ATTENUATION OF LONGITUDINAL SHOCK WAVES IN A ROD   Revision B

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Distance attenuation curves for pyrotechnic-induced shock pulses in launch vehicles are given in Reference 1. These curves were obtained from measured data. The purpose of this tutorial is to develop analytical attenuation curves for the case of longitudinal wave propagation[1] in a rod. The results are obtained via finite element method, using References 2 and 3.

The sample rod has the following characteristics:

Parameter	Value
Boundary Conditions	Free-Free
Material	Aluminum
Diameter	0.25 inch
Length	200 inch
Damping for Each Mode	%1 for case 1 %5 for case 2
Number of Elements	100
Sample Rate	500K samples/sec

The rod has a rigid-body mode with a frequency of zero due to the free-free boundary condition. Otherwise, the fundamental frequency is 496 Hz, with integer harmonics thereof.

The excitation source is an initial displacement of 0.001 inch at one of the free ends of the rod. All other points of the rod have zero initial displacement. Furthermore, the initial velocity is zero at all points. This set of initial conditions would be nearly impossible to produce with real hardware; but this approach was determined to be very useful for shock propagation analysis, as determined by trial-and-error with a variety of excitation methods.

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[1] Note that a pyrotechnic shock can excite many other types of modes, particularly bending modes and the ring mode.

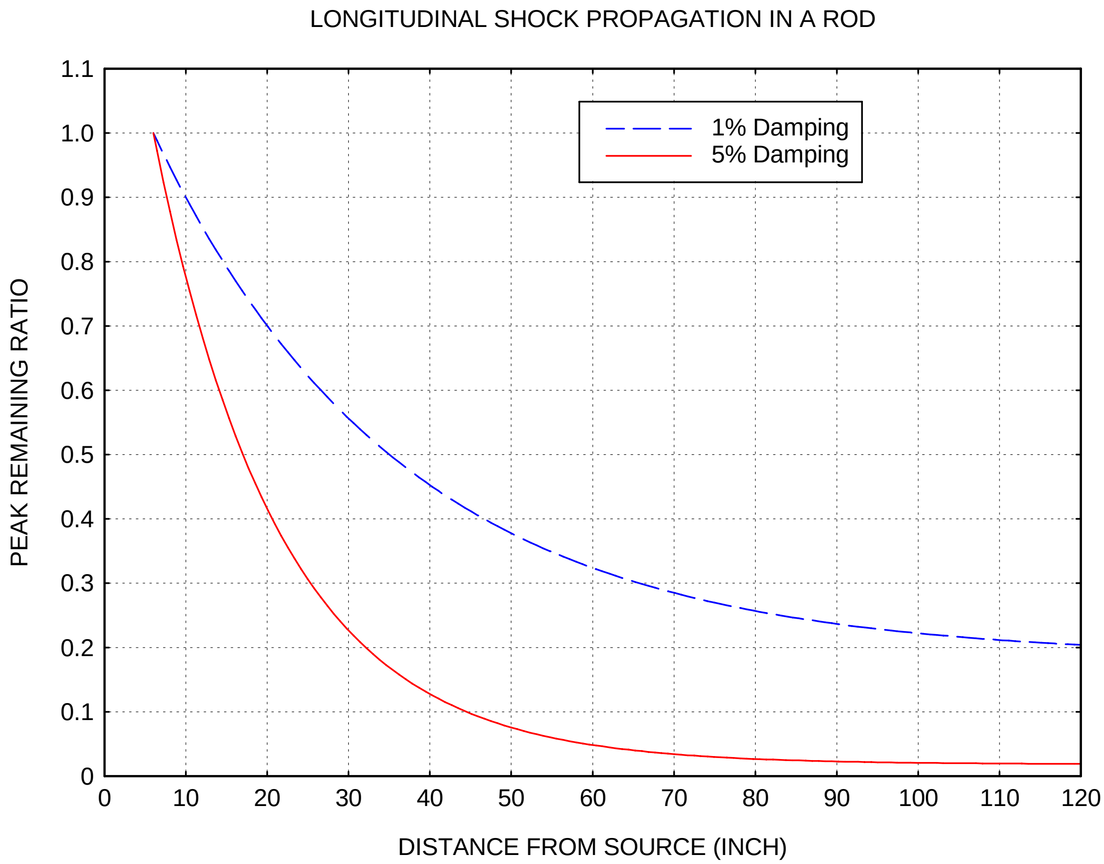

Figure 1.

A modal transient solution was then applied to the finite element method using Matlab script: beam_long_initial_disp.m, ver 1.3. The modal transient method for systems subjected to initial conditions is given in Reference 3.

The curves are calculated from the respective peak acceleration values in the time domain and in the longitudinal axis. Each is a smoothed curve, but with only a slight amount of smoothing.

The curves are normalized so that the peak remaining ratio is unity at 6 inches. This follows the convention in Reference 1. The attenuation curves in Reference 1, however, are not given in terms of a damping ratio.

The results show that damping has a significant effect on the shock wave propagation.

Each of the curves assumes uniform damping. Reality is more complicated given that each mode in a real system would have its own damping ratio.

The curves are characterized by mathematical curve-fitting formulas as follows.

x is the distance from the source (inches)

y is the acceleration peak remaining ratio

The domain is constrained for each curve by

$6 \leq x \leq 120$ inches

The respective peak remaining ratio formulas are

$$y = 0.815 \exp(-0.0327 * (x - 6)) + 0.185 \quad \text{for 1\% damping} \tag{1}$$

$$y = 0.981 \exp(-0.0646 * (x - 6)) + 0.019 \quad \text{for 5\% damping} \tag{2}$$

Sample time histories are given in Appendix A for the 1% damping case.

This study could be expanded further by considering different dimensions, materials, boundary conditions, excitation methods, etc.

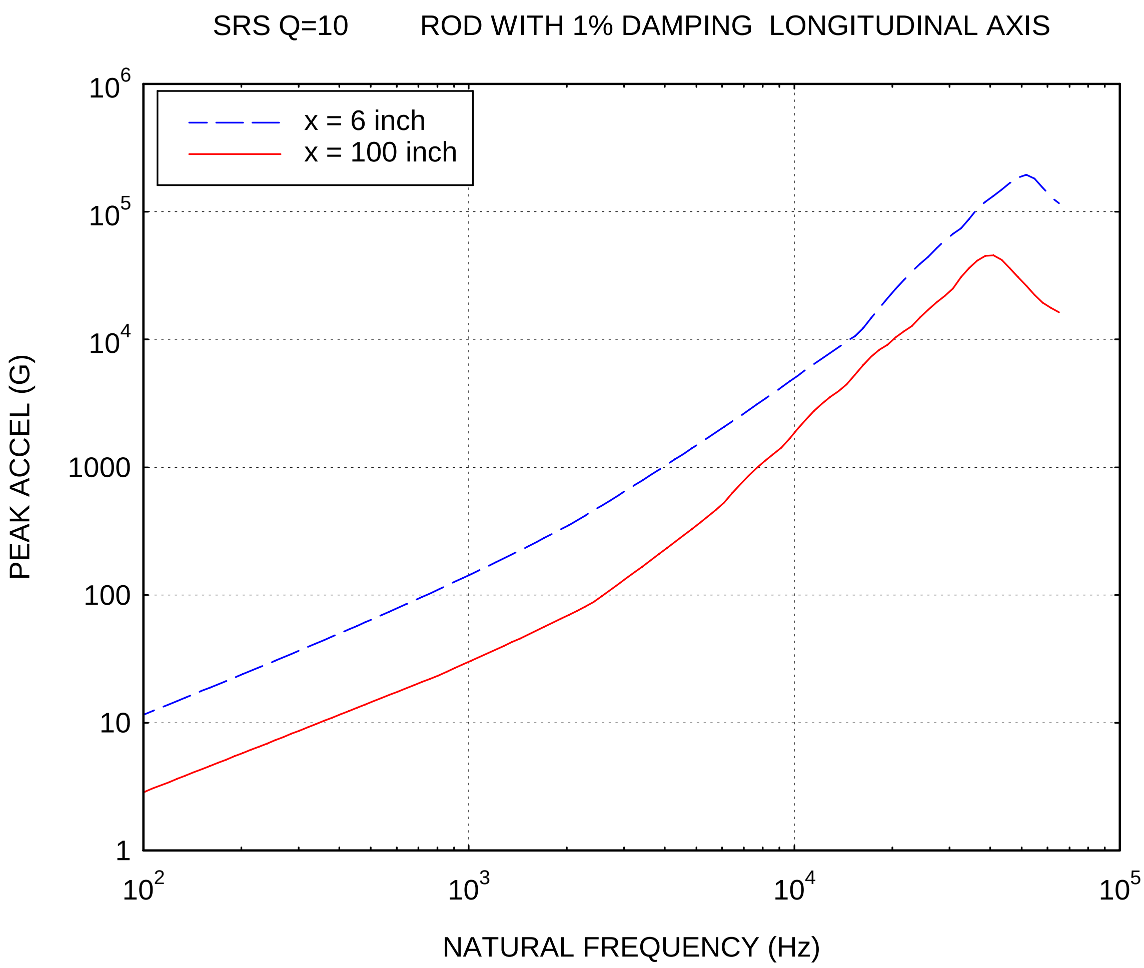

Figure 2.

The SRS comparison shows that the attenuation is rather uniform with respect to natural frequency below 10 KHz.

Note that the Q=10 values assumes that a component with 5% damping would be mounted to the rod. The rod itself has a damping value of 1%.

References

1. W. Kacena, M. McGrath, A. Rader; Aerospace Systems Pyrotechnic Shock Data, Vol. VI, NASA CR 116406, Goddard Space Flight Center, 1970.
2. T. Irvine, Longitudinal Vibration of a Rod via the Finite Element Method, Rev B, Vibrationdata, 2008.
3. T. Irvine, Free Vibration of a Two-Degree-of-Freedom System Subjected to Initial Velocity and Displacement, Vibrationdata, 2005.

APPENDIX A

Time History Plots, 1% Damping Case

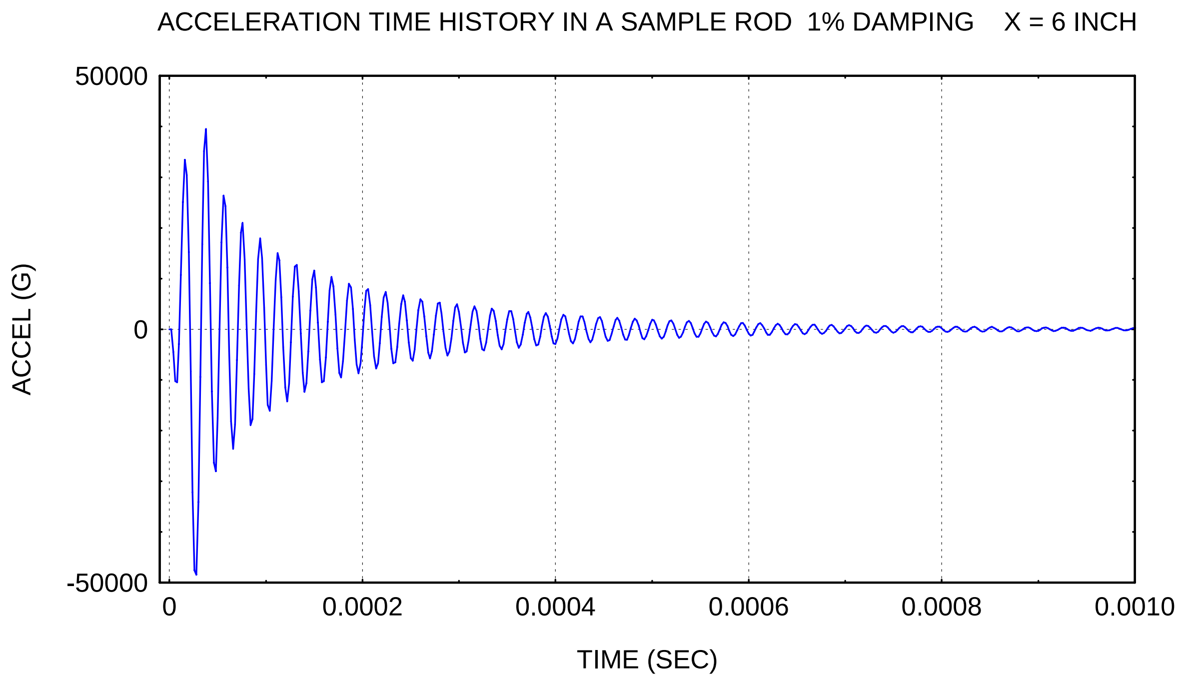

Figure A-1.

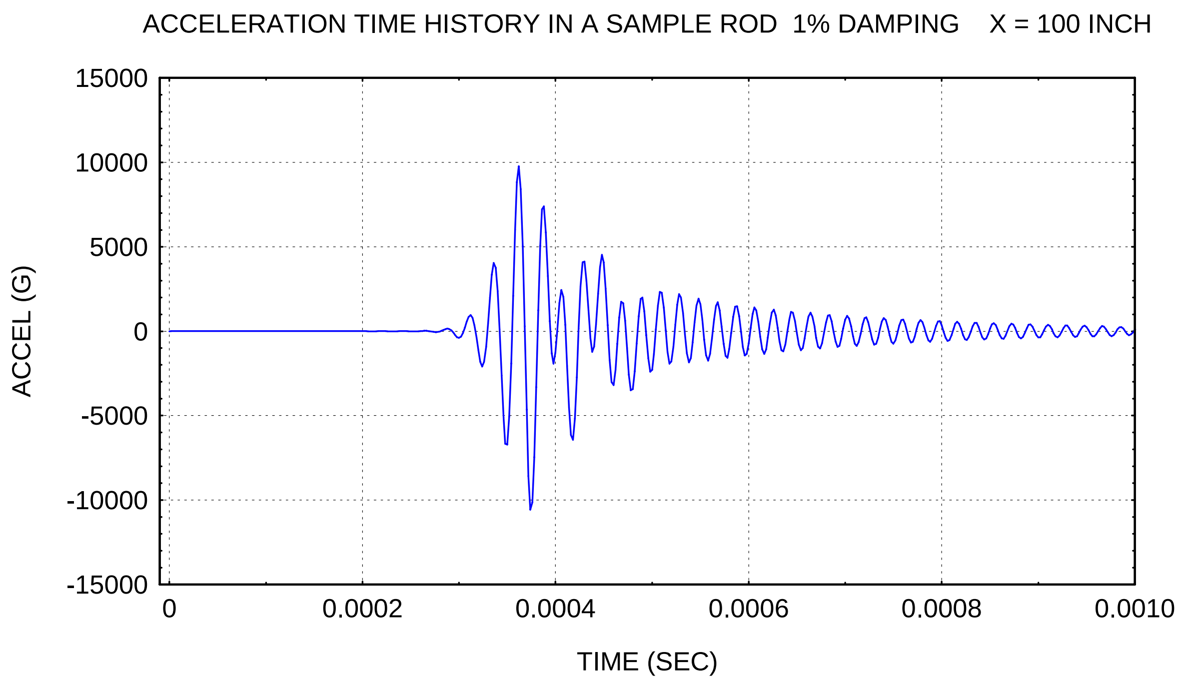

Figure A-2.

Note that the longitudinal wave speed in aluminum is about 200,000 in/sec. The onset of the pulse should thus reach the 100 inch mark at 0.0005 seconds. This occurs at about 0.0003 seconds in Figure A-2, however.

The reason for this error is that the finite element method natural frequencies are higher than the true natural frequencies, particularly at higher mode numbers. This causes the apparent wave speed to be higher than reality.