ENVELOPING DATA VIA THE VIBRATION RESPONSE SPECTRUM

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October 11, 1999

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## Introduction

The vibration response spectrum was introduced in Reference 1 as a method for enveloping vibration data. The purpose of this tutorial is to give further techniques for efficient use of this tool. These advanced techniques are also given in Reference 2.

Recall that vehicles and machines are subjected to random vibration. Here are a few examples:

1. A missile encounters aerodynamic buffeting as it accelerates through the transonic velocity and encounters its maximum pressure condition.
2. The turbine engines on a jet aircraft produce both structural-borne vibration and an acoustic pressure field.
3. An automobile is subjected to base excitation as it travels down a washboard road.
4. The motors and gears on a silicon wafer polishing machine produce vibration.
5. A network server has a number of hard drives, each of which is both a source and recipient of vibration.

These vehicles and machines have electronic components which must withstand the vibration environments. The components must be designed and tested accordingly.

The design and test levels must meet two requirements:

1. The levels must envelop the maximum expected level, which typically includes a safety factor.
2. The levels must also envelop minimum workmanship screening levels.

For simplicity, a single test level is typically derived to cover both of the above requirements. Specific guidelines for deriving test levels are given in References 3 through 7, for example.

The maximum expected level should be derived from measured vibration data. The measured data is typically represented in terms of an acceleration power spectral density function. This function is considered as the base input to nearby electrical components.

In some cases, the function may reveal that the data is composed of several narrowband peaks superimposed on a broadband random background. The maximum expected level, however, should consist of a few simple segments on a log-log power spectral density plot. For example, a maximum of seven segments is suggested in Reference 6.

One reason for this simplicity is that a vehicle's spectral peaks might have frequency shifts under changing conditions. Enveloping an entire measured power spectral density function, however, can produce a very conservative maximum expected level.

The vibration response spectrum is an alternate approach which solves this dilemma. The vibration response spectrum shows how a simplified base input level with reasonable amplitude can serve as a maximum predicted level without enveloping each spectral peak of the measured data, as demonstrated in Reference 1.

This tutorial extends the method to obtain the envelope with the least possible overall level.

General Method

Both the Miles equation and the general method were considered in Reference 1. The Miles equation makes a number of restrictive assumptions. For example it assumes that the base input is white noise from a frequency of zero to infinity.

The general method is preferred since it allows the base input power spectral density to vary with frequency. This method is derived in Reference 1.

Typically, the function base input $P(f)$ is available in a digital format $P(f_i)$.

The general method gives the overall response shown in equation (1).

$$\ddot{x}_{GRMS} = \sqrt{\sum_{i=1}^{N} \frac{\left\{1+(2\xi\rho_i)^2\right\}}{\left\{\left[1-\rho_i^2\right]^2+\left[2\xi\rho_i\right]^2\right\}} P(f_i)\Delta f_i}, \tag{1}$$

$$\rho_i = f_i / f_n$$

where

$\ddot{x}_{GRMS}$ = the overall response,
$P(f_i)$ = the power spectral density level at frequency $f_i$,
$f_n$ = the natural frequency.

Equation (1) must be calculated for each natural frequency of interest. It can be easily implemented via a computer program.

Sample Power Spectral Density

Consider a commercial jet aircraft. A computer mounted on an avionics bulkhead is subjected to the sample flight vibration data is shown in Figure 1.

The data consists of three narrowband peaks superimposed on broadband random vibration. The peak at 240 Hz represents a bulkhead bending mode. The peak at 540 Hz is due to the engine shaft speed. The peak at 1080 Hz is a harmonic of the shaft speed.

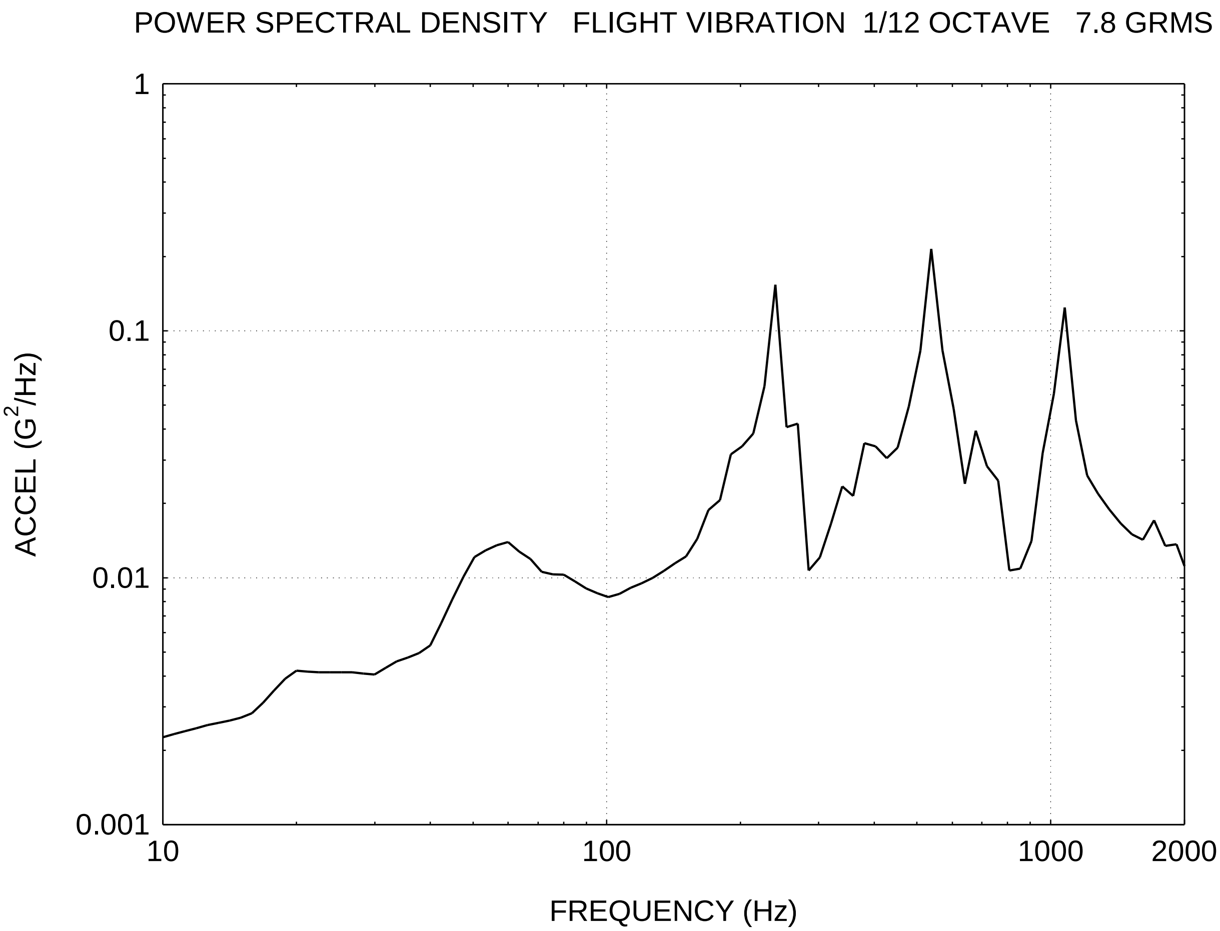


Figure 1. Flight Power Spectral Density

## Vibration Response Spectrum

The corresponding vibration response spectrum is shown in Figure 2. It was calculated via the general method, equation (1). Note that the response spectrum smoothes the spectral peaks. Also, note that the horizontal axis dimension is given in terms of natural frequency. For example a component with a natural frequency of 540 Hz would have a 30 GRMS response to the bulkhead vibration, assuming single-degree-of-freedom behavior and Q=10.

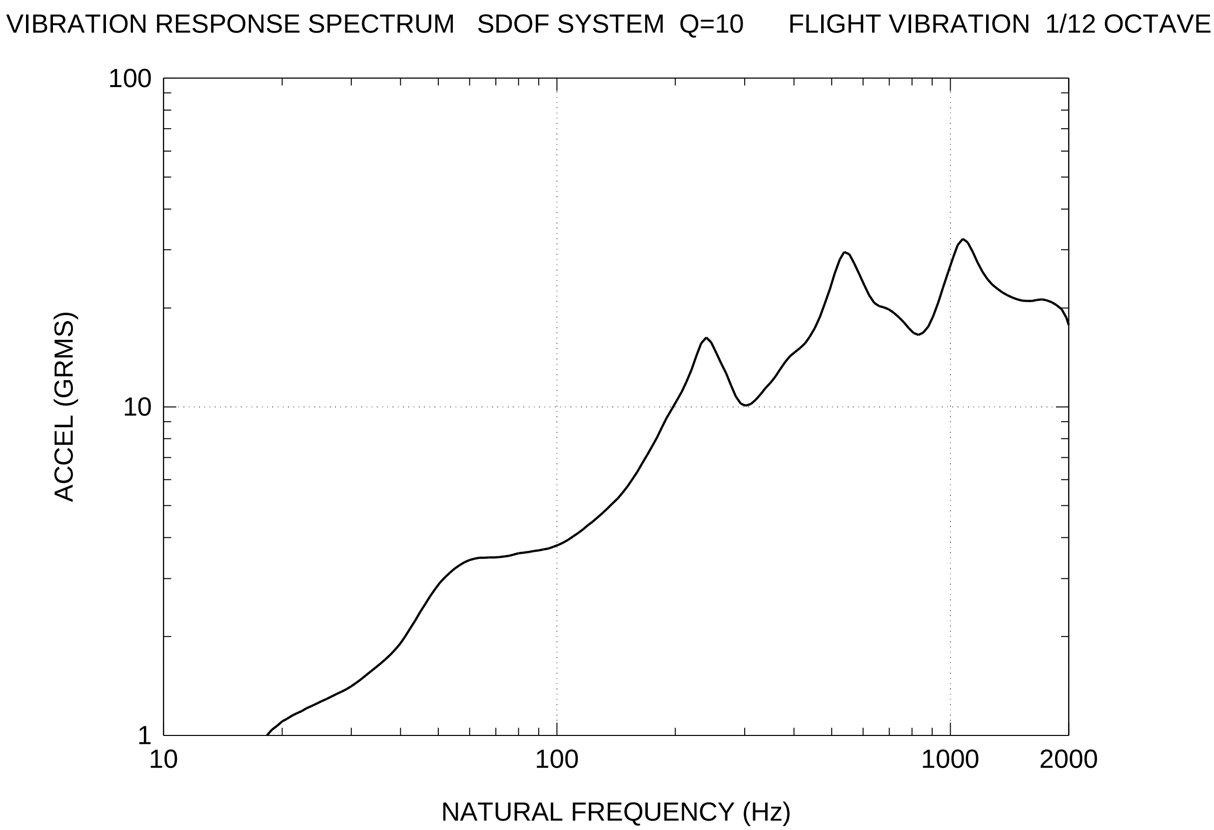


Figure 2. Flight Vibration Response Spectrum

## Flight Data Envelope

An envelope must be determined for the flight data in Figure 2.

The typical assumptions for an envelope are:

1. The equipment-structure interaction is negligible, such that the force input can be ignored.
2. The bulkhead flight vibration in Figure 1 was stationary.
3. Spatial variation across the bulkhead can be neglected.
4. A simplified power spectral density is required to represent the flight data, even though a sine-on-random function might be more appropriate.

Note that each of these assumptions has been the subject of numerous papers, as documented in Reference 6, for example.

## Traditional Envelope

A typical envelope with three segments is shown in Figure 3.

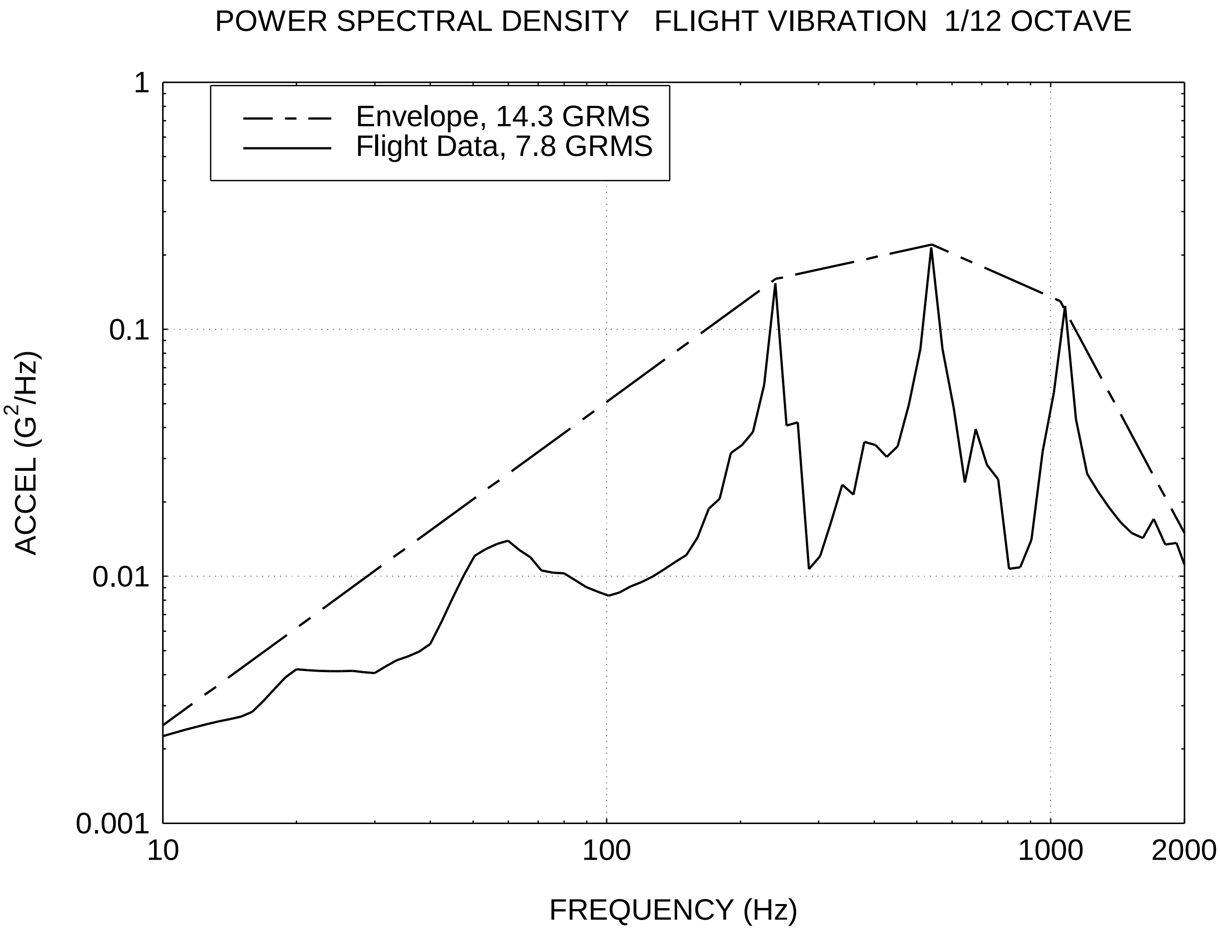


Figure 3. Traditional Envelope

There is no need to calculate a vibration response spectrum since the envelope level clearly envelops the flight data. The envelope has an overall acceleration level of 14.3 GRMS

Note that the overall envelope acceleration level is 5.3 dB greater than that of the flight level. Such a high margin is unnecessary.

Optimal Envelope Steps

Again, the simplified level does not need to envelop each of the flight level's spectral peaks. Rather, the vibration response spectrum of the simplified level must envelop that of the flight level.

A number of additional assumptions are made for this approach:

1. The bulkhead avionics components can be considered as independent, single-degree-of-freedom systems.
2. The damping of each system is fixed at 5%, which is equivalent to Q=10.

Obtaining an “optimal envelope” is a trial-and-error process. Furthermore, there may not be a unique solution.

Again, the general method, equation (1), should be used for this process.

The process is carried out by the following steps:

1. Calculate the vibration response spectrum of the flight power spectral density.
2. Select the desired number of segments for the simplified envelope level.
3. Generate a set of arbitrary power spectral density coordinates. The number of coordinates is equal to the number of segments plus one.
4. Impose optional constraints, such as each slope must have an absolute value of 6 dB/octave or less.
5. Interpolate the sample power spectral density function.
6. Calculate the vibration response spectrum of the interpolated arbitrary power spectral density.
7. Compare the vibration response spectrum of step 6 with that of the flight data from step 1.
8. Scale the arbitrary vibration response spectrum so that it fully envelops that of the flight data.
9. Apply the square of the scale factor to the arbitrary power spectral density function.
10. Calculate the overall RMS acceleration level of the scaled power spectral density.
11. Repeat steps 3 through 8 “many” times.
12. Select the power spectral density level which yields the lowest acceleration RMS.

As an alternative, other selection criteria can be used in step 12. For example, the overall velocity and displacement could be considered.

Optimal Envelope Example

An optimal envelope was derived using the above steps, based on four segments. The result is a shown in Table 1 and Figure 4.

Table 1.
Optimal Envelope Power Spectral Density

| Frequency (Hz) | Acceleration (G^2/Hz) |
|---|---|
| 10 | 0.0024 |
| 40 | 0.0084 |
| 105 | 0.0285 |
| 400 | 0.1380 |
| 2000 | 0.0370 |

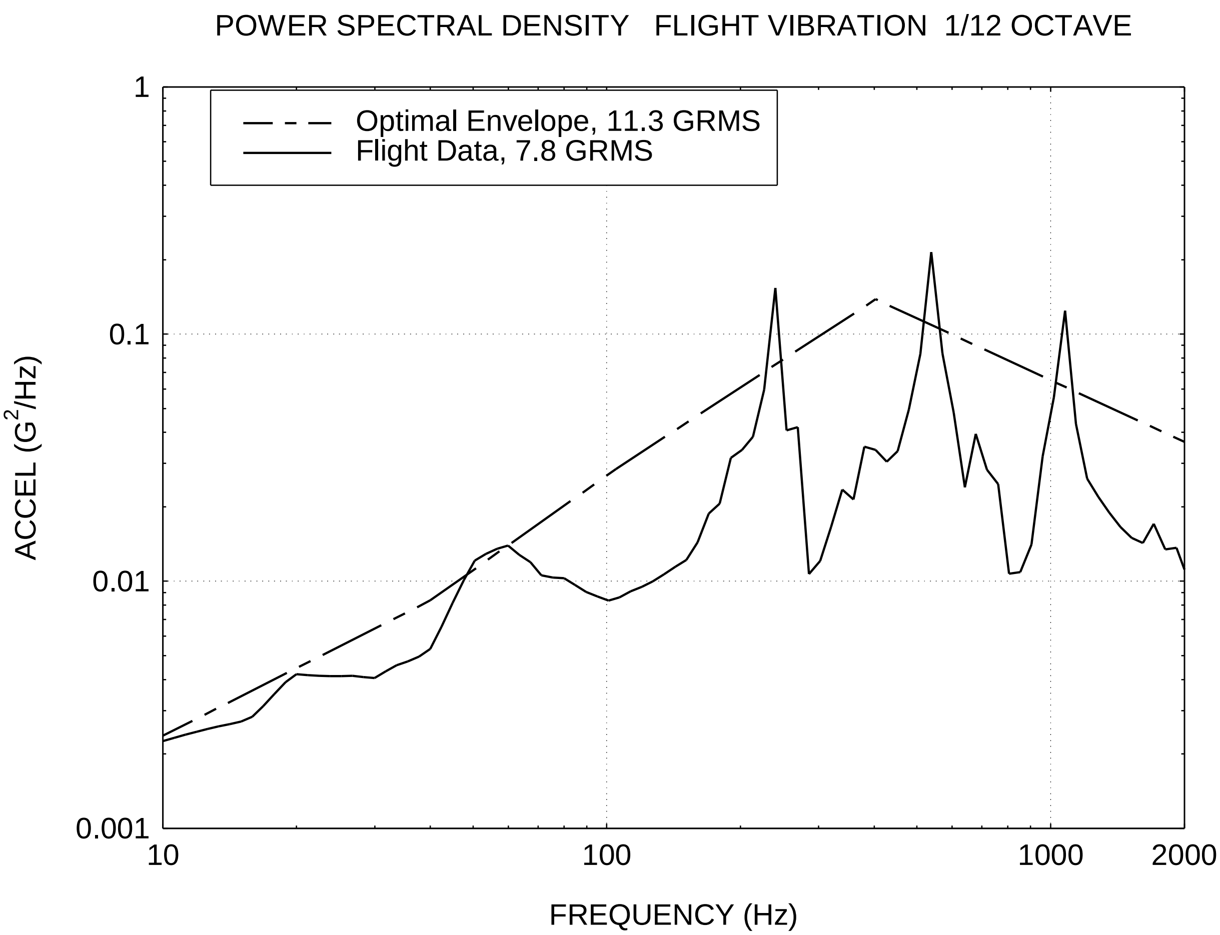


Figure 4. Optimal Envelope

The optimal envelope has an overall acceleration level of 11.3 GRMS.

This overall level is 3.2 dB greater than the flight data overall level. Note that an optimal envelope level is always greater than the flight level in terms of the overall acceleration. Nevertheless, the optimal envelope allows for peak clipping. The spectral peaks at 240 Hz and 540 Hz in Figure 4 are each clipped by approximately 6 dB.

Furthermore, the optimal envelope overall level is 2.0 dB less than the traditional envelope overall level, which was shown in Figure 3.

The respective overall levels are shown in Table 2.

| Table 2.<br>Overall Acceleration Levels | |
|---|---|
| PSD | GRMS |
| Traditional Envelope | 14.3 |
| Optimal Envelope | 11.3 |
| Flight Data | 7.8 |

The justification for the optimal envelope level is given by the vibration response spectra comparison in Figure 5. The optimal envelope in Figure 4 can thus be used as a basis for deriving design and test levels for the computer in the commercial jet example.

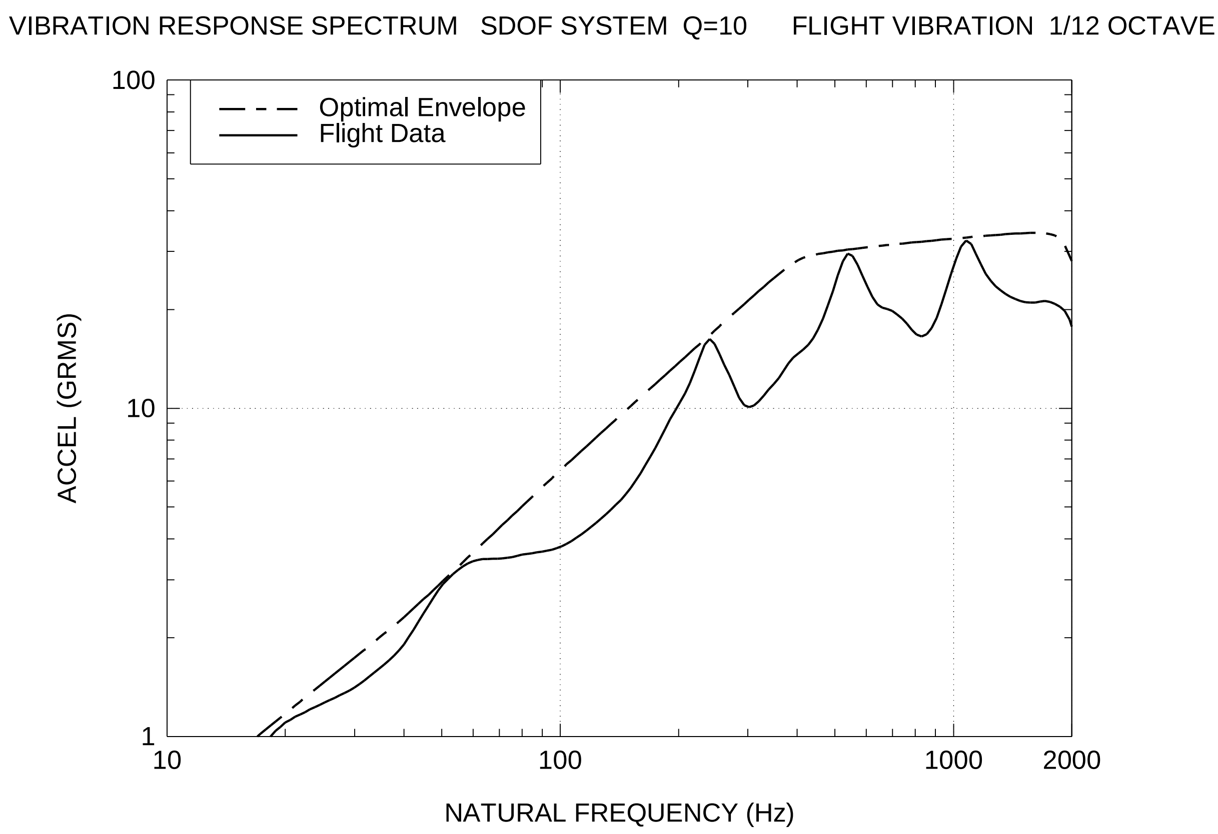


Figure 5. Optimal Envelope Vibration Response Spectrum

## Conclusion

This paper presented an optimal method for enveloping vibration data. An example was given whereby the optimal method was used to achieve 2.0 dB reduction in the overall envelope level, compared to the traditional method.

Additional steps would be required to determine appropriate design and test levels. In particular, a safety or statistical uncertainty factor might be required per the guidelines in References 3 through 7. Nevertheless, optimizing the envelope level is desirable in order to prevent design and test levels from becoming overly conservative.

## References


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