RADIATION & DRIVING POINT IMPEDANCE OF A THIN, ISOTROPIC PLATE
Revision B

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Variables

Z_{dp}	=	driving point impedance
Z_{rad}	=	radiation impedance
C	=	speed of sound in air
C_L	=	longitudinal wave speed
C_B	=	bending wave speed
σ_{rad}	=	radiation efficiency
ρ	=	mass/volume
h	=	thickness
S	=	surface area
n	=	modal density
M	=	mass
ω	=	excitation frequency (rad/sec)
f	=	excitation frequency (Hz)
f_c	=	critical frequency
$\left\langle v^2 \right\rangle$	=	normal component of the space-time average mean-square vibration velocity of the radiating surface
W_{rad}	=	radiated sound power

Note that the critical frequency is the frequency where the bending wave speed in a structure equals the speed of sound. The bending wave speed from Reference 1, chapter 3 is

$$C_B \approx \sqrt{1.8\, C_L\, h\, f} \tag{1}$$

Critical Frequency

The critical frequency is also the frequency at which the airborne acoustic wavelength matches the panel bending wavelength. The critical frequency f_c for a homogeneous panel is

$$f_c \approx \frac{c^2}{1.8\, c_L\, h} \tag{3}$$

Driving Point Impedance Derivation

The following impedance formula is taken from Reference 1, p 317.

$$Z_{dp} \approx 2.3\, C_L\, \rho\, h^2 \tag{4}$$

The longitudinal wave speed is related to the modal density by the formula given in Reference 1, page 489.

$$C_L = \frac{S}{3.6\, n(\omega)\, h} \tag{5}$$

Substitute equation (4) into (3).

$$Z_{dp} \approx 2.3 \left[\frac{S}{3.6\, n(\omega)\, h} \right] \rho\, h^2 \tag{6}$$

$$Z_{dp} \approx 2.3\left[\frac{S\,\rho\,h}{3.6\,n(\omega)}\right] \tag{7}$$

$$Z_{dp} \approx 0.64\left[\frac{M}{n(\omega)}\right] \tag{8}$$

$$Z_{dp} \approx \frac{4M}{n(f)} \tag{9}$$

<u>Radiation Impedance</u>

The radiation impedance is the ratio of the force exerted by the radiator on the medium to the velocity of the radiator.

The radiation impedance for frequencies above the critical frequency is

$$Z_{rad} = \frac{S\,(\rho c)_{air}}{\sqrt{1-(f_c\,/\,f)}} \tag{10}$$

Equation (10) is taken from Reference 2.

Note that bending waves travel faster than the speed of sound at frequencies above the critical frequency. Again, the bending wave speed formula is given in equation (1).

<u>Radiation Efficiency</u>

The radiation efficiency for supersonic bending waves in structures is

$$\sigma_{rad} = \frac{1}{\sqrt{1-(f_c\,/\,f)}} \tag{11}$$

Equation (10) is taken from Reference 2.

The radiation efficiency of a vibrating body generating sound energy in air is

$$\sigma_{rad} = \frac{W_{rad}}{\left\langle v^2 \right\rangle (\rho c)_{air} S} \tag{12}$$

Equation (12) is taken from Reference 3, section 9.6.

References

1. L. Cremer and M. Heckl, Structure-Borne Sound, Springer-Verlag, New York, 1988.

2. Byam & Radcliffe, Statistical Energy Analysis Model and Connectors for Automotive Vibration Isolation Mounts.

3. L. Beranek & I. Ver, Noise and Vibration Control Engineering, Principles and Applications, Wiley, New York, 1992.