THE HALF POWER BANDWIDTH METHOD FOR DAMPING CALCULATION
Revision A

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Introduction

Damping in mechanical systems may be represented in numerous formats. The most common forms are Q and ξ, where

- Q is the amplification or quality factor
- ξ is the viscous damping ratio or fraction of critical damping

These two variables are related by the formula

$$Q = \frac{1}{2\xi} \tag{1}$$

An amplification factor of Q=10 is thus equivalent to 5% damping.

The Q value is approximately equal to the peak transfer function magnitude for a single-degree-of-freedom subjected to base excitation at its natural frequency. This simple equivalency does not necessarily apply if the system is a multi-degree-of-freedom system, however.

Another damping parameter is the frequency width Δf between the -3 dB points on the transfer magnitude curve. The conversion formula is

$$Q = \frac{f_n}{\Delta f} \tag{2}$$

where f_n is the natural frequency.

The -3 dB points are also referred to as the “half power points” on the transfer magnitude curve.

Equation (2) is useful for determining the Q values for a multi-degree-of-freedom system as long as the modal frequencies are well separated.

Single-degree-of-freedom System Example

Consider the single-degree-of-freedom system in Figure 1.

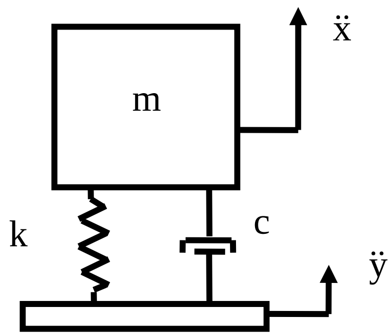

Figure 1.

Given:

1. The mass is 1 lbm (0.00259 lbf sec^2/in).
2. The spring stiffness is 1000 lbf/in.
3. The damping value is 5%, which is equivalent to Q=10.

The natural frequency equation is

$$fn = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{3}$$

The resulting natural frequency is 98.9 Hz.

Now consider that the system is subjected to base excitation in the form of a sine sweep test. The resulting transfer function magnitude is given in Figure 2, as calculated using the method in Reference 1.

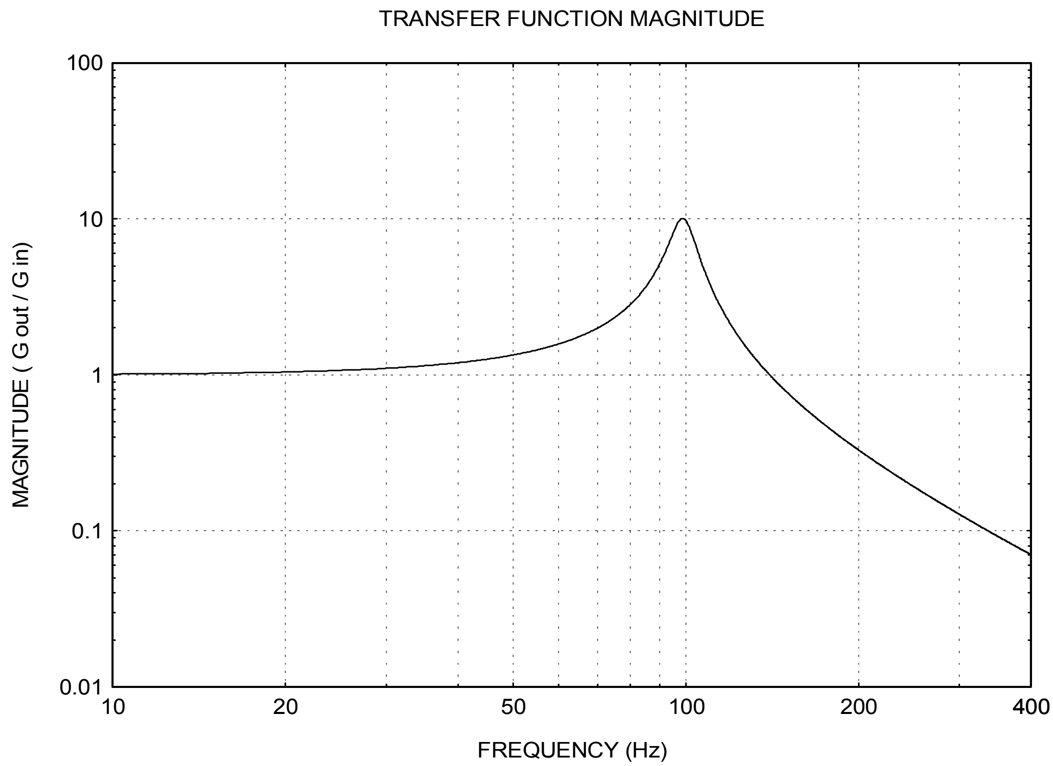

Figure 2. Single-degree-of-freedom System

The peak transfer function magnitude is equal to the Q value for this case, which is Q=10.

Two-degree-of-freedom System Example

Consider the two-degree-of-freedom system in Figure 3.

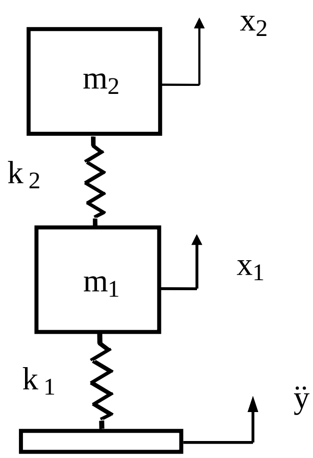

Figure 3.

(The dashpots are omitted from Figure 3 for brevity).

Given:

1. Each mass is 1 lbm (0.00259 lbf sec^2/in).
2. Each spring stiffness is 1000 lbf/in.
3. Each mode has a damping value of 5%, which is equivalent to Q=10.

The resulting natural frequencies are 61.1 Hz and 160.0 Hz, as calculated using the method in Reference 2.

Now consider that the two-degree-of-freedom system is subjected to base excitation in the form of a sine sweep test. The resulting transfer function magnitude is given in Figure 4.

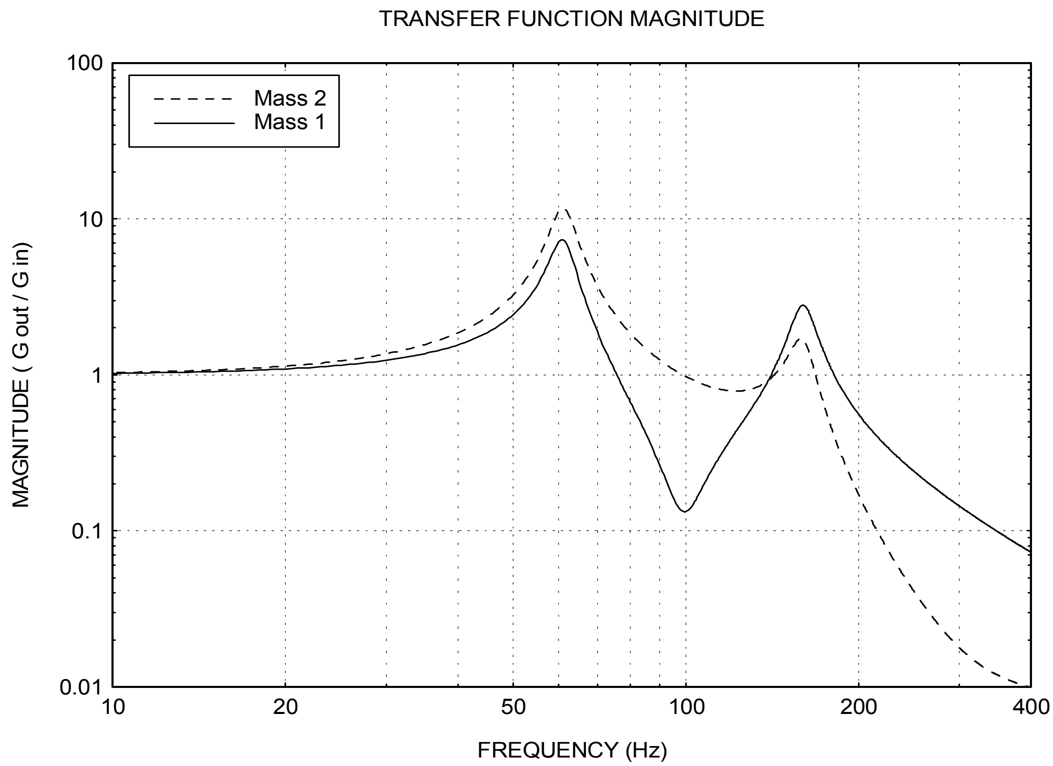

Figure 4. Two-degree-of-freedom System

Each mass is represented by a separate curve in the transfer function plot. The Q value for each mode cannot be determined by simple inspection for this case.

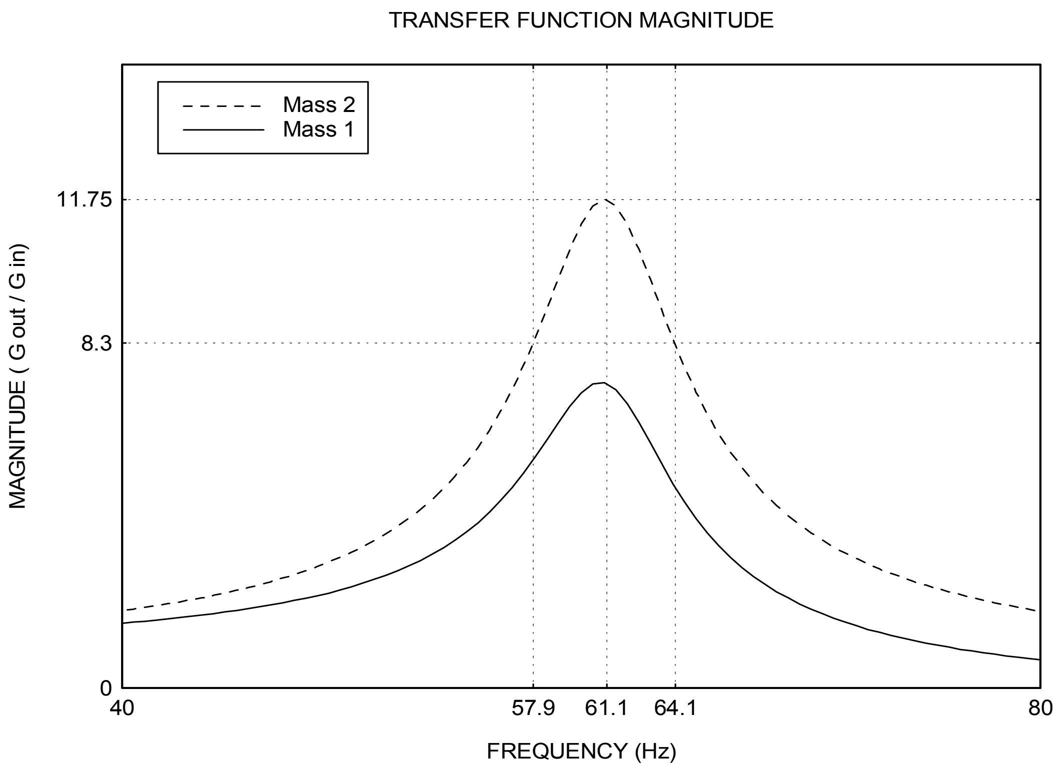

Figure 5. Two-degree-of-freedom System, First Mode

The -3 dB points occur at 57.9 Hz and at 64.1 Hz

The Q value for the first mode is calculated as

$$Q = \frac{f_n}{\Delta f} \tag{4}$$

$$Q = \frac{61.1}{6.2} = 9.9 \approx 10 \tag{5}$$

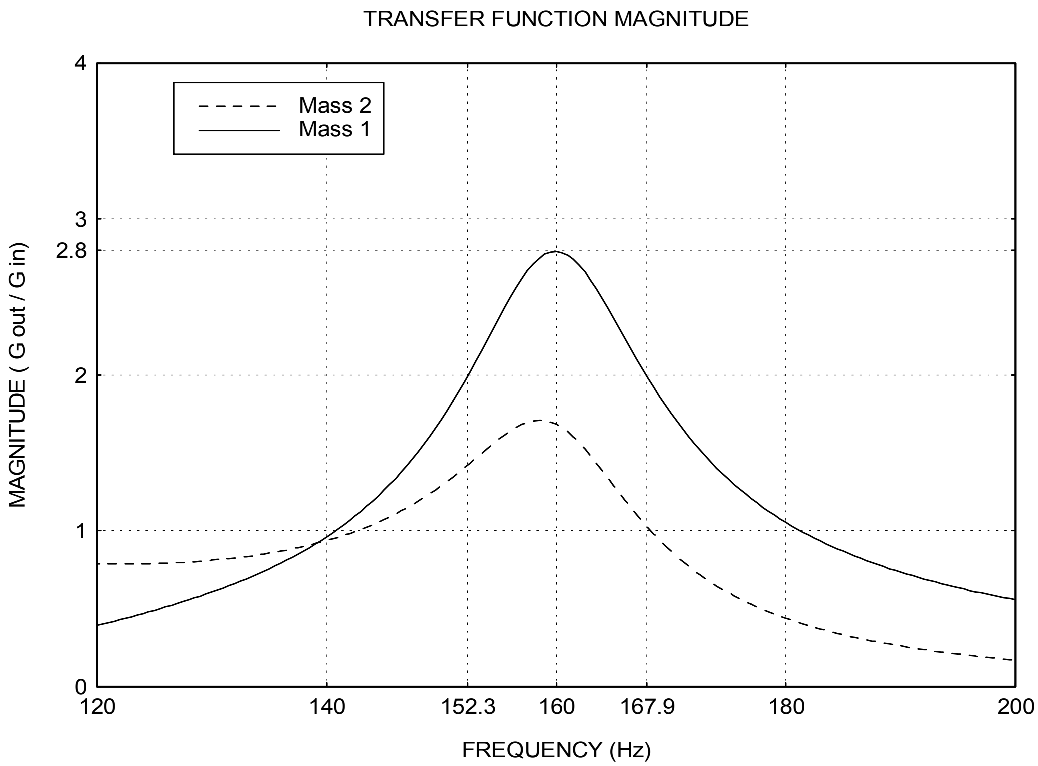

Figure 6. Two-degree-of-freedom System, Second Mode

The -3 dB points occur at 152.3 Hz and at 167.9 Hz

The Q value for the second mode is calculated as

$$Q = \frac{f\,n}{\Delta f} \tag{6}$$

$$Q = \frac{160}{15.6} = 10.3 \approx 10 \tag{7}$$

References

1. T. Irvine, The Steady-state Response of Single-degree-of-freedom System to a Harmonic Base Excitation, Vibrationdata, 2004.

2. T. Irvine, The Generalized Coordinate Method for Discrete Systems Subjected to Base Excitation, Vibrationdata, 2004.

APPENDIX A

Half-Power Points for a SDOF System

The receptance function (displacement/force) for an SDOF system is

$$\frac{\omega_n^2}{k\sqrt{\left(\omega_n^2-\omega^2\right)^2+\left(2\xi\omega\omega_n\right)^2}} \tag{A-1}$$

The peak value at resonance is

$$\frac{\omega_n^2}{k\sqrt{\left(2\xi\omega_n{}^2\right)^2}} \tag{A-2}$$

The frequencies at which the half-power points occur are determined as follows

$$\frac{\dfrac{\omega_n^2}{k\sqrt{\left(\omega_n^2-\omega^2\right)^2+\left(2\xi\omega\omega_n\right)^2}}}{\dfrac{\omega_n^2}{k\sqrt{\left(2\xi\omega_n{}^2\right)^2}}}=\frac{\sqrt{2}}{2} \tag{A-3}$$

$$\frac{\dfrac{1}{\sqrt{\left(\omega_n^2-\omega^2\right)^2+\left(2\xi\omega\omega_n\right)^2}}}{\dfrac{1}{\sqrt{\left(2\xi\omega_n{}^2\right)^2}}}=\frac{\sqrt{2}}{2} \tag{A-4}$$

$$\frac{\dfrac{1}{\sqrt{\left(\omega_n^2-\omega^2\right)^2+\left(2\xi\omega\omega_n\right)^2}}}{\dfrac{1}{2\xi\omega_n{}^2}}=\frac{\sqrt{2}}{2} \tag{A-5}$$

$$\frac{2\xi\omega_n^{\ 2}}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\xi\omega\omega_n\right)^2}} = \frac{\sqrt{2}}{2} \tag{A-6}$$

$$2\xi\omega_n^{\ 2} = \frac{\sqrt{2}}{2}\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\xi\omega\omega_n\right)^2} \tag{A-7}$$

$$\frac{4}{\sqrt{2}}\xi\omega_n^{\ 2} = \sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\xi\omega\omega_n\right)^2} \tag{A-8}$$

$$8\xi^2\omega_n^{\ 4} = \left(\omega_n^2 - \omega^2\right)^2 + \left(2\xi\omega\omega_n\right)^2 \tag{A-9}$$

$$8\xi^2\omega_n^{\ 4} = \omega_n^4 - 2\omega^2\omega_n^{\ 2} + \omega^4 + 4\xi^2\omega^2\omega_n^{\ 2} \tag{A-10}$$

$$\omega_n^4 - 2\omega^2\omega_n^{\ 2} + \omega^4 + 4\xi^2\omega^2\omega_n^{\ 2} - 8\xi^2\omega_n^{\ 4} = 0 \tag{A-11}$$

$$\omega^4 + \left(-2\omega_n^{\ 2} + 4\xi^2\omega_n^{\ 2}\right)\omega^2 + \omega_n^4 - 8\xi^2\omega_n^{\ 4} = 0 \tag{A-12}$$

$$\omega^4 + 2\omega_n^{\ 2}\left(-1 + 2\xi^2\right)\omega^2 + \omega_n^4\left(1 - 8\xi^2\right) = 0 \tag{A-13}$$

The roots of equation (A-13) can be determined by the quadratic formula. The positive ω roots are the frequencies corresponding to the half-power points.