A FATIGUE EQUIVALENCE METHOD FOR ENVELOPING NONSTATIONARY RANDOM VIBRATION

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Introduction

Electronic components are subjected to random vibration testing to verify the integrity of the design and workmanship. The random vibration tests are specified in terms of a base input power spectral density function, along with the duration.

Consider a component which must withstand a number of different vibration environments, where each is characterized by a power spectral density (PSD) and a duration. The goal is to derive a single test level which can be performed on a shaker table for some reasonable duration which may be substantially shorter than the expected service life.

Time Scaling Method

MIL-STD-1540C, paragraph 3.3.3 states:

> The fatigue damage potential is taken to be proportional to the fourth power of amplitude, unless another basis can be justified.

The same statement is made in MIL-HDBK-340A, paragraph 3.3.3.

The amplitude is assumed to be the GRMS amplitude.

Fatigue is thus assumed to be proportional to the GRMS^4 level, which is equivalent to {PSD (G^2/Hz)}^2.

Further information is given in References 1 through 3.

Enveloping Multiple PSD Curves

The equivalent PSD_{eq} for all events is assumed to be:

$$T_1 PSD_1^2 + T_2 PSD_2^2 + \ldots + T_n PSD_n^2 = T_{eq} PSD_{eq}^2 \quad (1)$$

$$\{ T_1 PSD_1^2 + T_2 PSD_2^2 + \ldots + T_n PSD_n^2 \} / T_{eq} = PSD_{eq}^2 \quad (2)$$

where T_{eq} is the equivalent duration.

Example

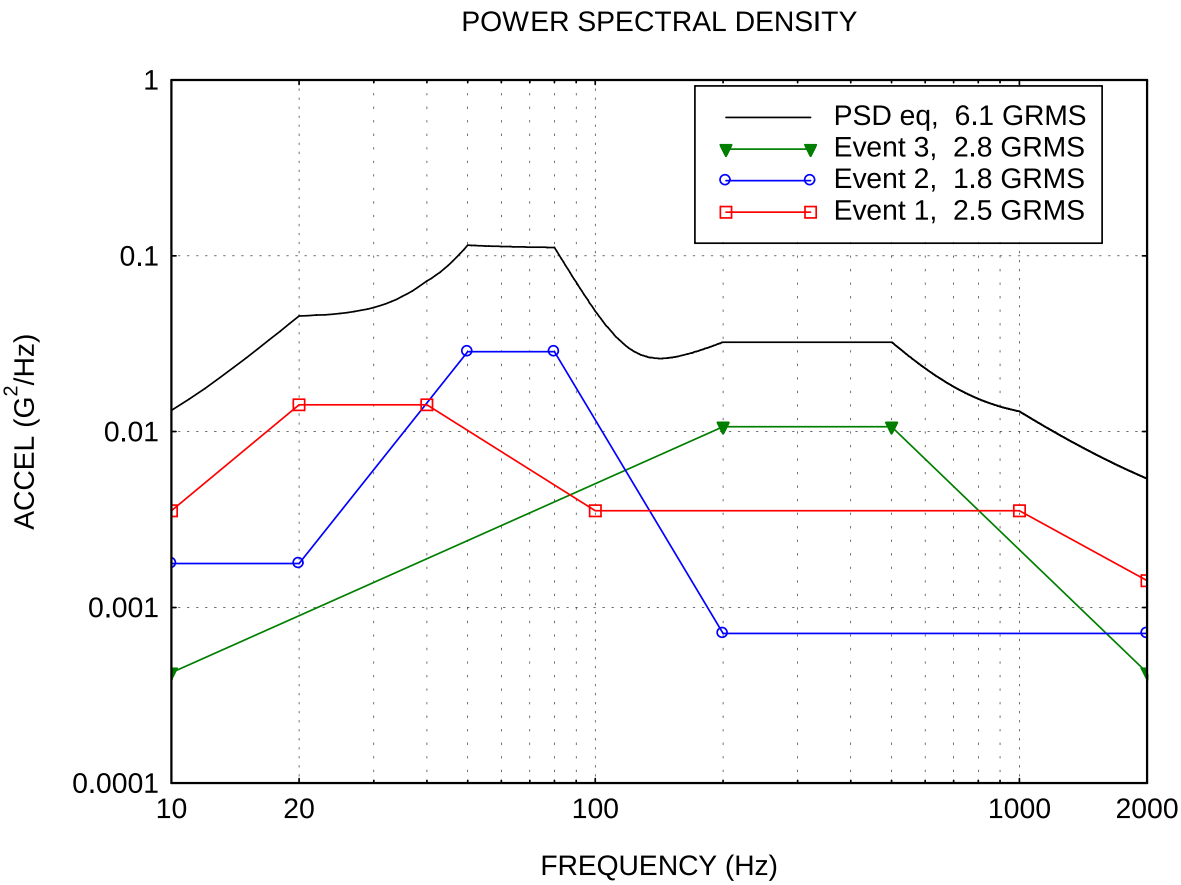

Figure 1.

Event 1

Freq (Hz)	PSD (G^2/Hz)
10	3.5E-03
20	1.4E-02
40	1.4E-02
100	3.5E-03
1000	3.5E-03
2000	1.4E-03

Event 2

Freq (Hz)	PSD (G^2/Hz)
10	1.8E-03
20	1.8E-03
50	2.8E-02
80	2.8E-02
200	7.1E-04
2000	7.1E-04

Event 3

Freq (Hz)	PSD (G^2/Hz)
10	4.3E-04
200	1.1E-02
500	1.1E-02
2000	4.3E-04

Consider a component that must survive the three vibration events as shown in Figure 1. Assume that these are the Maximum Predicted Environment (MPE) levels. The corresponding durations and time scaling are shown in Table 1. The goal is to derive a single 8-hour test to cover all three events.

Table 1. Time Scaling Factors			
Event	Ti (hr)	Teq (hr)	Ti/Teq (hr)
1	80	8	10
2	120	8	15
3	64	8	8

The equivalent PSD is also shown in Figure 1, as calculated from equation (2). The coordinates are given in Appendix A, simplified to one-sixth octave format.

The equivalent PSD represents an MPE.

Further steps would be needed to convert this to a test level. For example, there may be a requirement for uncertainty margin, or for some minimum test level.

References

1. T. Irvine, Time-Scaling Equivalence Methods for Random Vibration Testing, Revision F, Vibrationdata, 2002.

2. MIL-STD-1540C (USAF), Military Standard Test Requirements for Launch, Upper-Stage, and Space Vehicles, Department of Defense, Washington, D.C., 1994.

3. MIL-HDBK-340A (USAF), Department of Defense Handbook, Test Requirements for Launch, Upper Stage, and Space Vehicles, Washington, D.C., 2000.

APPENDIX A

Equivalent PSD Coordinates

Freq (Hz)	PSD (G^2/Hz)
10.0	1.32E-02
11.2	1.53E-02
12.5	1.89E-02
14.1	2.47E-02
16.0	2.96E-02
17.9	3.51E-02
20.0	4.41E-02
22.4	4.60E-02
25.0	4.70E-02
28.1	4.89E-02
31.5	5.27E-02
35.5	5.98E-02
40.0	7.17E-02
44.7	8.89E-02
50.0	1.11E-01
56.1	1.13E-01
63.0	1.13E-01
71.0	1.12E-01
80.0	1.06E-01
89.4	7.31E-02
100	4.89E-02
112	3.52E-02
125	2.82E-02
141	2.62E-02

Freq (Hz)	PSD (G^2/Hz)
160	2.71E-02
179	2.93E-02
200	3.18E-02
224	3.22E-02
250	3.22E-02
281	3.22E-02
315	3.22E-02
355	3.22E-02
400	3.22E-02
447	3.22E-02
500	3.13E-02
561	2.58E-02
630	2.10E-02
710	1.76E-02
800	1.54E-02
894	1.40E-02
1000	1.29E-02
1118	1.11E-02
1250	9.45E-03
1414	8.03E-03
1600	6.94E-03
1789	6.10E-03
2000	5.56E-03