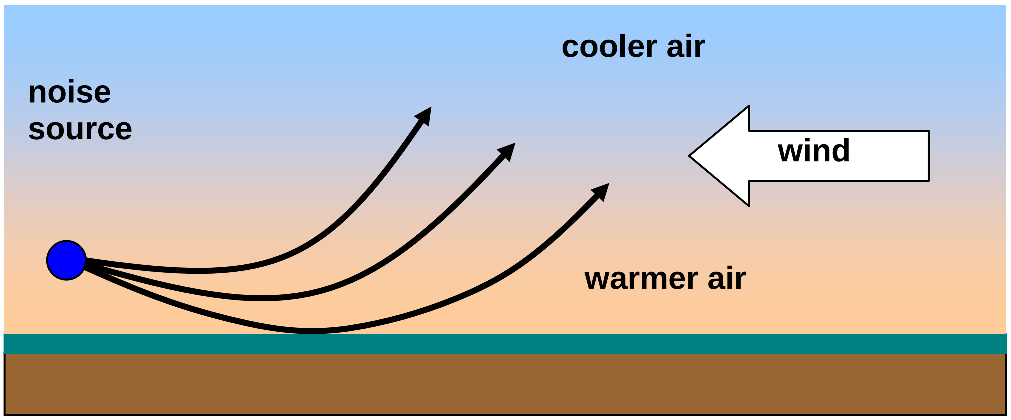


Figure E-1.

The speed of sound typically changes with the height above the ground. Usually, the temperature decreases with height, which is the adiabatic lapse condition. This cause the sound waves to bend, or refract, upward.

A similar effect occurs if the sound waves propagate upwind.

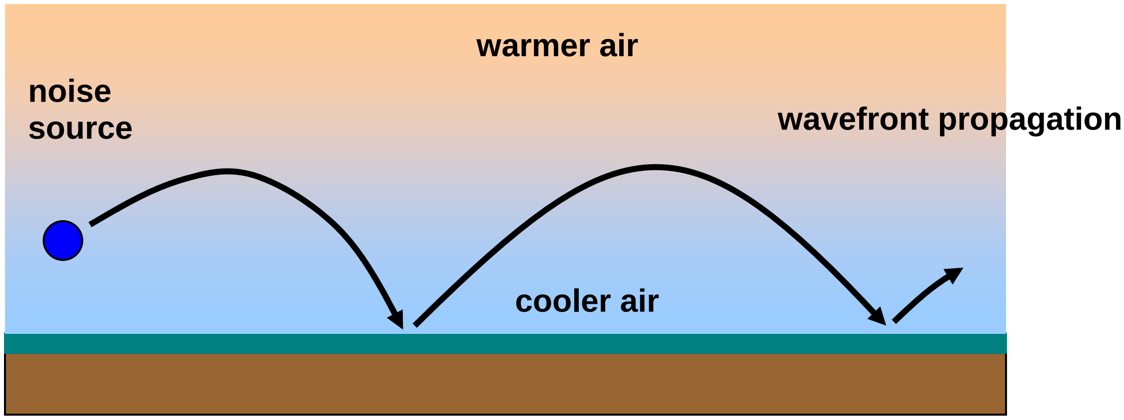


Figure E-2.

The effect of a temperature inversion is shown in Figure E-2. The ground-level air temperature is cooler than the high-level air.

Only a sample acoustic ray is shown in Figure E-2, but others would have similar patterns.

An inversion occurs when radiation from the surface of the earth exceeds the amount of radiation received from the sun, which commonly occurs at night, early morning, or during the winter when the angle of the sun is very low in the sky.