Deriving a Random Vibration Maximum Expected Level with Consideration for Kurtosis Revision C

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Acronyms

PSD	-	Power Spectral Dens	ity
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- SRS Shock Response Spectrum
- VRS Vibration Response Spectrum

Introduction

Consider the case where a component is to be subjected to base input random vibration in the field or during a rocket vehicle flight. The component must be designed and tested accordingly.

The typical approach is to derive a maximum expected level in terms of an acceleration PSD function. The maximum expected PSD may be derived from measured data or from empirical scaling of a reference PSD. Margin may be added as appropriate for statistical uncertainty, etc.

The component is then subjected to a shaker table test which is controlled to the PSD specification. The corresponding time history usually has a normal distribution, with a kurtosis of 3.

Real-world random vibration environments, however, may have kurtosis value greater than 3. This would be the case if transient pulses or high-sigma peaks appeared on the random time history data. In this case, the field vibration would most likely have a higher damage potential than the shaker table test, assuming that the test specification did not have margin. This is a possible explanation for components which pass the vibration test but still fail in the field, as discussed in Reference 1.

Kurtosis Enveloping Method

A method for enveloping measured data for the purpose of deriving a maximum expected level was given in Reference 2. The method uses a VRS comparison assuming single-degree-of-freedom behavior.

A limitation of the method in Reference 2 was that it effectively assumes that the random environment could be replicated with a normal distribution on a shaker table. As a simplification, the Reference 2 method assumes, or at least implies, that the random source environment itself has a normal distribution.

A conservative approach is needed for a random environment with high kurtosis peaks.

An approach which may be suitable for some cases¹ is as follows:

- 1. Take the SRS of the measured environment time history.
- 2. Derive a candidate PSD which has a 3-sigma VRS that envelops the measured SRS.
- 3. Perform step 2 using optimization techniques similar to those in Reference 2 to achieve the least PSD which still satisfies the goal of step 2.

The resulting optimized PSD would then cover the source environment's high-kurtosis peaks in terms of a shaker test time history with a normal distribution.²

Note that a higher-sigma VRS may be allowable in step 2 if the resulting PSD specification has a long duration. i.e. - Increasingly higher sigma peaks are likely to occur as the test duration is increased. See Reference 3, Appendix B. Allowing for higher-sigma peaks would result in a lower optimized PSD amplitude, as shown in Appendix B of this paper.

Example



Figure 1. Terrier-Black Brant, Rocket Vehicle Launch

The vehicle has two solid motor stages. It flew a suborbital trajectory.

¹ Engineering judgment is required to determine whether this approach is suitable. A separate SRS specification may be appropriate in cases with "very high" kurtosis peaks.

² Additional steps might be needed to derive a test level from the optimized PSD, such as adding an uncertainty margin and covering a minimum workmanship level.



Figure 2.

A measured flight accelerometer time history from the Terrier-Black Brant vehicle is shown in Figure 2.

The time history has high kurtosis peaks beginning at 31 seconds. The peaks may have been driven by rough combustion during the Black Brant, second stage motor burn.

POWER SPECTRAL DENSITY



Figure 3.

The power spectral density from the flight data³ is shown in Figure 3. It covers the second stage motor burn duration from 12 to 44 seconds.⁴

The optimized PSD covers the flight environment PSD which is a basic requirement. The optimized PSD also has margin.

Furthermore, the optimized PSD has a three-sigma VRS which covers the SRS of the fight data, as shown in Figure 4. Note that the SRS was calculated for the second stage motor burn duration from 12 to 44 seconds. This duration does not include any pyrotechnic shock events.

The optimization was performed using a C/C++ program: envelope_kurtosis.cpp.

The optimized PSD could then be used to derive a maximum expected flight level. Acceptance and qualification test levels could be derived from the maximum level. The component tests could then be performed on a shaker using a random time history with a normal distribution and with a kurtosis of 3.

³ The spectral peak at 110 Hz and its integer harmonics are due to pressure oscillations in the solid rocket motor cavity. The oscillations never formed into a sinusoidal pattern, but rather had a highly transient, narrowband characteristic, perhaps due to rough combustion.

⁴ A more thorough approach would be to divide the motor burn duration into segments. Then calculate a PSD for each segment. Then take a maximum envelope of the segment PSDs.



Figure 4.

References

- 1. T. Irvine, Kurtosis in Vibration Analysis, Vibrationdata, 2008.
- 2. T. Irvine, Enveloping Data via the Vibration Response Spectrum, Vibrationdata, 1999.
- 3. T. Irvine, Equivalent Static Loads for Random Vibration, Vibrationdata, 2009.

APPENDIX A

Kurtosis

Kurtosis is a parameter that describes the shape of a random variable's histogram or its equivalent probability density function (PDF).

The kurtosis for a time series Y_i is

$$\text{Kurtosis} = \frac{\displaystyle\sum_{i=1}^{n} [Y_i - \mu]^4}{n\sigma^4}$$

where

$$\mu$$
 = mean

 σ = standard deviation

n = number of samples

The term in the numerator is the "fourth moment about the mean."

A pure sine time history has a kurtosis of 1.5.

A time history with a normal distribution has a kurtosis of 3.

Some alternate definitions of kurtosis subtract a value of 3 so that a normal distribution will have a kurtosis of zero.

A kurtosis larger than 3 indicates that the distribution is more peaked and has heavier tails than a normal distribution with the same standard deviation.

APPENDIX B

Rayleigh Distribution, n-sigma Method



Figure B-1.

The example from the main text is repeated assuming that the envelope level has a duration of 60 seconds, which could represent a test duration.

The n-value, or crest factor, is calculated from the natural frequency fn and duration T as follows using the formula in Reference 3 for a Rayleigh distribution of the peaks.

$$n = \sqrt{2\ln(fnT)} \tag{B-1}$$

As an example, the expected peak value at 200 Hz for a 60-second duration is

$$n = \sqrt{2 \ln (\text{fn T})} = \sqrt{2 \ln (200 \text{ Hz})(60 \text{ sec})} = 4.33$$
 (B-2)

Thus, the highest peak expected for this case is 4.33σ .



Figure B-2.

The frequency-dependent n-value from equation (1) was applied the $n\sigma$ VRS.