EQUIVALENT STATIC LOADS FOR SINE VIBRATION

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Introduction

A particular engineering design problem is to determine the equivalent quasi-static load for equipment subjected to base excitation sine vibration. The goal is to determine peak values for

1. Relative Displacement
2. Absolute Acceleration
3. Transmitted Force

The resulting peak values may be used in a quasi-static analysis, or perhaps in a fatigue calculation.

Model

The first step is to determine the acceleration response of the equipment or structural component. As a first approximation, model the component as a single-degree-of- freedom system. Consider the single-degree-of-freedom system subjected to base excitation shown in Figure 1.

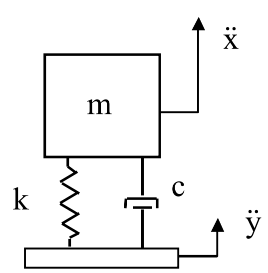

Figure 1.

where

m	is the mass
c	is the viscous damping coefficient
k	is the stiffness
x	is the absolute displacement of the mass
y	is the base input displacement

The complete derivation of the governing equation of motion for the system in Figure 1 is given in Reference 1.

Acceleration Response

The resulting steady-state magnitude function from Reference 1 is

$$\left|\frac{\ddot{x}}{\ddot{y}}\right| = \sqrt{\frac{1+(2\xi\rho)^2}{\left(1-\rho^2\right)^2+(2\xi\rho)^2}} \quad , \qquad \rho = f/f_n \tag{1}$$

where

fn is the natural frequency
f is the forcing frequency
ξ is the damping ratio

Note that the damping may be represented in terms of the quality factor Q.

$$Q = \frac{1}{2\xi} \tag{2}$$

Sine Dwell

Consider that the input acceleration is

$$\ddot{y}(t) = A\sin\left(2\pi f\, t\right) \tag{3}$$

The resulting steady-state response magnitude is

$$\left|\ddot{x}\right| = A\sqrt{\frac{1+(2\xi\rho)^2}{\left(1-\rho^2\right)^2+(2\xi\rho)^2}} \quad , \qquad \rho = f/f_n \tag{4}$$

The response acceleration in equation (4) can be considered as the equivalent peak quasi-static acceleration.

Sine Sweep

Equation (4) can be readily extended for the case of a sine sweep. Assume that the sweep rate is slow enough that the system reaches a steady-state response at each forcing frequency.

The calculation may need to be repeated for several forcing frequencies if the base input amplitude varies with frequency.

Only one calculation is required if the base input amplitude is constant. The highest response will occur for $\rho = 1$ if the system's natural frequency is within the limits of the sine sweep. Otherwise, the calculation should be performed at the forcing frequency that is nearest to the natural frequency.

Relative Displacement

The relative displacement z for sine vibration is

$$|z| = \frac{A}{4\pi^2 \, fn^2} \frac{1}{\sqrt{1 + \frac{(f/fn)^2}{Q^2}}} \tag{5}$$

Equation (5) is taken from Reference 2.

Transmitted Force

The transmitted force Ft applied against the mass is

$$|Ft| = A\,m \sqrt{\frac{1 + (2\xi\rho)^2}{\left(1 - \rho^2\right)^2 + (2\xi\rho)^2}} \tag{6}$$

References

1. T. Irvine, The Steady-state Response of Single-degree-of-freedom System to a Harmonic Base Excitation, Vibrationdata, 2004.
2. T. Irvine, Avionics Isolation Design Guidelines, Vibration, 2005.