ACCELERATED VIBRATION TESTING
BASED ON FATIGUE DAMAGE SPECTRA

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ABSTRACT

Failure of an aerospace component can arise through the long term exposure to fatigue damaging events such as large numbers of low amplitude random events and/or relatively fewer high amplitude events. Mission profiling and test synthesis is a process for deriving a simple laboratory test that has at least the same damage potential as the real environment but in a fraction of the real time. In this paper we introduce the technical concepts and present a case study showing how new technology has dramatically reduced the time it takes to prepare and reduce the original test data.

KEYWORDS

Fatigue Damage Spectrum, Mission Profiling, Test Synthesis, Shock Response Spectrum, Extreme Response Spectrum, Accelerated Vibration Test

INTRODUCTION

A successful durability rig test should satisfy the following criteria:-

1. The test must be suitable for the item in question, be that a single component, sub-assembly or airframe.
2. The test must replicate the same failure mechanisms observed in the real loading environment.
3. The test should be representative of the real loading environment within known statistical margins.
4. The test should be accelerated where possible to improve development schedules and reduce cost; however, it should not incur unrealistically high loads that might alter the failure mechanism.
5. The test specification should be suitable for laboratory based testing and Finite Element (FE) based analysis.

In this paper we are interested in vibration testing employing electrodynamic shaker or hydraulic shaker rigs. The method derives a test signal in the form of acceleration PSD that can be used as input to the test rig or a suitable FE based analysis. To understand the approach we must first review the Shock Response Spectrum (SRS), Extreme Response Spectrum (ERS), and Fatigue Damage Spectrum (FDS). These are discussed in the following sections. We can then see how each representation is used to describe the lifetime fatigue damage and shock response during the Mission (or Duty) Profiling stage and how a synthesised test PSD is derived. The paper concludes with a short case study demonstrating the design appraisal of an underslung stores pod. The design approach is based on the French military standard GAM EG-13 [1] and NATO AECTP 200 [2].

REVIEW OF BACKGROUND THEORY

The Shock Response Spectrum (SRS)

Consider a typical sub-component within an underslung stores pod; for example, an electronic control unit. How would we develop an accelerated vibration test for the pod that would still ensure at least the same damage being applied to the control unit as if it were in-service? The measured acceleration on the control unit itself is generally not available because the unit is one of many minor components and it's not possible to measure each of them. In addition, many test schedules are fixed before real flight test data is available so we must rely on more generic data. The most likely source of data comes from generic acceleration measurements taken on the host aircraft. We must therefore find a way of using this data as a means to establish the test specification.

Using measured acceleration it is possible to determine the acceleration levels seen by the control unit. For this calculation we need to know the frequency response of the control unit and all the various brackets, etc: then we can filter the input acceleration by the frequency transfer function and, provided the system responds linearly, establish the acceleration levels witnessed by the unit.

In reality the frequency response is usually quite complicated and without a prototype or CAE model is quite impossible to determine. Furthermore, the control unit is only one of many sub-components that all require testing. We must therefore make a working assumption. In 1932, the American engineer Biot [3] was researching the effect of earthquakes and made the assumption of a simple Single Degree of Freedom (SDOF) response. This response function is illustrated in Figure 1. The response is dominated by a single spike located at the natural frequency. At frequencies below the natural frequency the component behaves quasi-statically while at frequencies exceeding the natural frequency the response is significantly attenuated. Around the natural frequency the component will respond dynamically and will become greatly amplified with its maximum response being limited only by the damping in the system. The ratio of the maximum dynamic response to the static response is known as the 'Dynamic Amplification (Q)' factor. For typical 5% structural damping, this has the value of Q = 10. Biot reasoned that as he didn't know the actual natural frequency of his component beforehand, he could create a Spectrum of response by sweeping the natural frequency and plotting maximum response over a range of natural frequencies.

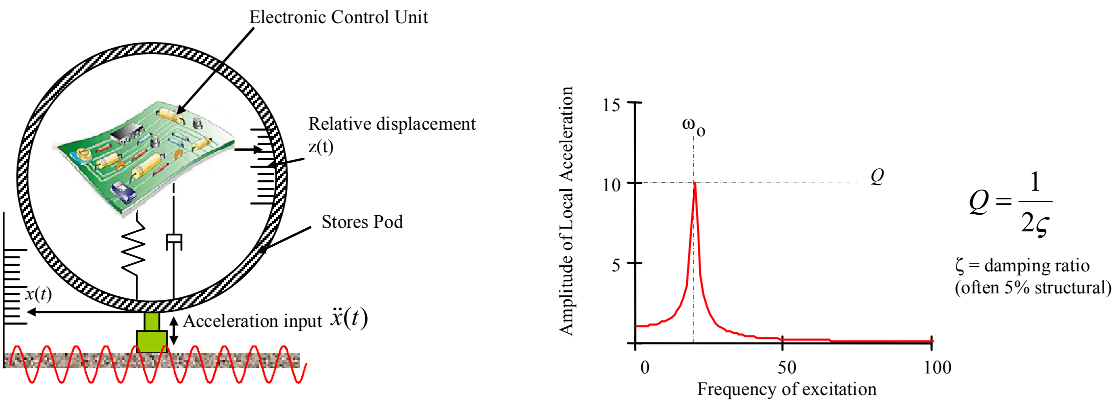

Figure 1: Single Degree of Freedom (SDOF) System

To compute Biot's Shock Spectrum the input signal is filtered by a SDOF transfer function as illustrated in Figure 2 and the maximum of the response calculated. The calculation is repeated a number of times over a range of natural frequencies and a plot made of the maximum response V's the natural frequency. In 1934, Biot [4] published a paper on Earthquake analysis and used the term 'Shock Spectrum' for the first time. The Shock Response Spectrum (SRS), as it is now known, can be expressed in terms of acceleration or displacement response depending on the frequency response function used. For fatigue purposes we are most interested in the displacement response. Fatigue cracks initiate and grow through the cyclic release of strain energy and therefore the displacement response provides a proportional relationship with the energy driving fatigue failure. Acceleration might be the origin of the load but it's the resulting strain (displacement) that drives the structural failure. The SRS of displacement can therefore be used to quantify the damaging effect of the input acceleration for any SDOF system over a range of natural frequencies.

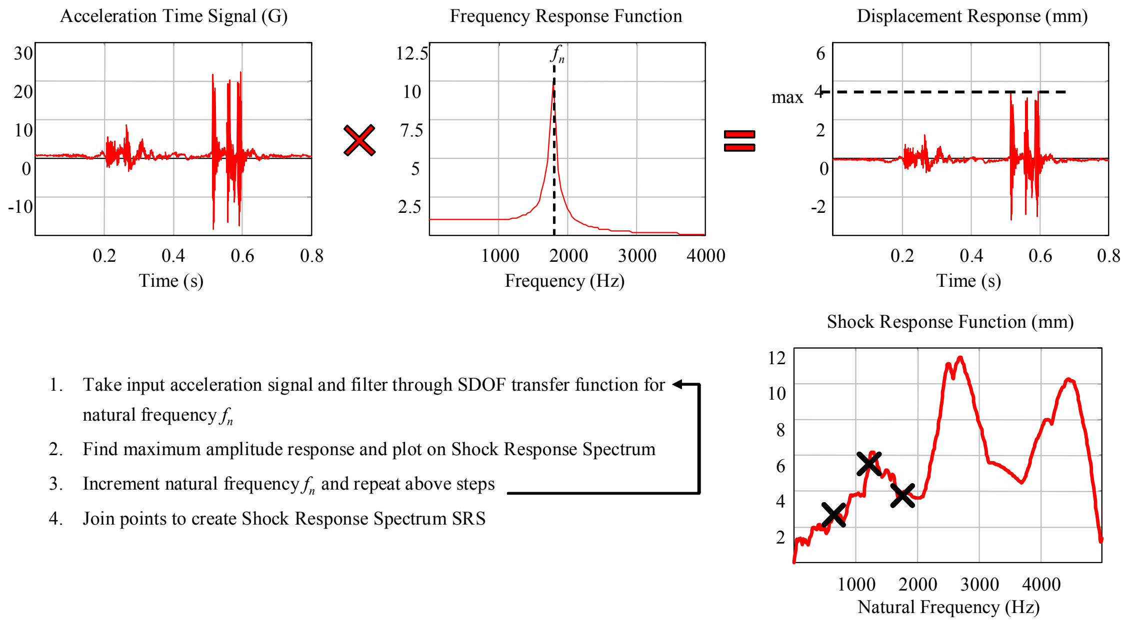

Figure 2: Calculating the Shock Response Spectrum (SRS)

Biot proposed the SDOF assumption be made for all components under excitation regardless of the actual frequency response. Over the past years many have contested the conservatism of this assumption. Lalanne [5] documents a number of these studies which all conclude that the SDOF response, used in conjunction with a frequency sweep, is a suitably conservative assumption for all practical cases.

The arrival of digital computers has made it possible to calculate the SRS for long time signals very rapidly. Using the Z-transform, Irvine [6] derives the equations for a very efficient Infinite Impulse Response (IIR) filter.

The Extreme Response Spectrum (ERS)

To this point we have derived the Shock Response Spectrum (SRS) from a time signal of acceleration input. For random vibration data it is usually more efficient to represent the acceleration loading by means of a PSD. In 1953, Miles [7] presented an equation similar in nature to the SRS. Using the simple formula expressed in Eqn. 1, he derived a spectrum of the RMS (Root Mean Square) acceleration response to a random PSD applied to a SDOF system of natural frequency f_n. The corresponding displacement spectrum can also be determined using the author's modification given in Eqn. 2.

$$\text{RMS acceleration response, } G_{\ddot{X}\,RMS}(f_n) = \sqrt{\frac{\pi}{2} \cdot f_n \cdot Q \cdot G_{\ddot{z}}(f_n)} \tag{1}$$

$$\text{RMS displacement response, } G_{X\,RMS}(f_n) = \frac{G_{\ddot{X}\,RMS}(f_n)}{(2\pi \cdot f_n)^2} \tag{2}$$

Where $G_{\ddot{z}}(f_n)$ is the value of the acceleration input PSD at frequency f_n and Q is the dynamic amplification factor.

Miles' equation is used to determine the RMS acceleration response for a particular natural frequency. For a zero mean process, the RMS and standard deviation are the same; therefore Miles proposed using a Gaussian approximation to the amplitude distribution as an estimate for the extreme response. If you want to estimate the most likely 99.97% highest amplitude local acceleration or displacement response, you could effectively multiply the spectrum by a factor of 3, (i.e. 3 standard deviations).

In 1978, Lalanne [8] proposed a refinement to Miles' equation. For a narrow banded response, typical of a SDOF system, the amplitude distribution was found by Bendat [9] to be Rayleigh and not Gaussian as proposed by Miles. Lalanne therefore rederived the equation this time substituting the Rayleigh probability function, Eqn. 3. The resulting equation is known as the Maximax Response Spectrum (MRS) or the Extreme Response Spectrum (ERS). It represents the most likely extreme amplitude response witnessed by a SDOF system through exposure to a random PSD excitation of duration T seconds. The response can also be expressed in terms of relative displacement using the author's modification, Eqn. 4.

ERS acceleration response, $$ERS_{accel}(f_n) = \sqrt{\pi \cdot f_n \cdot Q \cdot G_{\ddot{z}}(f_n) \cdot \ln(f_n \cdot T)} \tag{3}$$

RMS displacement response, $$ERS_{disp}(f_n) = \frac{ERS_{accel}(f_n)}{(2\pi \cdot f_n)^2} \tag{4}$$

The ERS is therefore analogous to the time domain SRS. However, whereas the SRS is usually used to determine the maximum response to a highly damaging transient shock, the ERS is used to represent the expected response witnessed over more typical long term vibratory loading. It is therefore beneficial to maintain the differing terminology even though both spectra are essentially providing the same information.

The Fatigue Damage Spectrum (FDS)

Lalanne [10] working on the hypothesis of the Extreme Response Spectrum (ERS), proposed an equivalent Fatigue Damage Spectrum (FDS). Following initial work by Bendat [9] and Rice [11] to determine the fatigue damage directly from a PSD of stress, Lalanne was able to utilise this technology to create a closed form calculation for the FDS directly from the acceleration PSD, this is given in Eqn. 5. An explanation of vibration fatigue theory is beyond the scope of this paper and for more details you are referred to Halfpenny [12] and Bishop et al. [13].

Fatigue Damage Spectrum $$FDS(f_n) = f_n \cdot T \cdot \frac{K^b}{C} \cdot \left[\frac{Q \cdot G_{\ddot{z}}(f_n)}{2(2\pi \cdot f_n)^3}\right]^{b/2} \cdot \Gamma(1 + b/2) \tag{5}$$

Where K is the spring stiffness of the SDOF system, $\Gamma()$ is the Gamma function defined by $\Gamma(g) = \int_0^\infty x^{(g-1)} \cdot e^{-x} dx$, and b and C are fatigue parameters describing the Wöhler line such that $N = C \cdot S^{-b}$ where N is the number of cycles to failure of cyclic stress amplitude S.

Using an analogous approach to the Shock Response Spectrum (SRS), the author has derived an approach to calculate the FDS using time series input. For a transient shock, the FDS is calculated in the same way as the SRS but rather than simply finding the maximum displacement response, the filtered displacement response is now rainflow cycle counted and the fatigue damage obtained using a Wöhler calculation. An explanation of fatigue theory and rainflow analysis is beyond the scope of this paper; for more details you are referred to Halfpenny [14] and Downing et al. [15] respectively.

MISSION PROFILING

An accelerated test PSD is required that yields at least the same fatigue damage content as that seen by a component through its whole life. The total lifetime fatigue damage is calculated from the sum of the Fatigue Damage Spectra (FDS) observed over the life of the component. In the previous section we discussed how the FDS could be determined from a PSD and measured time signal. Representative data is measured or calculated for all events likely in the service of the component. This data is then used to create a 'Mission (or Duty) Profile'. A Mission Profile comprises several measured events that are assumed typical of various real-life situations: examples include; taxiing, takeoff, climb, decent, landing, and various in-flight manoeuvres. It should also take account of component packaging and transportation as many components are only occasionally mounted to the aircraft and significant damage is accrued through storage and transportation. Representative PSDs or time signals are measured for each situation along with estimates of how long the component might be expected to see these in-service. The total Lifetime fatigue damage is therefore obtained by summing the FDS for all events over the time they are assumed to act. The FDS calculated from a PSD yields the fatigue damage accumulated per second of exposure; therefore, the FDS for each PSD event should be scaled by the exposure time in seconds. The FDS calculated from a time signal estimates the likely damage seen over that measurement period; therefore the FDS for each time signal event should be scaled by the number of times the event is likely in the lifetime of the component. The total Lifetime fatigue damage is then found by summing the scaled FDS for each event as illustrated in Figure 3.

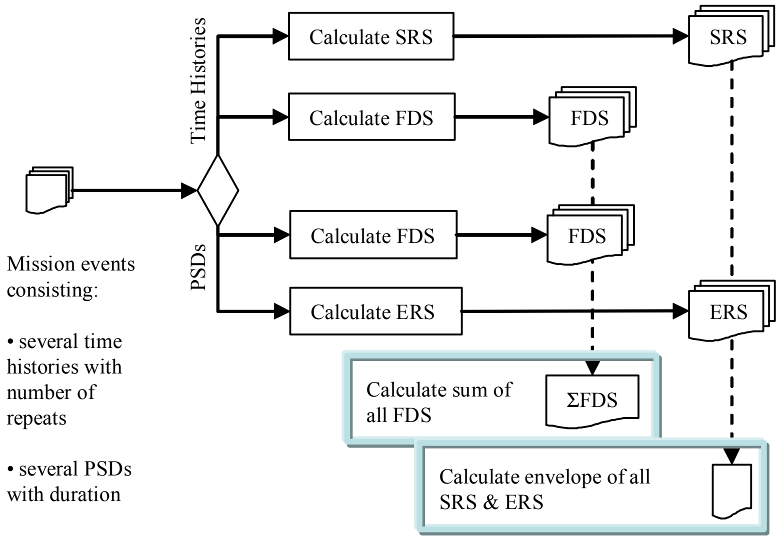

Figure 3: Obtaining the Lifetime FDS and SRS

The lifetime SRS and ERS are determined using an envelope of all the measured SRS and ERS respectively, as illustrated in Figure 3. As most critical shocks are deterministic in nature they will be captured in the time domain and represented in the SRS. The lifetime SRS therefore represents the worst local displacement/acceleration likely to be seen by the SDOF system during the entire life of the component. The ERS is used to represent the expected maxima arising through ordinary vibratory loading and is therefore much lower in amplitude than the SRS. These two spectra will be used to help validate the synthesised Test PSD later.

In some circumstances we might want to test a component to make sure it's suitable for installation on several aircraft platforms. In this case we would measure the acceleration input for each platform as it undertakes the same mission event. We can obtain an effective FDS for the event by simply enveloping all the FDS; thereby obtaining the worst possible situation. This process is illustrated in Figure 4a.

It is also common to have a number of measurements taken of the same event to obtain a statistical distribution. In the above example we could have measured data for a number of landing events. In this case the effective FDS for the event could be taken as the envelope of the FDS for each. Alternatively, if sufficient measurements are available it would prove more valuable to derive a statistical FDS based on the mean and standard deviation of the FDS. In this case the mean FDS is used in the lifetime calculation and a Safety Factor is calculated based on the statistical variability of the measured results and the required confidence parameters. This is illustrated in Figure 4b.

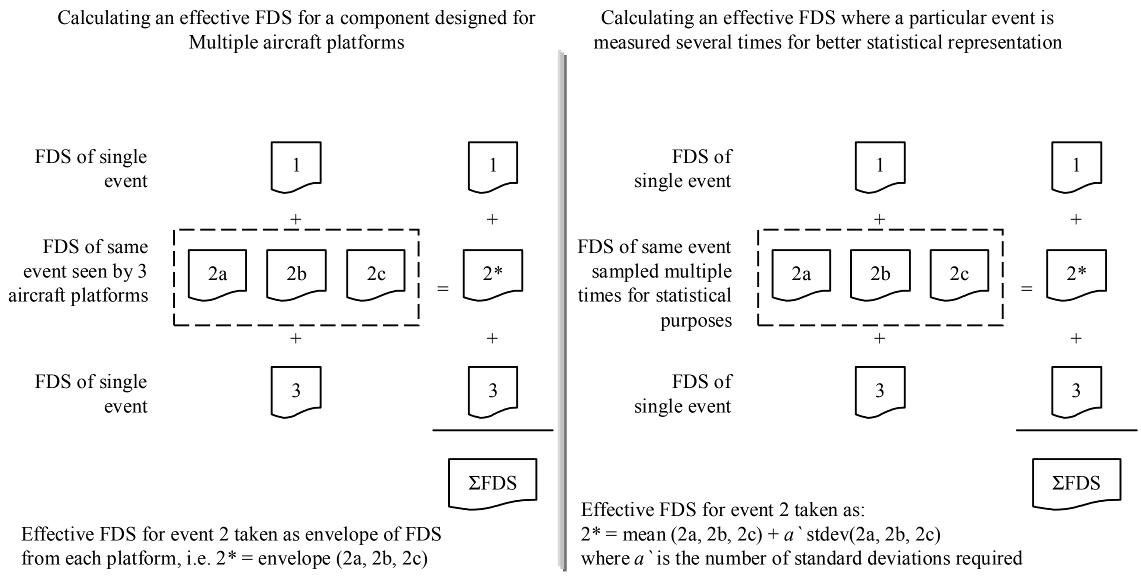

Figure 4: Dealing with multiple variants

TEST SYNTHESIS

Any synthesised test signal should ideally include a statistical safety factor that accounts for variations in applied loading and fatigue strength of the component. Figure 5 illustrates how the safety factor relates to the probability of in-service failure. Depending on whether a Gaussian-normal or Log-normal distribution is assumed, the safety factor 'k' is found using Eqn. 6 or Eqn. 7 respectively.

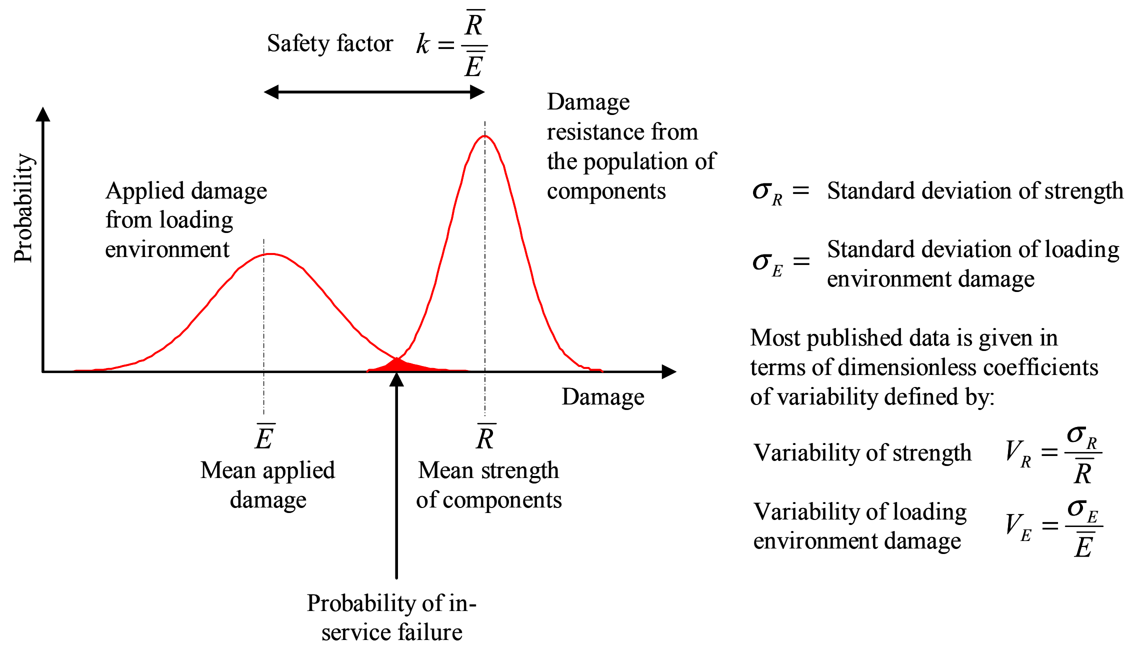

Figure 5: Probability of in-service failure

$$\text{Gaussian-normal probability } k = \frac{1+\sqrt{1-\left(1-a'^2\cdot V_R^2\right)\cdot\left(1-a'^2\cdot V_E^2\right)}}{\left(1-a'^2\cdot V_R^2\right)} \tag{6}$$

$$\text{Log-normal probability } k = \exp\left\{a'\cdot\sqrt{\ln\left[\left(1+V_E^2\right)\left(1+V_R^2\right)\right]}-\ln\left[\sqrt{\frac{1+V_E^2}{1+V_R^2}}\right]\right\} \tag{7}$$

Where a` is the probability of success (1-probability of failure) expressed as a number of standard deviations.

A test factor is used to account for the limited number of durability tests to actually be undertaken. The total safety factor employed is therefore taken as the product of the Safety factor obtained above and the test factor determined using either Eqn. 8 or Eqn. 9, depending on the type of distribution required.

Gaussian-normal probability $k_{test} = 1 + \frac{a'}{\sqrt{n}} \cdot V_R$ (8)

Log-normal probability $k_{test} = \exp\left\{ \frac{a'}{\sqrt{n}} \cdot \sqrt{\ln\left(1 + V_R^2\right)} \right\}$ (9)

Where n is the scheduled number of tests

The synthesised test PSD is obtained by inverting Eqn. 5 as shown in Eqn. 10 for required test duration T_{eq}.

Synthesised vibration PSD $G_{synth}(f_n) = \frac{2(2\pi \cdot f_n)^3}{Q} \cdot \left[\frac{k \cdot \Sigma FDS(f_n) \cdot C}{K^b \cdot f_n \cdot T_{eq} \cdot \Gamma\left(1 + b/2\right)} \right]^{2/b}$ (10)

Where $\Sigma FDS(f_n)$ is the lifetime FDS and k is the combined safety factor.

This equation can lead to a PSD that is unnecessarily complicated and so it is usual practice for an engineer to produce an idealised representation. One approach involves enveloping the PSD over a reduced number of frequency steps, thus forming a stepped PSD. The engineer can verify that the test PSD is adequate by comparing its FDS with the lifetime FDS.

The ERS of the test PSD should be compared with the ERS and SRS measured over the lifetime of the component. The test ERS should ideally be greater than the lifetime ERS to ensure that all likely maxima are covered by the test. It should also be less than the lifetime SRS to minimise the risk of failure through overly severe loading conditions during the test. If the test ERS exceeds the lifetime SRS then you have probably accelerated the test too much and are applying loads that are unreasonably high. If the test ERS is below the lifetime SRS then you are not applying loads beyond those the component was originally designed to survive. You have effectively scaled the low level vibratory loads so they accumulate damage more rapidly without exceeding the design threshold established for the critical shock load.

It is unlikely that the test ERS will fit neatly between the lifetime ERS and lifetime SRS at all points. GAM EG-13 [1] considers the condition acceptable if the mean of the test ERS is therefore less than the mean of the lifetime SRS and greater than the mean of the lifetime ERS.

Occasionally we find it impossible to reproduce the Shock response of some severe transient shock events using a PSD; in these cases we might also have to augment the PSD testing with some time domain tests to ensure all eventualities are considered.

CASE STUDY

A large aerospace Tier-1 supplier was contracted to provide underslung stores for use on a fast jet aircraft. An existing and proven stores pod was used as the basis for this new design; however, the pod had only previously been used on large sub-sonic tanker aircraft and concern was voiced over its suitability for flight trials. The flight envelope of the fast jet significantly exceeded the original design spectrum of the pod and the supplier had to assess whether the pod, originally designed for 1000 hours on a tanker, would safely endure the required 20 hours of flight trials on the fast jet.

An assessment of the main structure and mounting system showed this to be adequate for the proposed loading; however, it would be impossible to perform a full CAE analysis of every internal component before the flight trials. An engineering approach was devised by the supplier which would combine frequent structural inspections of the pod with a mathematical analysis of the accumulated damage using a Fatigue Damage Spectrum (FDS) approach. The accumulated damage was calculated from the measured acceleration data and compared with the certified capacity of the pod to ensure that it did not encroach too far towards the certified level. A Shock Response Spectrum (SRS) analysis was also performed to ensure that no single high acceleration event would induce loads in excess of the design envelope.

nCode International were contracted to provide the necessary signal processing, fatigue analysis and Mission Profiling software. Together with the supplier they performed analyses to compare the FDS given for the fast jet flight envelope over a 20 hour flight trial with that of the certified sign-off test. A comparison between the PSDs is given in Figure 6; however, this simple comparison does not account for the differences in loading duration or the frequency response of any internal components. The ERS and FDS analyses are able to account for these parameters and are illustrated in Figure 7.

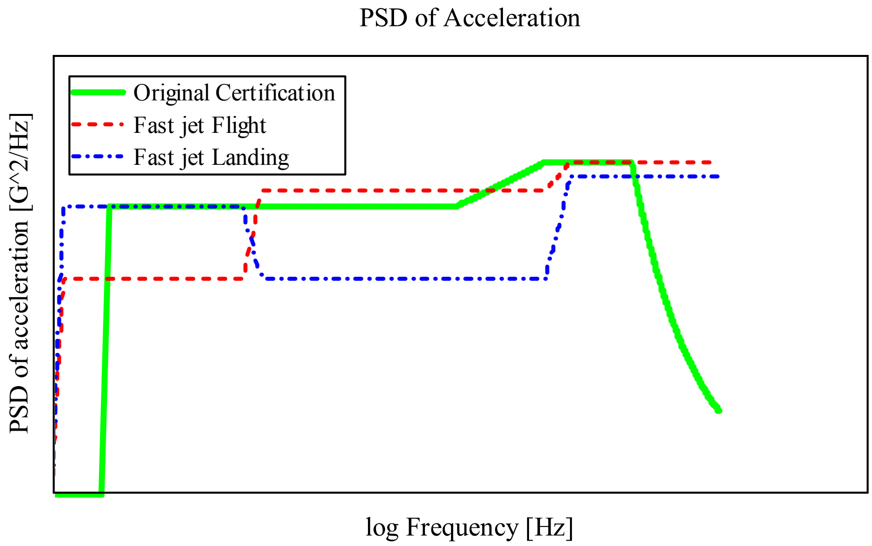

Figure 6 Comparison between PSDs of original certification test and fast jet design spectrum

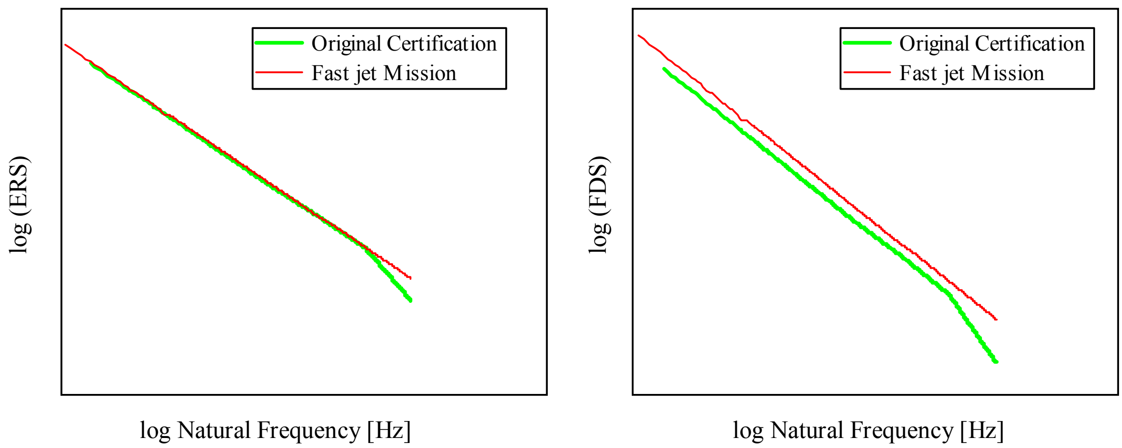

Figure 7 Comparison between ERS and FDS of original certification test and fast jet design spectrum

The ERS and FDS plots are not easily interpreted on account of their logarithmic scaling; it is therefore preferable to convert these into equivalent PSDs over a common time base. This is accomplished using Eqn. 10 and the inverse of Eqn. 2 for the FDS and ERS respectively; the plots shown in Figure 8.

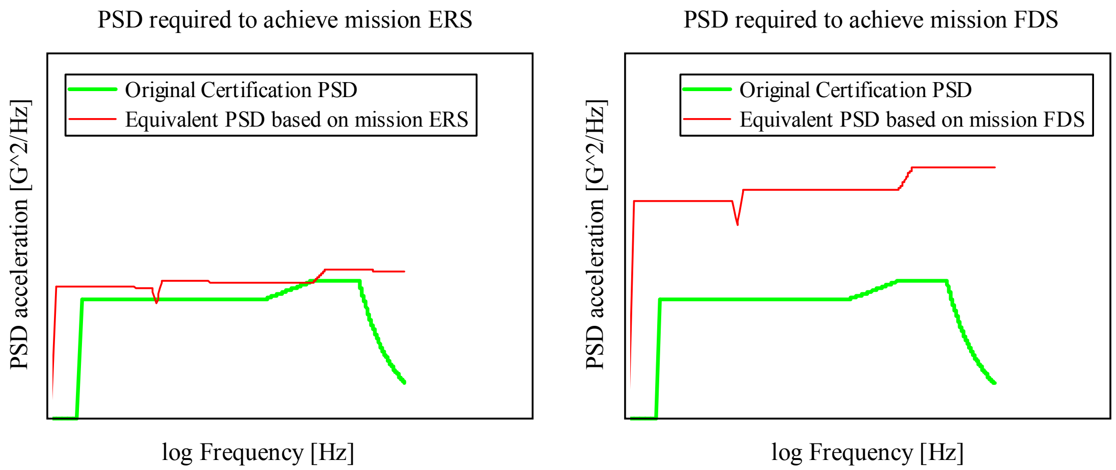

Figure 8 Comparison between equivalent PSDs

The above analysis revealed an anticipated overload in the peak acceleration of 70% with a consequential reduction in fatigue life from 1000 hours to 125 hours. The supplier proposed a change in material for all key structural items to improve the fatigue performance. The resulting improvements were substantial and offered several orders of magnitude greater fatigue resistance. Improvement, however, is restricted only to those components using the new material and careful observation was given to areas of stress concentration along with other internal components to ensure there were no signs of distress throughout the trials. The trials were scheduled carefully to allow engineers time to inspect the pod after each sortie and sorties were arranged in terms of increasing severity. The cumulative FDS was calculated using the measured acceleration data at each inspection interval and the damage compared with that of the certified sign-off test. The trials never exceeded the design FDS or ERS of the pod and in all cases the manoeuvres were found to be significantly less damaging than the original flight spectrum had predicted. Operational constraints were imposed on flying the aircraft with the pod fitted and this, in conjunction with the inboard mounting of the pod on the aircraft, contributed to significantly lower service loads. Using the measured acceleration data at various locations on the pod, the supplier derived an equivalent Mission PSD for comparison with the certified sign-off test. The comparison is given in Figure 9 and shows the Mission response to be lower than the existing certified test in all but the most severe case. Even then, the Mission only encroaches on the certified test at one measurement point and only in the high frequency region and is not considered to be a problem.

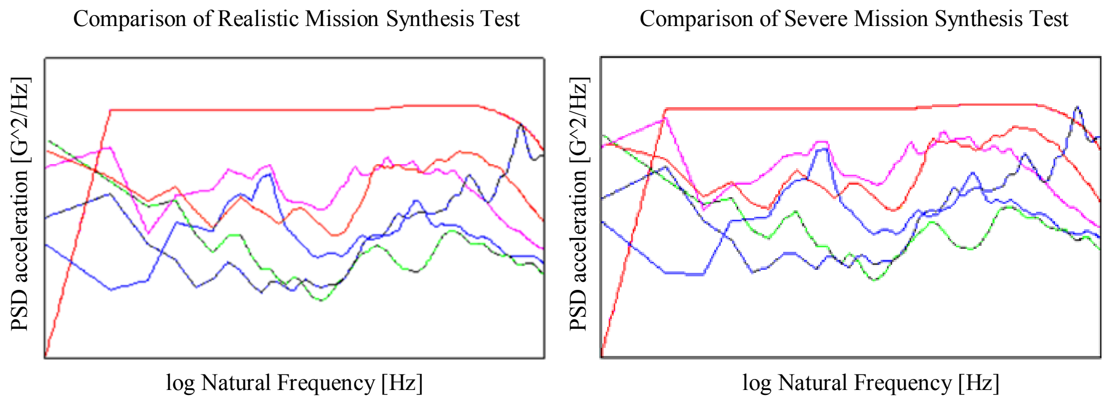

Figure 9 Comparison between Mission and certified sign-off test

Following a successful round of flight trials, the supplier provided an instrumented pod to the customer to measure the typical acceleration loads over more representative operational use. These were then used to create an accelerated vibration sign-off test. The sign-off test is far superior to any previous test as it is rigorously derived using real load measurements with statistically representative safety factors accounting for variability in loading, component strength and the finite number of scheduled sign-off tests performed.

CONCLUSION

In this paper we have introduced the concept of Mission Profiling and Test Synthesis. We have reviewed several spectra that contain information on the damage content of a signal as it is seen by various components of different natural frequency. We have shown how Fatigue Damage Spectra (FDS) can be summed over all events in the life of a component and how the resultant lifetime FDS can be inverted to produce a PSD suitable for use in an accelerated vibration test. The test PSD contains at least the same damage as that accrued over the anticipated life of the component and can be scaled to account for statistical variability in loading, component strength and the number of components being tested. The test PSD is also verified to ensure that it doesn't excessively accelerate the test by using unrealistic loads that are beyond that which the components were originally designed to sustain. A case study is discussed illustrating how the technology was used to assess the suitability of underslung stores for use on a fast jet and how a statistically representative sign-off test was synthesised.

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