GM Truck Group

Desert Proving Ground

**SUBJECT:** Exponential Windows

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The following analysis seeks to determine the implied criterion for setting an exponential window in the time domain, given the requirement for adequate resolution of a resonant peak in the frequency domain.

Let the modal time - domain response function be $y(t) = Ae^{-t/\tau}\cos(2\pi f_d t)$.
The relative amplitude at the end of the time record is

$$c = |y(t_c)| \big/ |y(t_0)| = e^{-t_c/\tau}$$

For a modal response, we have $\tau = 1/\zeta\omega_n = 1/2\pi\zeta f_n = Q/\pi f_n$, where $Q = 1/2\zeta = f_r/\Delta f_r$ is the height of resonance. Note that we are making a distinction between $f_d$, $f_n$, $f_r$ (the damped, natural, and resonant frequencies, respectively), but the distinction is not significant for this discussion and later we will take them as being approximately equal.
Therefore we have :

$$c = e^{-\pi f_n t_c/Q}.$$

Now consider the problem from the frequency domain. The width of the resonant peak is

$$\Delta f_r = f_r/Q$$

This resonance bandwidth will contain n analysis $\Delta f's$. Therefore, in terms of $\Delta f$,

$$n\Delta f = \Delta f_r = f_r/Q.$$

Recalling that $T = 1/\Delta f$, we have

$$T = nQ/f_r$$

Using the above expression for decay, this implies that the signal will have decayed to

$$c = e^{-\pi f_n t_c/Q} = e^{-(\pi f_n nQ)/f_r Q} \cong e^{-n\pi}$$

For minimal representation of the resonant peak, there should be at least $n = 2$ $\Delta f's$ defining it. This provides three frequency lines within the peak. Therefore, with $n = 2$, we have

$$c = e^{-2\pi} \cong 0.002.$$

This is zero for all intents and purposes when dealing with the signal analysis equipment. Therefore, this analysis indicates that the time record length should always be set so as to allow the signal to decay fully to zero. Therefore, no windowing should be required on the response channel