NATURAL FREQUENCIES OF A HONEYCOMB SANDWICH PLATE
Revision F

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Bending Stiffness of a Honeycomb Sandwich Plate

A diagram of a honeycomb plate cross-section is shown in Figure 1.

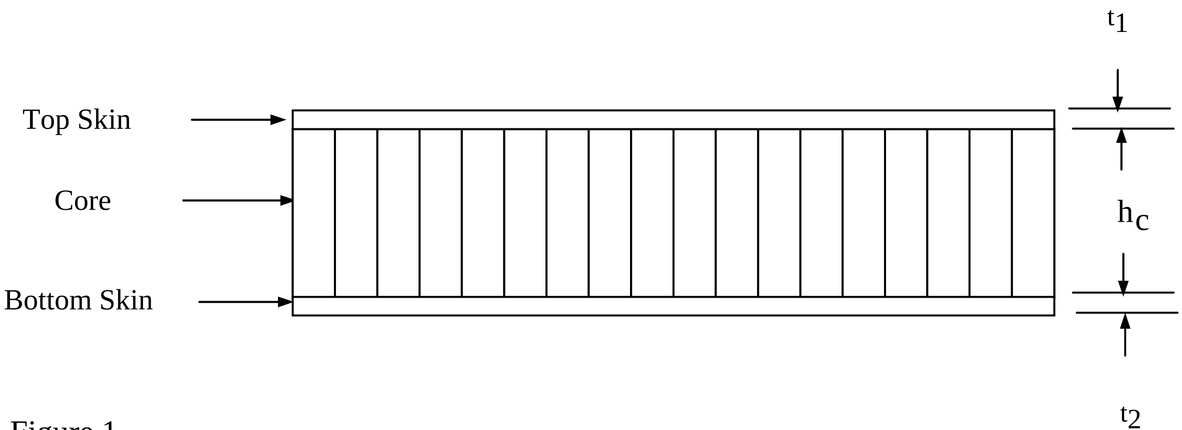


Figure 1.

Let

D = bending stiffness
E_f = elastic modulus of skin facings
t_1 = thickness of top skin
t_2 = thickness of bottom skin
h_c = honeycomb core thickness
v = Poisson’s ratio

Assume

1. The skin elastic modulus is much greater than the core modulus.
2. Each skin has the same material.
3. Each skin is "thin" relative to the core.

The bending stiffness from Reference 1 and 2 is

$$D = \left[\frac{E_f}{1-v^2}\right]\left[\frac{t_1\ t_2\ \ h^2}{t_1+t_2}\right] \tag{1}$$

where

$$h = h_c + \frac{1}{2}\left[t_1 + t_2\right]$$

Now assume that the core thickness is much greater than the skin or face sheet thickness. Also assume that each skin has the same thickness. Reference 7 gives the following approximation for this case.

$$D \approx E_f\left[\frac{{h_c}^2}{4}\right]h_f \tag{2}$$

where h_f is the total face sheet thickness.

Example 1

A rocket vehicle has a circular bulkhead made from honeycomb. The material is aluminum. The bulkhead properties are shown in Table 1.

Table 1. Bulkhead Parameters for Example 1	
Parameter	Value
Boundary Condition	Simply Supported
Diameter	40 inch
Core Thickness	1.0 inch
Thickness of Each Skin	0.063 inch
Total Thickness	1.125 inch
Skin Elasticity	10.0e+06 lbf/in^2
Core Elasticity	Negligible
Skin Density	0.10 lbm/in^3
Core Density	0.01 lbm/in^3
Poisson's Ratio	0.3

Calculate the fundamental bending frequency.

The bending stiffness is

$$D = \left[\frac{E_f}{1 - v^2}\right]\left[\frac{t_1\ t_2\ \ h^2}{t_1 + t_2}\right] \tag{3}$$

where

$$h = h_c + \frac{1}{2}\left[t_1 + t_2\right]$$

$$h = 1.0 + \frac{1}{2}\left[\ 0.063 + 0.063\ \right]\ \text{in} \tag{4}$$

$$h = 1.063\ \ \text{in} \tag{5}$$

$$D = \left[\frac{10\left(10^6\right)\dfrac{\text{lbf}}{\text{in}^2}}{1 - (0.3)^2}\right]\left[\frac{(0.063\ \text{in})(0.063\ \text{in})(1.063\ \text{in})^2}{0.063\ \text{in} + 0.063\ \text{in}}\right] \tag{6}$$

$$D = 391{,}143\ \ \text{lbf in} \tag{7}$$

$$D = \left[391{,}143\ \ \text{lbf in}\right]\left[\frac{\text{slugs ft}}{\text{lbf sec}^2}\right]\left[\frac{32.2\ \text{lbm}}{1\text{slug}}\right]\left[\frac{12\,\text{in}}{\text{ft}}\right] \tag{8}$$

$$D = 1.511\left(10^8\right)\left[\frac{\text{lbm in}^2}{\text{sec}^2}\right] \tag{9}$$

The total skin mass ms is

$$ms = \pi\,\frac{(40\ \text{in})^2}{4}\left(\ 0.063\ \text{in} + 0.063\ \text{in}\right)\left(0.1\,\frac{\text{lbm}}{\text{in}^3}\right) \tag{10}$$

$$ms = 15.83\ \text{lbm} \tag{11}$$

The total core mass mc is

$$mc = \pi \frac{(40\ \text{in})^2}{4}(1.00\ \text{in})\left(0.01\frac{\text{lbm}}{\text{in}^3}\right) \tag{12}$$

$$mc = 12.57\ \text{lbm} \tag{13}$$

The total mass mt is

$$mt = ms + mc \tag{14}$$

$$mt = 28.4\ \text{lbm} \tag{15}$$

The total volume V is

$$V = \pi \frac{(40\ \text{in})^2}{4}(1.125\ \text{in}) \tag{16}$$

$$V = 1414\ \text{in}^3 \tag{17}$$

The overall mass per volume ρ is

$$\rho = mt\ /\ V \tag{18}$$

$$\rho = \frac{28.4\ \ \text{lbm}}{1414\ \ \text{in}^3} \tag{19}$$

$$\rho = 0.020\ \frac{\text{lbm}}{\text{in}^3} \tag{20}$$

The mass per area μ is

$$\mu = \rho T \tag{21}$$

where T is the total thickness.

$$\mu = \left[0.020 \ \frac{\text{lbm}}{\text{in}^3} \right] [1.125 \text{ in}] \tag{22}$$

$$\mu = 0.023 \ \frac{\text{lbm}}{\text{in}^2} \tag{23}$$

The fundamental bending frequency for a simply supported circular plate with radius a is

$$f_n = \frac{4.9744}{2\pi a^2} \sqrt{\frac{D}{\mu}} \tag{24}$$

Equation (24) is taken from Reference 3.

$$f_n = \frac{4.9744}{2\pi (20 \text{ in})^2} \sqrt{\frac{1.511 \left(10^8\right) \left[\frac{\text{lbm in}^2}{\text{sec}^2} \right]}{0.023 \ \frac{\text{lbm}}{\text{in}^2}}} \tag{25}$$

$$f_n = 160.4 \ \text{Hz} \tag{26}$$

<u>Example 2</u>

Equation (25) is the result for a bare honeycomb bulkhead. Now assume that 100 lbm of avionics components are added to the bulkhead. Neglect the geometry and stiffness of the avionics.

Repeat the natural frequency calculation.

The total mass mt is

$$\text{mt} = \text{ms} + \text{mc} + \text{avionics mass} \tag{26}$$

$$\text{mt} = 28.4 \text{ lbm} + 100 \text{ lbm} \tag{27}$$

$$\text{mt} = 128.4 \text{ lbm} \tag{28}$$

The overall mass per volume ρ is

$$\rho = mt \;/\; V \tag{29}$$

$$\rho = \frac{128.4 \;\; lbm}{1414 \;\; in^3} \tag{30}$$

$$\rho = 0.091 \, \frac{lbm}{in^3} \tag{31}$$

The mass per area μ is

$$\mu = \rho T \tag{32}$$

where T is the total thickness.

$$\mu = \left[0.091 \;\; \frac{lbm}{in^3} \right] [1.125 \, in] \tag{33}$$

$$\mu = 0.102 \;\; \frac{lbm}{in^2} \tag{34}$$

Again, the fundamental bending frequency for a simply supported circular plate with radius a is

$$f_n = \frac{4.9744}{2\pi \, a^2} \sqrt{\frac{D}{\mu}} \tag{35}$$

$$f_n = \frac{4.9744}{2\pi \, (20 \; in)^2} \sqrt{\frac{1.511 \left(10^8\right) \left[\dfrac{lbm \; in^2}{sec^2} \right]}{0.102 \; \dfrac{lbm}{in^2}}} \tag{36}$$

$$f_n = 76.1 \;\; Hz \qquad\qquad \text{(with added avionics mass)} \tag{37}$$

Example 3

A different rocket vehicle has an annular bulkhead made from honeycomb. The material is aluminum. The bulkhead properties are shown in Table 2.

Table 2. Bulkhead Parameters for Example 3	
Parameter	Value
Boundary Conditions	Outside: Simply Supported Inside: Free
Diameter	O.D: 63 inch I.D: 42 inch I.D. / O.D. = 0.67
Core Thickness	1.0 inch
Thickness of Each Skin	0.063 inch
Total Thickness	1.125 inch
Skin Elasticity	10.0e+06 lbf/in^2
Core Elasticity	Negligible
Skin Density	0.10 lbm/in^3
Core Density	0.004 lbm/in^3
Poisson's Ratio	0.3
Added Avionics Mass	150 lbm

Calculate the fundamental bending frequency.

The bending stiffness D is the same as that in Example 1.

$$D = 1.511\left(10^{8}\right)\left[\frac{\text{lbm in}^{2}}{\sec^{2}}\right] \tag{38}$$

The total skin mass ms is

$$ms = \pi\left[\frac{(63\text{ in})^{2}-(42\text{ in})^{2}}{4}\right]\left(0.063\text{ in}+0.063\text{ in}\right)\left(0.1\,\frac{\text{lbm}}{\text{in}^{3}}\right) \tag{39}$$

$$ms = 21.6\text{ lbm} \tag{40}$$

The total core mass mc is

$$mc = \pi \left[\frac{(63 \text{ in})^2 - (42 \text{ in})^2}{4} \right] (1.00 \text{ in}) \left(0.004 \frac{\text{lbm}}{\text{in}^3} \right) \tag{41}$$

$$mc = 6.9 \text{ lbm} \tag{42}$$

The total mass mt is

$$mt = ms + mc + avionics \tag{43}$$

$$mt = (21.6 + 6.9 + 150) \text{ lbm} \tag{44}$$

$$mt = 178.5 \text{ lbm} \tag{45}$$

The total volume V is

$$V = \pi \left[\frac{(63 \text{ in})^2 - (42 \text{ in})^2}{4} \right] (1.125 \text{ in}) \tag{46}$$

$$V = 1948 \text{ in}^3 \tag{47}$$

The overall mass per volume ρ is

$$\rho = mt / V \tag{48}$$

$$\rho = \frac{178.5 \text{ lbm}}{1948 \text{ in}^3} \tag{49}$$

$$\rho = 0.092 \frac{\text{lbm}}{\text{in}^3} \tag{50}$$

The mass per area μ is

$$\mu = \rho T \tag{51}$$

where T is the total thickness.

$$\mu = \left[0.092 \ \frac{\text{lbm}}{\text{in}^3}\right]\left[1.125 \text{ in}\right] \tag{52}$$

$$\mu = 0.104 \ \frac{\text{lbm}}{\text{in}^2} \tag{53}$$

The fundamental bending frequency for an annular plate simply supported on the outside and free on the inside with radius a is

$$f_n = \frac{6.222}{2\pi\, a^2}\sqrt{\frac{D}{\mu}} \quad , \qquad \text{I.D. / O.D.} = 0.67 \tag{54}$$

Equation (54) is taken from Reference 4, Table 2.27, with interpolation.

$$f_n = \frac{6.222}{2\pi\left(\frac{63}{2}\text{ in}\right)^2}\sqrt{\frac{1.511\left(10^8\right)\left[\frac{\text{lbm in}^2}{\text{sec}^2}\right]}{0.104 \ \frac{\text{lbm}}{\text{in}^2}}} \tag{55}$$

$$f_n = 38.0 \ \text{ Hz} \tag{56}$$

<u>References</u>

1. MIL-HDBK-23.
2. Baker, Structural Analysis of Shells, Krieger, Malabar, Florida, 1972.
3. T. Irvine, Natural Frequencies of Circular Plate Bending Modes, Vibrationdata.com Publications, 2000.
4. Arthur W. Leissa, Vibration of Plates, NASA SP-160, National Aeronautics and Space Administration, Washington D.C., 1969.
5. P. Bremner, “Finite Element Based Vibro-Acoustics of Panel Systems,” Finite Element Methods in Engineering: Proceedings of the Fifth International Conference in Australia on Finite Element Methods, Melbourne, 1987.

6. Grosveld et al., Finite element development of honeycomb panel configurations with improved transmission loss, INTER-NOISE 2006, Honolulu, Hawaii.

7. Paolozzi and Peroni, Response of Aerospace Sandwich Panels to Launch Acoustic Environment, Journal of Sound and Vibration, (1996) 196(1).

8. T. Irvine, Vibroacoustic Critical and Coincidence Frequencies of Structures, Vibrationdata, Rev A, 2008.

APPENDIX A

Honeycomb Sandwich Plate Properties for Nastran Input

Again, the bending stiffness is

$$D = \left[\frac{E_f}{1-v^2}\right]\left[\frac{t_1\ t_2\ \ h^2}{t_1 + t_2}\right] \tag{A-1}$$

where

$$h = c + \frac{1}{2}\left[t_1 + t_2\right] \tag{A-2}$$

c is the core thickness

Now assume that each face sheet thickness is equal to t.

$$D = \left[\frac{E_f}{1-v^2}\right]\left[\frac{t\ \ h^2}{2}\right] \tag{A-3}$$

$$h = c + t \tag{A-4}$$

$$D = \left[\frac{E_f}{1-v^2}\right]\left[\frac{t\ \left(c+t\right)^2}{2}\right] \tag{A-5}$$

Let I' be the effective area moment of inertia per width.

$$I' = \left[\frac{t\ (c+t)^2}{2} \right] \tag{A-6}$$

Let T_e be the effective thickness.

$$12\ \frac{I'}{{T_e}^3} = 12 \frac{1}{{T_e}^3} \left[\frac{t\ (c+t)^2}{2} \right] \tag{A-7}$$

$$12\ \frac{I'}{{T_e}^3} = \frac{6}{{T_e}^3} \left[t\ (c+t)^2 \right] \tag{A-8}$$

<u>Analysis Steps for Bending Modes Analysis via Nastran</u>

1. Assume that the core stiffness properties are negligible.

2. Set the thickness T_e in the Nastran PSHELL card equal to 1.

3. Calculate the $12\ \frac{I'}{{T_e}^3}$ parameter per equation (A-8). Insert this value in the PSHELL card.

4. Scale the mass density value to give the correct total mass value for a thickness of 1.

The thickness T_e could be set to the total face sheet thickness. The mass density value must be scaled regardless, since the core may have significant mass.

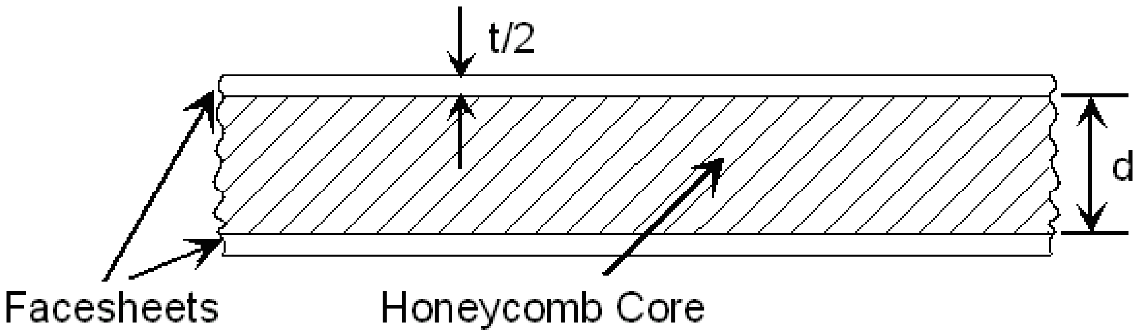
An alternate approach that includes membrane stiffness is given in Appendix B.

APPENDIX B

Note that the variables in this appendix are different than those in the previous sections in this report.

Creating the PSHELL Card for Honeycomb Panels

Jim Loughlin - NASA Goddard Space Flight Center



Information needed:

T - Total facesheet thickness - this should include both face sheets (the value "t" in the figure above)

D - Total core thickness (the value "d" in the figure above)

Facesheet - Material properties

Honeycomb - Material properties, cell size, and gage

Format of the PSHELL Card:

1	2	3	4	5	6	7	8	9	10
PSHELL	PID	MID1	T	MID2	$12I/T^3$	MID3	T_S/T	NSM	

PSHELL and PID are self explanatory.

MID1, T

These terms are for in plane load of a honeycomb panel. Assume that the face sheets take all the load. Therefore:

MID1 references the material for the face sheets.
T is the total thickness of the face sheets.

MID2, $12I/T^3$

These terms are for the bending load of a honeycomb panel. Assume the face sheets take all the load. Therefore:

MID2 references the material for the face sheets.

$12I/T^3$ is the inertia of a rectangular plate

I for Honeycomb is approximately $TD^2/4$ for thin face sheets and a thick core.

If the face sheets are thicker relative to the core, then I is $[2/3][(D/2)+(T/2)]^3-[D^3/12]$

T is the total thickness of the face sheets.

D is the thickness of the honeycomb core.

MID3, T_S/T

These terms are for the shear load of a honeycomb panel. Assume that the honeycomb core takes all the load. Therefore:

MID3 references the material for the honeycomb core. The shear modulus and the poisson's ratio is required on the MATERIAL card for MID3 and is obtained from the HEXEL manuals. You need to know the cell size and gage, or the desired nominal density.

T_S is the shear thickness, or for the case of honeycomb, it is the core thickness D.

T is the total thickness of the face sheets.

NSM

The non-structural mass is required for honeycomb panels. The density (rho) is ignored on the MID3 card, however, the mass of the honeycomb core and bonding material must be included. The NSM, which is given in mass per unit area, is determined by:

$NSM = (rho_{nominal})(D)$,

where D is the thickness and $rho_{nominal}$ is often given in pcf, pounds per cubic foot.

Be careful of your units when calculating NSM!

APPENDIX C

NASTRAN Laminate Element

Another approach is to use a Laminate or PCOMP element. This method simplifies the procedure because the software itself calculates the inertia.

The disadvantage of this method is that it effectively requires the elastic modulus of the core material. Setting the core modulus to some arbitrarily low value gives erroneous results. The output natural frequencies may be much lower than reality.

Furthermore, the core elastic modulus may vary per axis.

Repeat Example 3 using a finite element model with Laminate elements. Assume that the core modulus is 40 ksi in each axis.

The resulting fundamental frequency is 39.6 Hz.

The undeformed and deformed models are shown in Figures C-1 and C-2, respectively.

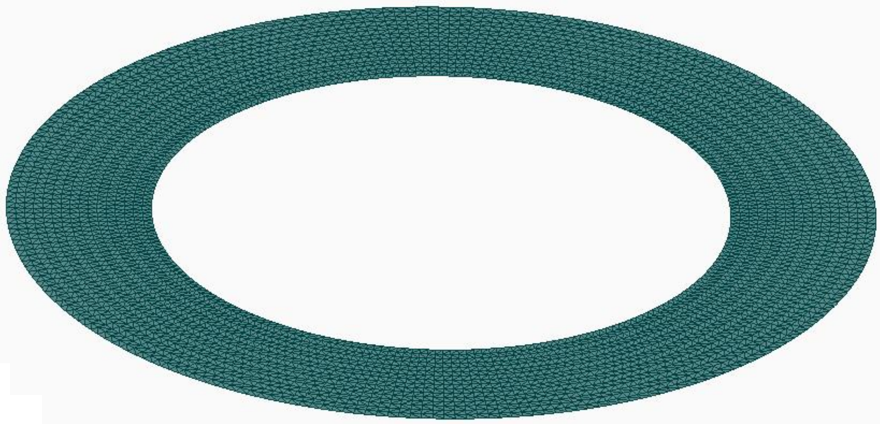


Figure C-1. Undeformed Model

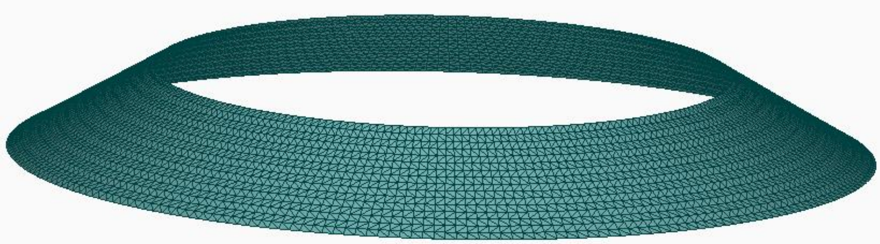


Figure C-2. Mode 1, fn = 39.6 Hz

The model name is: annular_bulkhead.nas

APPENDIX D

Bending-Shear Transition

The methods in Appendices A and B appear to be suitable for determining the fundamental bending mode of a honeycomb-sandwich structure. These same methods, however, may overestimate the natural frequencies of higher modes. The reason is that a “bending-shear transition occurs” as the mode number increases. The result is that the vibration modes tend to become less “panel-like” and more “skin-like” at higher modes due to shear effects. This transition is an important concern for vibroacoustic and hybrid FEA/SEA analyses.

Consider a sample panel as shown in Table D-1.

Table D-1. Honeycomb Sandwich Panel Parameters

Parameter	Value
Length	72 inch
Width	48 inch
Honeycomb Core Thickness	1.0 inch
Thickness of Each Skin	0.063 inch
Total Thickness	1.125 inch
Skin Elastic Modulus	10.0e+06 lbf/in^2
Core Elastic Modulus	40,000 lbf/in^2
Core Shear Modulus	15,385 lbf/in^2
Skin Density	0.10 lbm/in^3
Core Density	0.005 lbm/in^3
Poisson's Ratio	0.3

The panel is simply-supported on all four sides.

The resulting natural frequencies as a function of wavenumber are shown in Figure D-1.

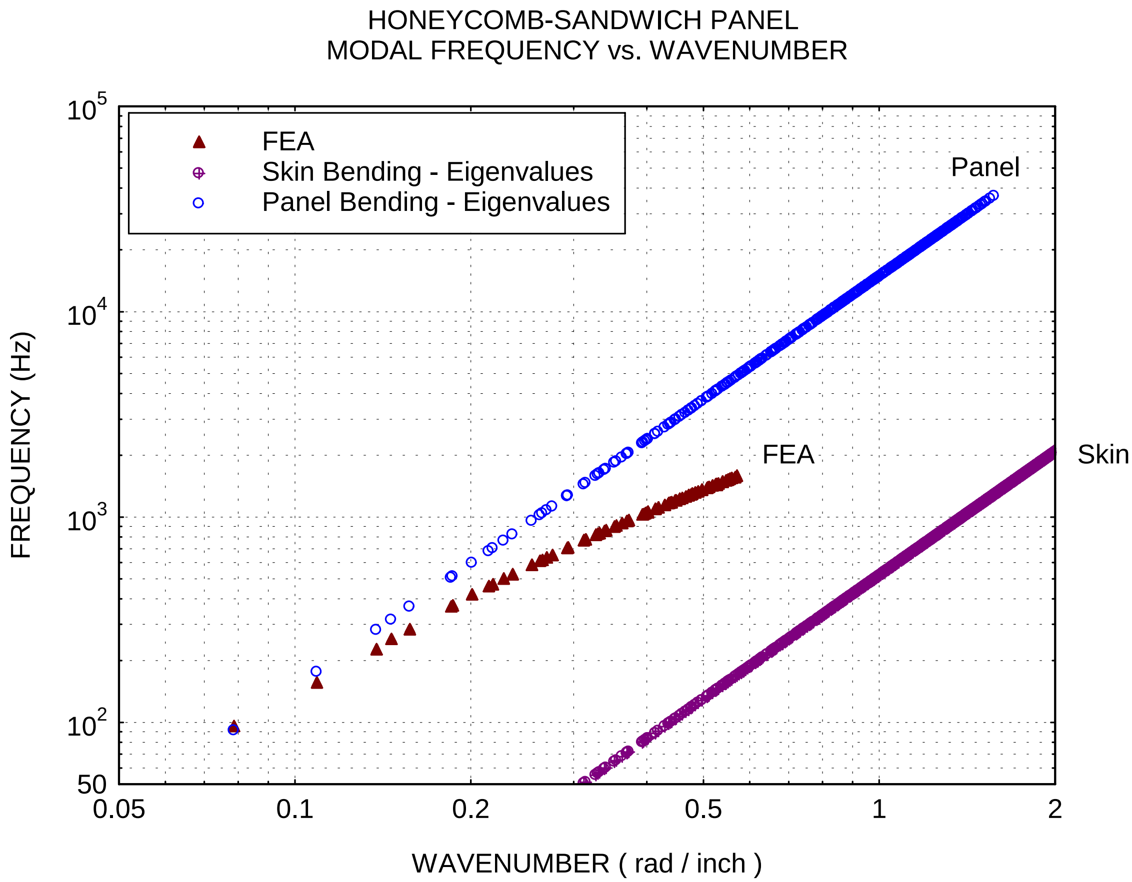


Figure D-1.

The Panel and Skin curves are calculated using the following for their respective natural frequencies ω_{mn} .

$$\omega_{mn} = \sqrt{\frac{D}{\rho h}} \left(\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right) \tag{D-1}$$

where a is the length and b is the width.

The indices m and n are integers with each greater than 1.

Equation (D-1) is taken from Reference 4. The natural frequencies are calculated from the governing differential equation's eigenvalues. The governing equation is a fourth-order differential equation which accounts for bending, but it does not account for any shear or membrane effects.

The plate bending stiffness D for the Panel curve is calculated using the method in Appendix A.

The D value for the Skin curve is simply calculated using a single face sheet with a thickness of 0.063 inch. The natural frequencies are then calculated via equation (D-1).

Note that the wave numbers are

$$k_m = m\pi / a \tag{D-2}$$

$$k_n = n\pi / b \tag{D-3}$$

The overall wave number k is

$$k = \sqrt{k_m{}^2 + k_n{}^2} \tag{D-4}$$

As an aside, the wave number k is related to the wavelength λ by

$$k = \frac{2\pi}{\lambda} \tag{D-5}$$

The Panel curve in Figure D-1 fails to account for the bending-shear transition effect. In contrast, the finite element analysis (FEA) curve demonstrates this effect.

The finite element results were calculated using an approach similar to the laminate method in Appendix C except that the “laminate” was constructed manually rather than using a PCOMP.

A sample “building block” element is shown in Figure D-2. The complete, undeformed model is shown in Figure D-3. A sample mode shape is shown in Figure D-4.

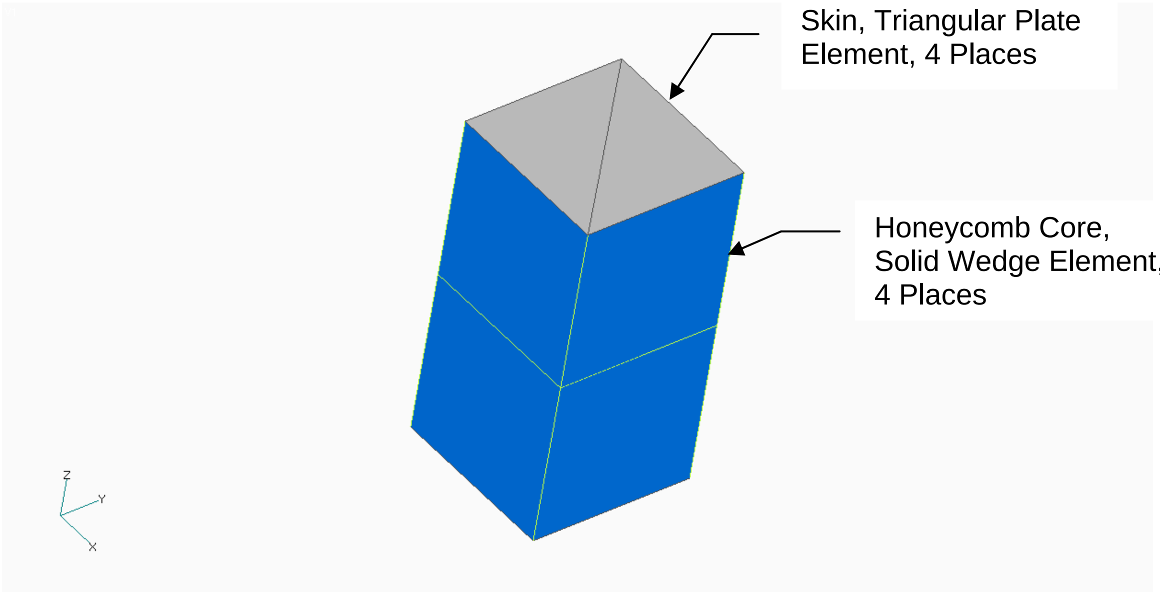


Figure D-2. Manually Constructed Laminate for FEA

The “building block” dimensions are 0.5” x 0.5” x 1.0”

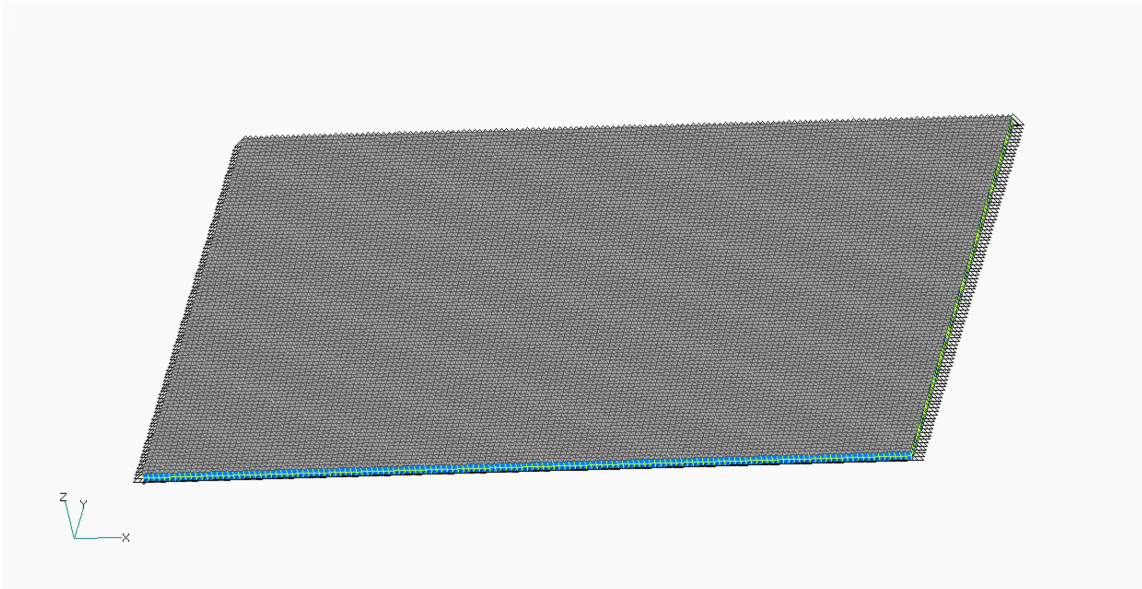


Figure D-3. Undeformed FEA Model

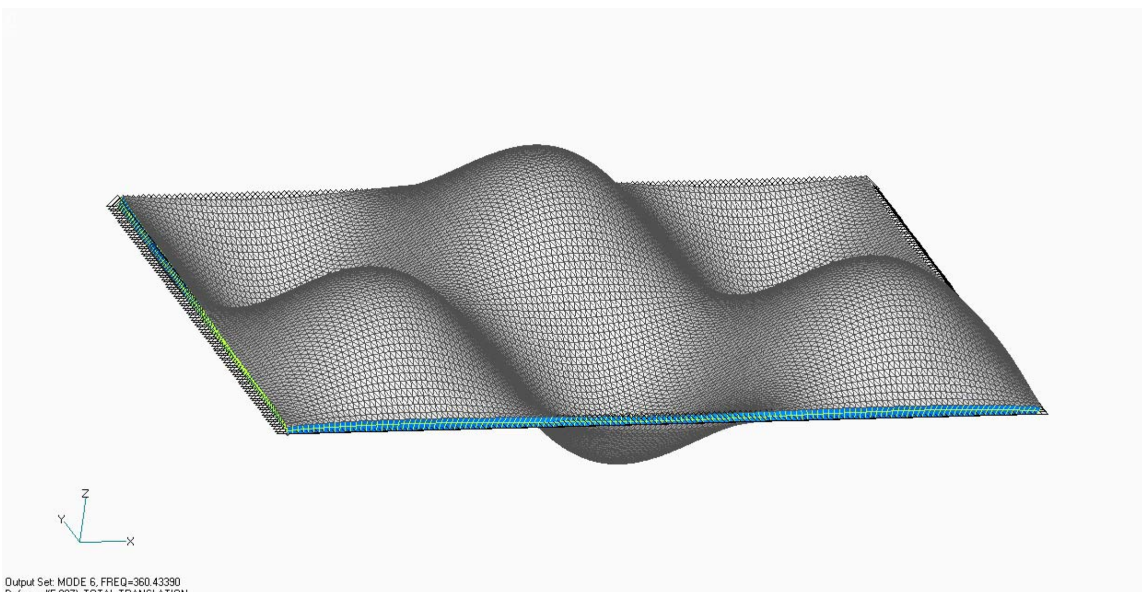


Figure D-4. Typical FEA Mode Shape, fn = 362 Hz

The FEA filename is: bending_shear.nas

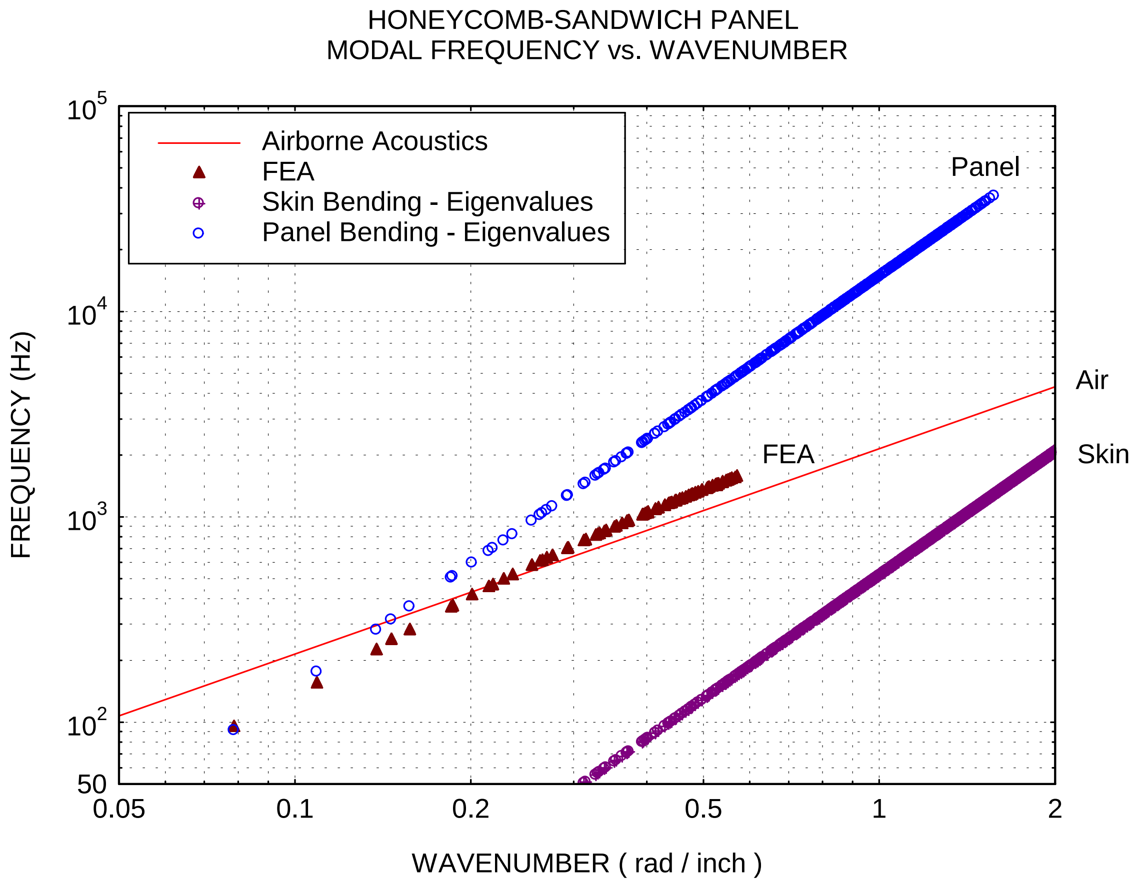


Figure D-5.

Figure D-5 is the same as Figure D-1 except that the Airborne Acoustic curve is added.

The “critical frequency” is the frequency at which the Air curve intersects the FEA curve. This frequency was about 500 Hz for the sample structure in Figure D-5.

The Air curve in Figure D-5 would also intersect the Skin curve if the plot limits were extended.

Note structures in general may or may not have a critical frequency, depending on their respective design parameters. See Reference 8 for further information.

Honeycomb Sandwich Panel Summary

The following summary is taken from References 6 and 7.

Range	Characteristic
Low Frequencies	Bending of the entire structure as if were a thick plate
Mid Frequencies	Transverse shear strain in the honeycomb core governs the behavior
High Frequencies	The structural skins act in bending as if disconnected