INTEGRATING AN ACCELERATION HALF-SINE PULSE

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Let

t = time

T = period

A = peak acceleration amplitude

$d(t)$ = displacement

$v(t)$ = velocity

$a(t)$ = acceleration

c_1 and c_2 = integration constants

The acceleration half-sine pulse is

$$a(t) = \begin{cases} A \sin\left(\frac{\pi t}{T}\right), & \text{for } 0 \le t \le T \\ 0, & \text{for } t > T \end{cases} \tag{1}$$

The initial conditions are

$$v(0) = v_o \tag{2}$$

$$d(0) = d_o \tag{3}$$

Integrate acceleration to obtain velocity.

$$v(t) = \int a(t)\,dt \tag{4}$$

$$v(t) = \int A \sin\left(\frac{\pi t}{T}\right) dt, \qquad \text{for } 0 \le t \le T \tag{5}$$

Apply velocity initial condition.

$$v(t) = -\left(\frac{A T}{\pi}\right)\cos\left(\frac{\pi t}{T}\right) + c_1 , \qquad \text{for } 0 \le t \le T \tag{6}$$

$$v(0) = v_o \tag{7}$$

$$v_o = -\left(\frac{A T}{\pi}\right) + c_1 , \qquad \text{for } 0 \le t \le T \tag{8}$$

$$c_1 = v_o + \left(\frac{A T}{\pi}\right) , \qquad \text{for } 0 \le t \le T \tag{9}$$

$$v(t) = -\left(\frac{A T}{\pi}\right)\cos\left(\frac{\pi t}{T}\right) + \left[v_o + \left(\frac{A T}{\pi}\right)\right], \qquad \text{for } 0 \le t \le T \tag{10}$$

Integrate velocity to obtain displacement.

$$d(t) = \int v(t)\,dt \tag{11}$$

$$d(t) = \int \left\{ -\left(\frac{A T}{\pi}\right)\cos\left(\frac{\pi t}{T}\right) + \left[v_o + \left(\frac{A T}{\pi}\right)\right]\right\} dt, \qquad \text{for } 0 \le t \le T \tag{12}$$

$$d(t) = -A\left(\frac{T}{\pi}\right)^2 \sin\left(\frac{\pi t}{T}\right) + \left[v_o + \left(\frac{A T}{\pi}\right)\right]t + c_2 , \qquad \text{for } 0 \le t \le T \tag{13}$$

Apply displacement initial condition.

$$d(0) = d_o \tag{14}$$

$$c_2 = 0 , \qquad \text{for } 0 \le t \le T \tag{15}$$

$$d(t) = -A\left(\frac{T}{\pi}\right)^2 \sin\left(\frac{\pi t}{T}\right) + \left[v_o + \left(\frac{A T}{\pi}\right)\right]t + d_o , \qquad \text{for } 0 \le t \le T \tag{16}$$

APPENDIX A

<u>Special Case</u>

Determine the required initial velocity and initial displacement such that the half-sine pulse will have zero velocity and zero displacement at t = T.

$$v(T) = -\left(\frac{A T}{\pi}\right)\cos\left(\frac{\pi T}{T}\right) + \left[v_o + \left(\frac{A T}{\pi}\right)\right] \tag{A-1}$$

$$v(T) = \left(\frac{A T}{\pi}\right) + \left[v_o + \left(\frac{A T}{\pi}\right)\right] \tag{A-2}$$

$$v(T) = \left(\frac{2 A T}{\pi}\right) + v_o \tag{A-3}$$

$$v(T) = 0 \tag{A-4}$$

$$\left(\frac{2AT}{\pi}\right) + v_o = 0 \quad \text{(A-5)}$$

$$v_o = -\left(\frac{2AT}{\pi}\right) \quad \text{(A-6)}$$

$$d(T) = -A\left(\frac{T}{\pi}\right)^2 \sin\left(\frac{\pi T}{T}\right) + \left[v_o + \left(\frac{AT}{\pi}\right)\right]T + d_o \quad \text{(A-7)}$$

$$d(T) = \left[v_o + \left(\frac{AT}{\pi}\right)\right]T + d_o \quad \text{(A-8)}$$

$$d(T) = \left[-\left(\frac{2AT}{\pi}\right) + \left(\frac{AT}{\pi}\right)\right]T + d_o \quad \text{(A-9)}$$

$$d(T) = \left[\frac{-AT}{\pi}\right]T + d_o \quad \text{(A-10)}$$

$$d(T) = \left[\frac{-AT^2}{\pi}\right] + d_o \quad \text{(A-11)}$$

$$d(T) = 0 \quad \text{(A-12)}$$

$$\left[\frac{-AT^2}{\pi}\right] + d_o = 0 \quad \text{(A-13)}$$

$$d_o = \left[\frac{AT^2}{\pi} \right] \qquad \text{(A-14)}$$

$$d(t) = -A\left(\frac{T}{\pi}\right)^2 \sin\left(\frac{\pi t}{T}\right) + \left[-\left(\frac{2AT}{\pi}\right)+\left(\frac{AT}{\pi}\right)\right]t + \left[\frac{AT^2}{\pi}\right], \qquad \text{for } 0 \le t \le T$$

(A-15)

$$d(t) = -A\left(\frac{T}{\pi}\right)^2 \sin\left(\frac{\pi t}{T}\right) + \left[\frac{-AT}{\pi}\right]t + \left[\frac{AT^2}{\pi}\right], \qquad \text{for } 0 \le t \le T$$

(A-16)

$$v(t) = -\left(\frac{AT}{\pi}\right)\cos\left(\frac{\pi t}{T}\right) + \left[-\left(\frac{2AT}{\pi}\right)+\left(\frac{AT}{\pi}\right)\right], \qquad \text{for } 0 \le t \le T$$

(A-17)

$$v(t) = -\left(\frac{AT}{\pi}\right)\cos\left(\frac{\pi t}{T}\right) + \left[-\frac{AT}{\pi}\right], \qquad \text{for } 0 \le t \le T$$

(A-18)

$$v(t) = -\left(\frac{AT}{\pi}\right)\left[1+\cos\left(\frac{\pi t}{T}\right)\right], \qquad \text{for } 0 \le t \le T$$

(A-19)