ELECTRICAL-MECHANICAL ANALOGY FOR DYNAMIC SYSTEMS

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Table 1. Analogous Parameters

Mechanical System		Electrical Systems: Voltage-Force Analogy		Electrical Systems: Current-Force Analogy	
	D'Alembert's principle		Kirchhoff's voltage law		Kirchhoff's current law
	Degree-of-freedom		Loop		Node
	Force Applied		Switch Closed		Switch Closed
F	Force	v	Voltage	i	Current
m	Mass	L	Inductance	C	Capacitance
x	Displacement	q	Charge		$\phi = \int v\,dt$
$\dot{x}$	Velocity	i	Loop Current	v	Node voltage
c	Damping	R	Resistance	1/R	Conductance
k	Spring	1/C	1/Capacitance	1/L	1/Inductance
	Coupling Element		Element common to two loops		Element between nodes

Impedance

Table 2. Electrical Impedance	
Element	Impedance
Capacitor	$Z_C = \frac{1}{j\omega C}$
Inductor	$Z_L = j\omega L$
Resistor	$Z_R = R$

Current-Force Analogy

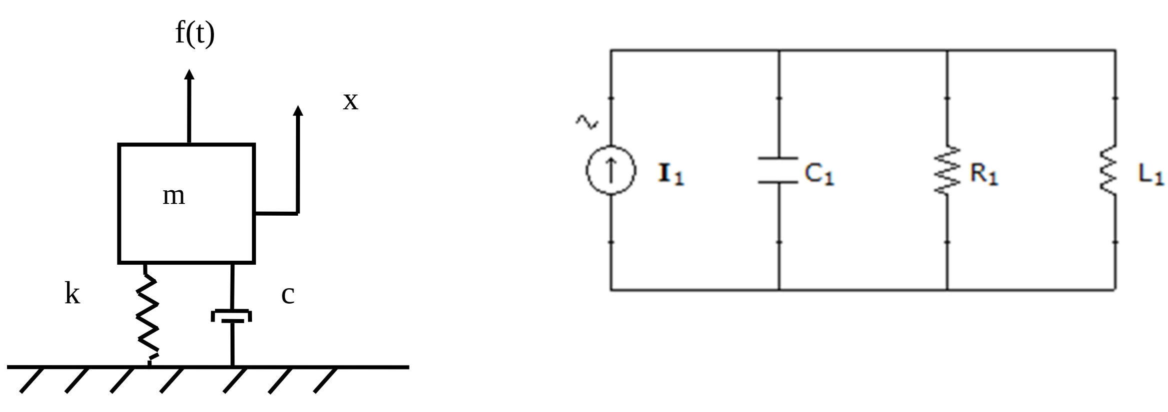

Figure 1. Equivalent Mechanical and Electrical Systems, SDOF System

The equation of motion for the mechanical oscillator is

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F(t) \tag{1}$$

Convert to velocity format.

$$m \frac{d}{dt}\left(\frac{dx}{dt}\right) + c \frac{dx}{dt} + k \int \left(\frac{dx}{dt}\right) dt = F(t) \tag{2}$$

Substitute the current-force analogy values from Table 1.

$$C \frac{dv}{dt} + \frac{1}{R} v + \frac{1}{L} \int v \, dt = i(t) \tag{3}$$

Take the derivative.

$$C \frac{d^2v}{dt^2} + \frac{1}{R} \frac{dv}{dt} + \frac{1}{L} v = \frac{d}{dt} i(t) \tag{4}$$

Voltage-Force Analogy

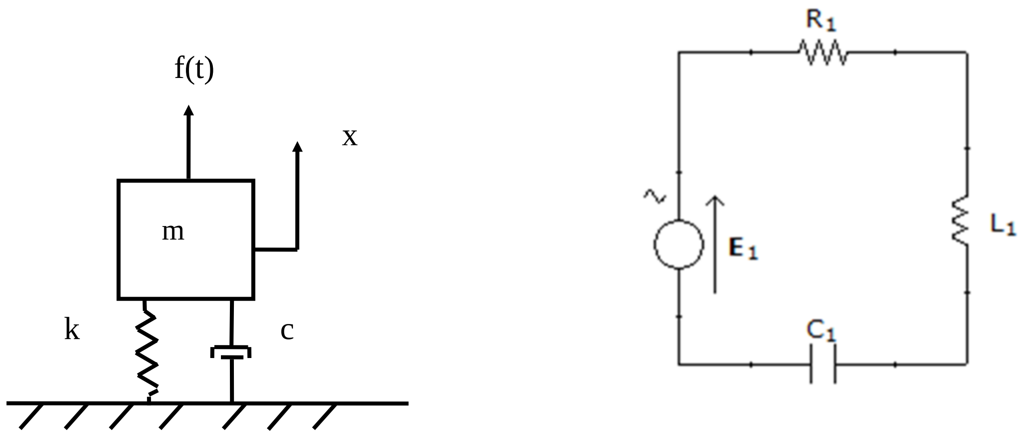

Figure 2. Equivalent Mechanical and Electrical Systems, SDOF System

Recall

$$m\frac{d}{dt}\left(\frac{dx}{dt}\right)+c\frac{dx}{dt}+k\int\left(\frac{dx}{dt}\right)dt=F(t) \tag{5}$$

Substitute the voltage-force analogy values from Table 1.

$$L\frac{di}{dt}+Ri+\frac{1}{C}\int i\ dt=v(t) \tag{6}$$

References

1. W. Seto, Mechanical Vibrations, McGraw-Hill, New York, 1964.
2. Cyril Harris, Shock and Vibration Handbook, 4th edition, McGraw-Hill, New York, 1995. See E. Hixson, Mechanical Impedance, Chapter 10.
3. T. Irvine, The Generalized Coordinate Method for Discrete Systems, Revision D, Vibrationdata, 2010.

APPENDIX A

Two-degree-of-freedom System

Voltage-Force Analogy

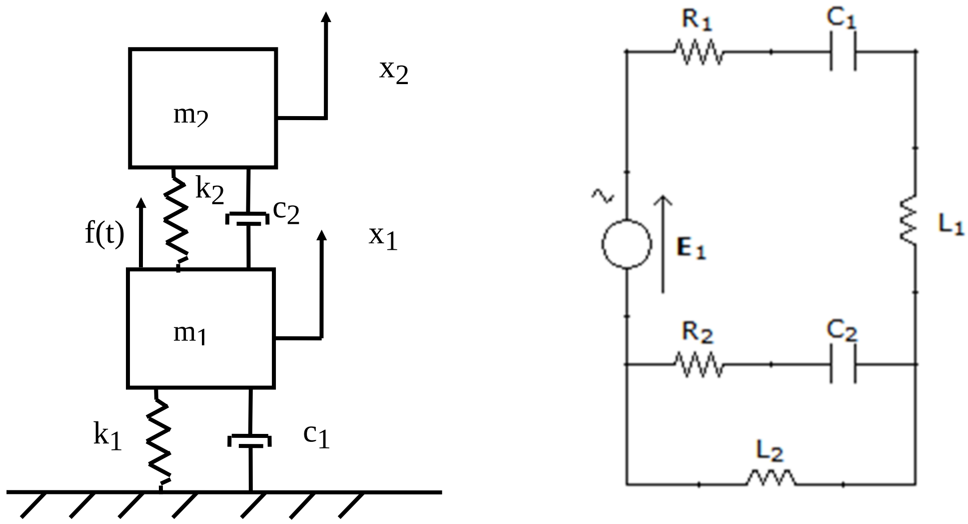

Figure A-1.

The equation of motion from Reference 3 is

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f(t) \\ 0 \end{bmatrix}$$

(A-1)

The velocity form is

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \left\{ \frac{d}{dt} \begin{bmatrix} dx_1/dt \\ dx_2/dt \end{bmatrix} \right\} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} dx_1/dt \\ dx_2/dt \end{bmatrix}$$

$$+ \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} \int \left(\frac{dx_1}{dt} \right) dt \\ \int \left(\frac{dx_2}{dt} \right) dt \end{bmatrix} = \begin{bmatrix} f(t) \\ 0 \end{bmatrix}$$

(A-2)

Substitute the voltage-force analogy values from Table 1.

$$\begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix} \begin{bmatrix} di_1/dt \\ di_2/dt \end{bmatrix} + \begin{bmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$+ \begin{bmatrix} 1/C_1 + 1/C_2 & -1/C_2 \\ -1/C_2 & C_2 \end{bmatrix} \begin{bmatrix} \int \left(\frac{di_1}{dt} \right) dt \\ \int \left(\frac{di_2}{dt} \right) dt \end{bmatrix} = \begin{bmatrix} v(t) \\ 0 \end{bmatrix}$$

(A-3)

APPENDIX B

SDOF System, Base Excitation

Voltage-Force Analogy

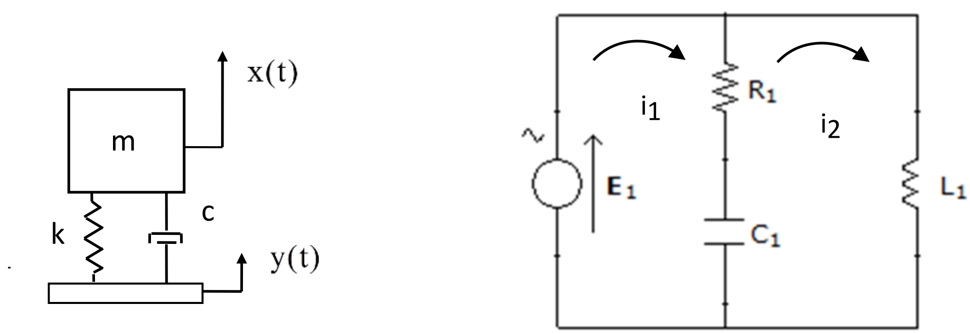

Figure B-1.

The equations of motion are

$$m\frac{d^2x}{dt^2}+c\left(\frac{dx}{dt}-\frac{dy}{dt}\right)+k(x-y)=0 \qquad \text{(B-1)}$$

and

$$-c\left(\frac{dx}{dt}-\frac{dy}{dt}\right)-k(x-y)=F(t) \qquad \text{(B-2)}$$

where F(t) is the force needed to maintain a prescribed $\frac{dy}{dt}$ or current by analogy.

The equations of motion in term of velocity are

$$m\frac{d}{dt}\left(\frac{dx}{dt}\right)+c\left(\frac{dx}{dt}-\frac{dy}{dt}\right)+k\left\{\int\left(\frac{dx}{dt}\right)dt-\int\left(\frac{dy}{dt}\right)dt\right\}=0 \qquad \text{(B-3)}$$

and

$$-c\left(\frac{dx}{dt}-\frac{dy}{dt}\right)-k\left\{\int\left(\frac{dx}{dt}\right)dt-\int\left(\frac{dy}{dt}\right)dt\right\}=F(t) \qquad \text{(B-4)}$$

Substitute the current-force analogy values from Table 1.

$$L\frac{d}{dt}\left(\frac{di_1}{dt}\right)+R\left(\frac{di_1}{dt}-\frac{di_2}{dt}\right)+\frac{1}{C}\left\{\int\left(\frac{di_1}{dt}\right)dt-\int\left(\frac{di_2}{dt}\right)dt\right\}=0 \qquad \text{(B-5)}$$

and

$$-R\left(\frac{di_1}{dt}-\frac{di_2}{dt}\right)-\frac{1}{C}\left\{\int\left(\frac{di_1}{dt}\right)dt-\int\left(\frac{di_2}{dt}\right)dt\right\}=F(t) \qquad \text{(B-6)}$$

<u>Kirchhoff's voltage law, Loop Current Method</u>

Solve for the current in Figure B-1.

$$E_1+(i_1-i_2)\left(R+\frac{1}{j\omega C}\right)=0 \qquad \text{(B-7)}$$

$$(i_1-i_2)\left(R+\frac{1}{j\omega C}\right)-i_2(j\omega L)=0 \qquad \text{(B-8)}$$

Let

$$Z_1 = \left(R + \frac{1}{j\omega C} \right) \tag{B-9}$$

$$Z_2 = j\omega L \tag{B-10}$$

$$E_1 + (i_1 - i_2)Z_1 = 0 \tag{B-11}$$

$$(i_1 - i_2)Z_1 - i_2 Z_2 = 0 \tag{B-12}$$

Simply as follows

$$i_1 Z_1 - i_2 Z_1 = -E_1 \tag{B-13}$$

$$i_1 Z_1 + i_2(- Z_1 - Z_2) = 0 \tag{B-14}$$

Thus,

$$i_1 Z_1 = i_2 (Z_1 + Z_2) \tag{B-15}$$

$$i_2(Z_1 - Z_2) - i_2 Z_1 = -E_1 \tag{B-16}$$

$$- i_2 Z_2 = -E_1 \tag{B-17}$$

$$i_2 = E_1 / Z_2 \tag{B-18}$$

$$i_1 = i_2 \frac{(Z_1 + Z_2)}{Z_1} \tag{B-19}$$

By substitution,

$$i_1 = E_1 \frac{(Z_1 + Z_2)}{Z_1 Z_2} \tag{B-20}$$

$$E_1 = i_1 \left[\frac{Z_1 Z_2}{Z_1 + Z_2} \right] \tag{B-21}$$

Thus the voltage source E_1 must be controlled to yield the desired i_1.

By analogy, the force must be controlled to yield the desired base displacement and velocity.