SHOCK & VIBRATION FATIGUE CRITERIA FOR ELECTRICAL COMPONENTS
Revision B

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## Introduction

*The author expresses appreciation to Jeilong Chung and Professor Steinberg for their guidance in the preparation of this tutorial.*

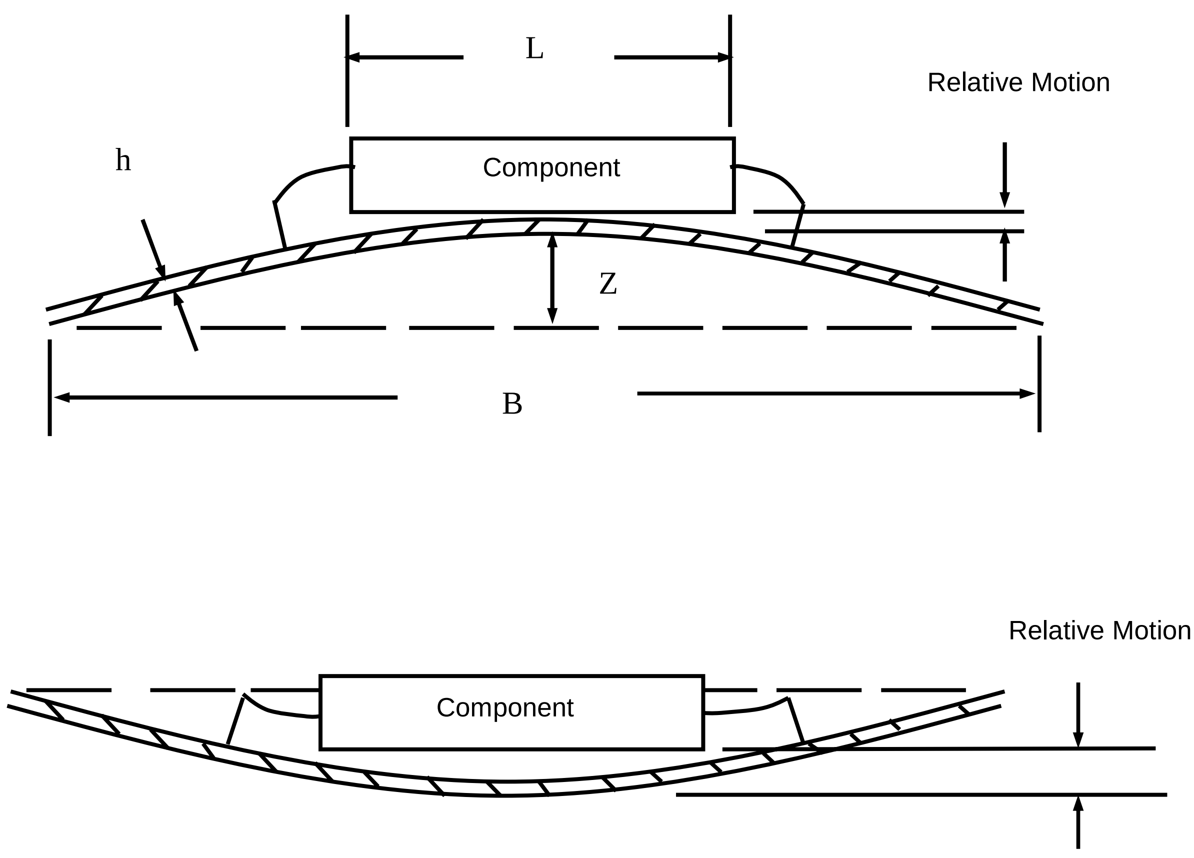



Figure 1.

Consider a circuit board that is simply supported about its perimeter. A concern is that repetitive bending of the circuit board will result in cracked solder joints or broken lead wires.

Let Z be the single-amplitude displacement at the center of the board that will give a fatigue life of about 20 million stress reversals in a random vibration environment, based upon the 3σ circuit board relative displacement.

An empirical formula for $Z_{3\sigma \text{ limit}}$ is

$$Z_{3\sigma \text{ limit}} = \frac{0.00022\ B}{C\,h\,r\,\sqrt{L}} \quad \text{inches} \tag{1}$$

where

B = length of the circuit board edge parallel to the component, inches

L = length of the electronic component, inches

H = circuit board thickness, inches

R = relative position factor for the component mounted on the board (Table 1)

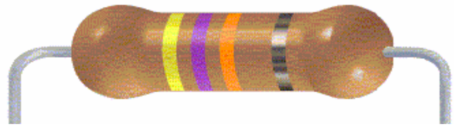
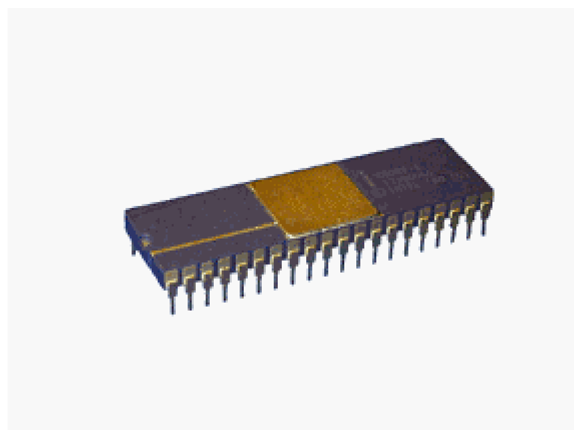
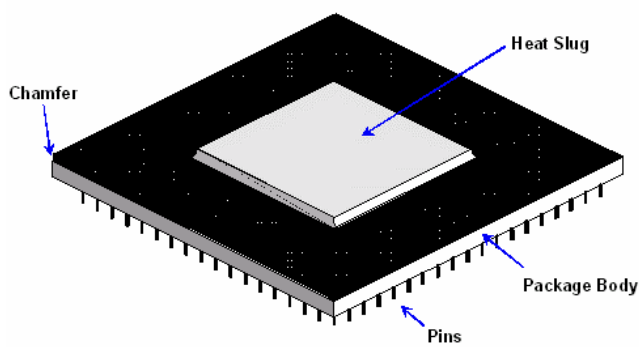
C = Constant for different types of electronic components (Table 2)
$0.75 \le C \le 2.25$

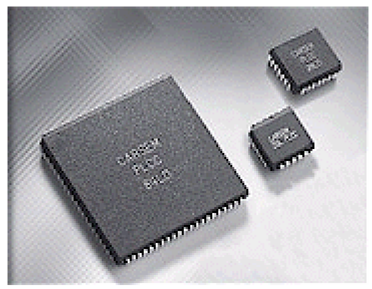
Equation (1) is taken from Reference 1. Further information on the B parameter is given in Appendix A.

Table 1.
Relative Position Factors for Component on Circuit Board

| R | Component Location (Board supported on all sides) |
|---|---|
| 1 | When component is at center of PCB (half point X and Y). |
| 0.707 | When component is at half point X and quarter point Y. |
| 0.50 | When component is at quarter point X and quarter point Y. |

Table 2. Constant for Different Types of Electronic Components

| C | Component | Image |
|---|---|---|
| 0.75 | Axial leaded through hole or surface mounted components, resistors, capacitors, diodes |  |
| 1.0 | Standard dual inline package (DIP) |   |
| 1.26 | DIP with side-brazed lead wires |  |
| 1.0 | Through-hole Pin grid array (PGA) with many wires extending from the bottom surface of the PGA |   |

| | | |
|---|---|---|
| 2.25 | Surface-mounted leadless ceramic chip carrier (LCCC).<br><br>A hermetically sealed ceramic package. Instead of metal prongs, LCCCs have metallic semicircles (called castellations) on their edges that solder to the pads. |  |
| 1.26 | Surface-mounted leaded ceramic chip carriers with thermal compression bonded J wires or gull wing wires. |  |
| 1.75 | Surface-mounted ball grid array (BGA).<br><br>BGA is a surface mount chip carrier that connects to a printed circuit board through a bottom side array of solder balls. |  |
| 0.75 | Fine-pitch surface mounted axial leads around perimeter of component with four corners bonded to the circuit board to prevent bouncing | — |
| 1.26 | Any component with two parallel rows of wires extending from the bottom surface, hybrid, PGA, very large scale integrated (VLSI), application specific integrated circuit (ASIC), very high scale integrated circuit (VHSIC), and multichip module (MCM). | — |

The resulting acceleration $\ddot{x}_{GRMS}$ can be determined by Miles equation.

$$\ddot{x}_{GRMS}(f_n, Q) = \sqrt{\left(\frac{\pi}{2}\right) f_n \; Q \; A} \tag{2}$$

where

| | |
|---|---|
| $f_n$ | is the natural frequency (Hz) |
| Q | is the amplification factor |
| A | is the input power spectral density amplitude ($G^2$ / Hz), assuming a constant input level. |

Note that

$$Q = \frac{1}{2\xi} \tag{3}$$

where $\xi$ is the viscous damping ratio.

Equation (2) is taken from Reference 1.

Now consider the realistic case where the acceleration power spectral density level varies with frequency. Nevertheless, constrain the acceleration power spectral density level to be constant within 1 octave of the natural frequency.

$$A = \hat{Y}_{APSD}(f_n) \tag{4}$$

where $\hat{Y}_{APSD}$ is the base input power spectral density.

$$\ddot{x}_{GRMS}(f_n, Q) = \sqrt{\left(\frac{\pi}{2}\right) f_n \; Q \; \hat{Y}_{APSD}(f_n)} \tag{5}$$

Note that the Miles equation may lead to error if the acceleration power spectral density function has significant variation with frequency. An alternative method, called the general method, overcomes this limitation. The general method is given in Appendix B.

The relative displacement is calculated by

$$Z_{RMS} = [386]\left[\frac{\ddot{x}_{GRMS}}{4\pi^2 f_n^2}\right] \quad \text{inches} \tag{6}$$

$$Z_{RMS} = [9.8]\left[\frac{\ddot{x}_{GRMS}}{f_n^2}\right] \quad \text{inches} \tag{7}$$

The mean relative displacement is zero, thus

$$Z_{3\sigma} = 3\,[\,Z_{RMS}\,] \quad \text{inches} \tag{8}$$

Substituting equations (2) and (8) into (9) yields

$$Z_{3\sigma} = [29.4]\left[\frac{1}{f_n^{1.5}}\right]\sqrt{\left(\frac{\pi}{2}\right) Q\ A} \quad \text{inches} \tag{9}$$

Thus, the relative displacement can be reduced by increasing the natural frequency assuming that

1. The base input power spectral density level A is constant with frequency.
2. The amplification factor Q is also constant.

Vibration Example

Consider a resistor mounted at the center of an electronics board. The parameters are given in Table 3.

Table 3. Example Parameters

| | | | |
|---|---|---|---|
| B | = | 6.0 inch | Circuit board length |
| L | = | 0.80 inch | Resistor length |
| H | = | 0.093 inch | Circuit board thickness |
| R | = | 1.0 | Center of circuit board from Table 1 |
| C | = | 0.75 | Axial lead component from Table 2 |
| Fn | = | 200 Hz | Circuit board natural frequency |
| Q | = | 10 | Amplification factor |

Calculate the relative displacement limit

$$Z_{3\sigma \text{ limit}} = \frac{0.00022\ B}{C\,h\,r\,\sqrt{L}} \quad \text{inches} \tag{10}$$

$$Z_{3\sigma \text{ limit}} = \frac{0.00022\ [6.0]}{[0.75][0.093][1.0]\sqrt{0.80}} \quad \text{inches} \tag{11}$$

$$Z_{3\sigma \text{ limit}} = 0.021 \quad \text{inches} \tag{12}$$

Now consider that the circuit board is subjected to the MIL-STD-1540C test level shown in Figure 2 and in Table 4.

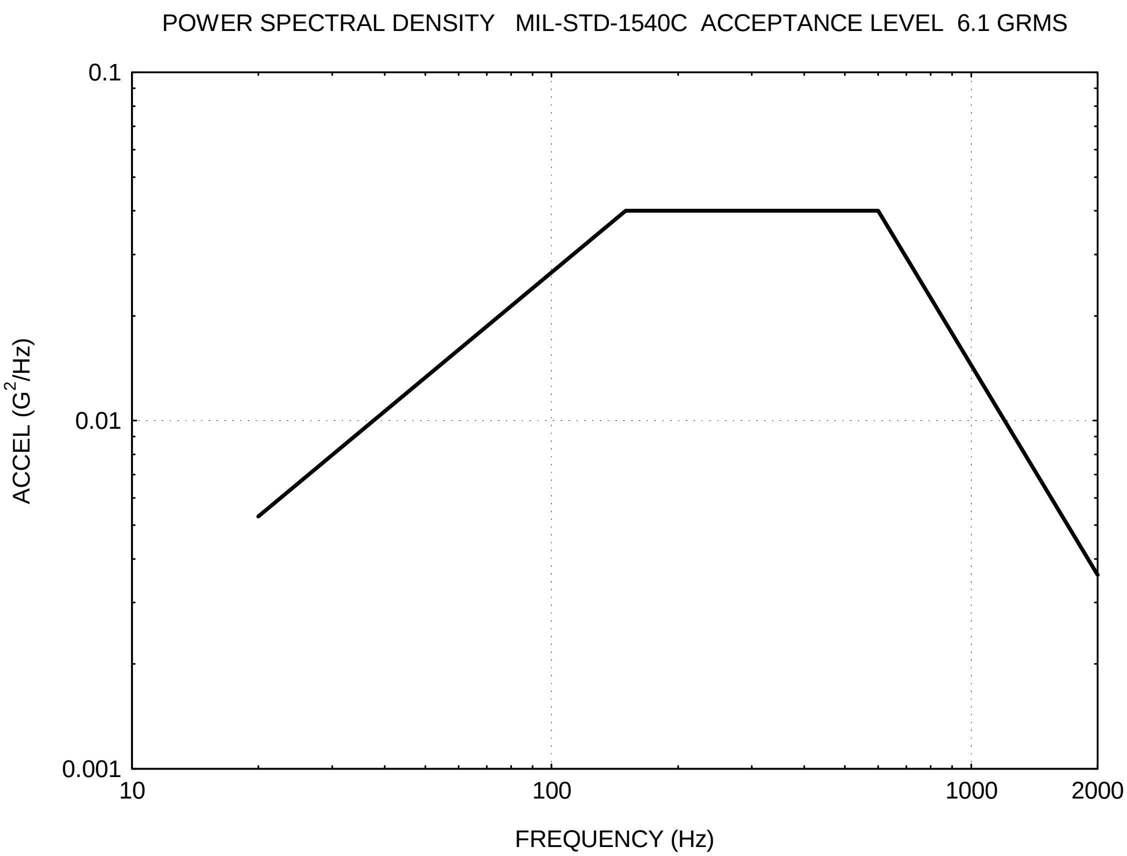



Figure 2.

| Table 4. MIL-STD-1540C, Acceptance Level, 6.1 GRMS Overall, 3 minutes/axis | |
|---|---|
| Frequency (Hz) | PSD ($G^2$/ Hz) |
| 20 | 0.0053 |
| 150 | 0.04 |
| 600 | 0.04 |
| 2000 | 0.0036 |

Assume that the circuit board behaves as a single-degree-of-freedom system. The response parameters are calculated using equations (5) through (9). The results are shown in Table 5.

| Table 5. Response to MIL-STD-1540C Level (fn = 200 Hz, Q = 10) | |
|---|---|
| $Z_{RMS}$ | 0.0027 inch |
| $Z_{3\sigma}$ | 0.0082 inch |
| $\ddot{x}_{GRMS}$ | 11.2 G |

Compare the response level to the limit level.

$$Z_{3\sigma\ \text{limit}} = 0.021\,\text{inch} \tag{13}$$

$$Z_{3\sigma\ \text{test response}} = 0.0082\ \text{inch} \tag{14}$$

$$Z_{3\sigma\ \text{test response}} < Z_{3\sigma\ \text{limit}} \tag{15}$$

The response is thus well within the limit. Define a safety factor

$$SF = \left(\frac{Z_{3\sigma\ \text{limit}}}{Z_{3\sigma\ \text{test response}}}\right) \tag{16}$$

$$SF = \left(\frac{0.021\,\text{inch}}{0.0082\ \text{inch}}\right) \tag{17}$$

$$SF = 2.56 \tag{18}$$

Now consider fatigue. The test time is 3 minutes/axis. Calculate an endurance test time based on Reference 1, section 8.25.

The life of the component $N_2$ is governed by the following fatigue equation

$$N_2 = N_1 \left( \frac{Z_{3\sigma \text{ limit}}}{Z_{3\sigma \text{ test response}}} \right)^b \tag{19}$$

where

b is a material constant, the slope of the

$N_1 = 20\ (10^6)$ cycles from Reference 1

The exponent b is unknown. Unfortunately, the calculation is highly sensitive to the exponent value.

The exponent is related to the slope of the S-N curve of the lead material, as well as related to any stress concentration factor.

A value as high as b = 6.4 may be appropriate, per Reference 1, section 8.25.

Note that a smaller value would be more conservative for this example. Thus, a value of b = 4 is assumed.

$$N_2 = 20\,(10^6) \left( \frac{0.021 \text{ inch}}{0.0082 \text{ inch}} \right)^4 \tag{20}$$

$$N_2 = 8.6\ (10^8) \quad \text{cycles} \tag{21}$$

Recall that the natural frequency is 200 Hz, or 200 cycles/second.

The endurance time $T_2$ is

$$T_2 = \frac{N_2}{f_n} \tag{22}$$

$$T_2 = \frac{8.6\ (10^8) \text{ cycles}}{200 \ \text{ cycles/sec}} \tag{23}$$

$$T_2 = 4.3(10^6) \ \text{ sec} \tag{24}$$

$$T_2 = 50 \text{ days} \tag{25}$$

The endurance time thus far exceeds the test time, which is 3 minutes/axis.

See Reference 3 for a further discussion of the fatigue equation.

## Shock

Steinberg gives the following empirical formula for the maximum allowable relative displacement limit for a shock environment.

$$Z_{\text{peak}} = \frac{0.00132\ B}{C\,h\,r\,\sqrt{L}} \quad \text{inches} \tag{26}$$

The shock equation (26) is the same as the vibration equation (1) except that the shock equation coefficient is six times higher.

The vibration equation has a lower coefficient because stress concentration factors are more significant for vibration than for a shock environment. Furthermore, there is fatigue factor for vibration that is some fraction of the ultimate stress limit. As an example, the stress limit for some nonferrous materials at 20 million cycles is one-third of the ultimate limit.

Shock, however, is evaluated with respect to the ultimate limit. Further details regarding equation (26) are given in Reference 1.

## References


1. Dave S. Steinberg, Vibration Analysis for Electronic Equipment, Second Edition, Wiley-Interscience, New York, 1988.

2. T. Irvine, An Introduction to the Vibration Response Spectrum, Vibrationdata Publications, 1999.

3. T. Irvine, Time-Scaling Equivalence Methods for Random Vibration Testing, Vibrationdata Publications, 2002.

# APPENDIX A

Note from Professor Steinberg

Regarding the B dimension for circuit boards, it relates to the length of the supported edge. When the board is supported along the edges with edge guides, the B dimension is the length of the board parallel to the longest length of a rectangular component mounted at the CENTER of the board. When the board is fastened with screws around the perimeter, and the screws are spaced away from the edges of the board, the B dimension becomes the dimension BETWEEN the supports as you indicated.

By the way that equation is based upon extensive tests to failure. Also the 0.00022 factor has a safety factor of 1.15 included to account for tolerance variations in the manufacturing processes.

Sincerely, Prof. Dave S. Steinberg

# APPENDIX B

The general method is shown in equation (B-1).

The overall acceleration response to a base input power spectral density function is

$$\ddot{x}_{GRMS}(f_n, \xi) = \sqrt{\sum_{i=1}^{N} \frac{\left\{1+(2\xi\rho_i)^2\right\}}{\left\{\left[1-\rho_i^{\,2}\right]^2+\left[2\xi\rho_i\right]^2\right\}} \hat{Y}_{APSD}(f_i)\Delta f_i} , \quad \rho_i = f_i / f_n$$

(B-1)

Equation (B-1) is taken from Reference 2.

The general method allows the power spectral density to vary with frequency. It also allows for power spectral density inputs with finite frequency limits.

The overall relative displacement is

$$z_{RMS}(f_n, \xi) = \left[386 \frac{\text{in} / \text{sec}^2}{\text{G}}\right] \sqrt{\sum_{i=1}^{N} \frac{1/\left[2\pi f_n\right]^4}{\left\{\left[1-\rho_i^{\,2}\right]^2+\left[2\xi\rho_i\right]^2\right\}} \hat{Y}_{APSD}(f_i)\Delta f_i} ,$$

$$\rho_i = f_i / f_n$$

(B-2)

$$Z_{3\sigma} = 3\left[Z_{RMS}\right] \quad \text{inches} \tag{B-3}$$