THE SHOCK RESPONSE SPECTRUM AT LOW FREQUENCIES

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This paper shows that for small damping the correct acceleration shock response spectrum will roll off with a slope of 6 dB/octave on a log-log plot. An undamped acceleration shock spectrum and the shock spectrum based on a relative displacement model will roll off with a slope of 12 dB/octave.

Introduction

The slope of the shock spectrum at low frequencies is typically evaluated by using a well known relationship between the undamped residual shock response spectrum and the magnitude of the Fourier spectrum [1]. This has led to the common belief that the shock response of shocks with a low velocity change, like a pyrotechnic shock, will have a low frequency slope of 12 dB/octave. But anyone who has examined these shocks has seen data which violates this assumption. Several suggestions have been offered to explain the slopes which are different from 12 dB/octave including:

- The spectrum doesn't really roll off at 12 dB/octave
- Zero offsets cause the spectrum to roll off at a slope different from 12 dB/octave
- Truncation errors cause the problem
- Incorrect algorithms are being used to calculate the spectrum

Each of these suggestions are examined in this paper.

Models

Before I can discuss the shock response spectrum we must examine the single-degree-of-freedom (SDOF) models which are used to evaluate the shock response spectrum. A large number of models are used, but they can be reduced to variations of just two models, which I will discuss.

<u>Acceleration</u> <u>Model</u>- The input to this model is the absolute acceleration of the base of the SDOF system. The response of the SDOF system is the absolute acceleration of the mass. The transfer function of this system in the complex Laplace domain is given by

$$H_a(s) = \frac{2\,\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\,. \tag{1}$$

The impulse response of this model is given by

$$h_a(t) = \omega_n(1-\zeta^2)^{-1/2}\exp(-\zeta\omega_n t)\sin(\omega_d t+\phi) \tag{2}$$

where

$\phi = \tan^{-1}[2\zeta(1-\zeta^2)^{1/2}/(1-2\zeta^2)]$

ζ = the fraction of critical damping

ω_n = the natural frequency, $(K/M)^{1/2}$

ω_d = the damped natural frequency, $\omega_n(1-\zeta^2)^{1/2}$

<u>This is the most commonly used model in the aerospace industry.</u>

<u>Relative</u> <u>Displacement</u> <u>Model</u>- The input to this model is the absolute acceleration of the base of the SDOF system. The response of the system is the relative displacement between the base and the mass. If the relative displacement is expressed in terms of an equivalent static acceleration,

$$\ddot{y}_{eq} = (y-x)\omega_n^2, \tag{3}$$

the transfer function becomes

$$H_d(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}. \tag{4}$$

The impulse response of this system is given by

$$h_d(t) = \omega_n(1-\zeta^2)^{-1/2}\exp(-\zeta\omega_n t)\sin(\omega_d t). \tag{5}$$

This model is commonly used in the Navy and the seismic industry.

The Relationship Between the Fourier Spectrum and Shock Response Spectrum

A well known relationship exist between the undamped residual shock response spectrum and the magnitude of the Fourier spectrum[2].

$$S_r(\omega_n) = \omega_n|A(\omega_n)| \tag{6}$$

where

S_r = undamped residual-shock-response spectrum
ω_n = natural frequency (rad/s)
$A(\omega_n)$ = magnitude of the Fourier Spectrum of the acceleration input evaluated at ω_n

We will also use the relationship

$$\frac{d^n f(t)}{dt^n} \Leftrightarrow (j\omega)^n F(\omega) \tag{7}$$

where $\Leftrightarrow$ means a Fourier transform pair,

and
$f(t) \Leftrightarrow F(\omega)$.

We can now derive the slope of the undamped residual shock response spectrum. Note that both models reduce to the same form for the undamped case. The velocity change, Δv, of an input acceleration is given by its Fourier spectrum evaluated at zero frequency. The Fourier spectrum is continuous and smooth near the origin. Therefore, for frequencies near zero this gives

$$S_r(\omega_n) = \Delta v\omega_n, \tag{8}$$

which is a slope of 1 on a log-log plot or 6 dB/octave*.

If the velocity change is zero, but the displacement change, Δd, is nonzero, the Fourier spectrum of the velocity for small frequencies is given by Δd. The use of Eq (7) and then Eq (6) yields

$$S_r(\omega_n) = \omega_n^2\Delta d \tag{9}$$

for frequencies near zero. This gives the spectrum a slope of 2 on a log-log plot or 12 dB/octave near zero frequency.

If the displacement change is also zero but its integral is a nonzero value, Δc, the undamped residual shock spectrum near zero frequency is given by

$$S_r(\omega_n) = \omega_n^3\Delta c. \tag{10}$$

This results in a slope of 3 or 18 dB/octave at low frequencies.

Slope of the Damped Shock Spectrum at Low Frequencies

In this section I will investigate the correct slope for the damped shock response spectrum at low frequencies. If the velocity change of the transient is nonzero and the velocity monotonically increases to its final value, the transient can be represented by an impulse when the product of natural frequency of the SDOF system and the sampling interval is much less than one. I can then represent the waveform by the impulse

$$\ddot{x}(t) = \Delta v\delta(t). \tag{11}$$

Looking first at the acceleration model the residual response is given by

$$\ddot{y}(t) = \Delta v h_a(t). \tag{12}$$

The primary response for an impulsive input is zero. If I make the assumption that the damping is small the maximum of the impulse response is approximately

$$h_{max}(t) = \omega_n. \tag{13}$$

This gives

$$S(\omega_n) = \Delta v\omega_n. \tag{14}$$

which is the same result as Eq. 8. The same result is achieved for the relative displacement model. Thus the common assumption, that the residual response dominates the low frequency response and that the relationship between the Fourier spectrum and the undamped residual shock spectrum can be used to estimate that response, is valid for this case. We get a slope of 1 or 6 dB/octave at the low frequencies.

If the velocity does not increase monotonically to its final value (i.e. zero crossings of the acceleration waveform exist) this analysis is not valid. We must now represent the input waveform as a series of impulses. To examine this problem I will first consider the response of a damped SDOF system to an impulse when the time is small. The impulse response of the acceleration model can be approximated for small values of time and damping by

$$h_a(t) = \omega_n(\omega_n t + 2\zeta). \tag{15}$$

*A slope of 1 is not precisely 6 dB/oct, but this value will be used in this paper.

For the relative displacement model

$$h_d(t) = \omega_n^2 t, \quad (16)$$

which is the same as for the acceleration model except the second term is zero. The acceleration response for early times and small damping is then given by

$$\ddot{y}(t) = \Delta v \omega_n(\omega_n t + 2\zeta). \quad (17)$$

The response can be approximated by a straight line for early times. Equation (17) can be derived in a different way. If we assume that the mass of the oscillator remains essentially stationary with respect to the base, the force on the mass is given by

$$F = Kx + C\dot{x} \quad (18)$$

where

$$2\zeta\omega_n = C/M$$

and where x is the base input displacement. The acceleration of the mass is then approximately

$$\ddot{y}(t) = (K/M)\ x(t) + (C/M)\ \dot{x}(t)$$

or

$$\ddot{y}(t) = \omega_n^2 x(t) + 2\ \zeta\omega_n x(t) \quad (19)$$

If

$$\ddot{x}(t) = \Delta v \delta(t)$$

then

$$\dot{x}(t) = \Delta v \text{ and}$$
$$x(t) = \Delta v\ t.$$

These results give

$$\ddot{y}(t) = \Delta v \omega_n^2 t + \Delta v 2\zeta\omega_n$$

which is the same as Eq (17). Equations (17)-(19) show that the mass accelerations for early times is dominated by two terms: a stiffness force proportional to the base displacement and a damping force proportional to the base velocity.

These equations will now be used to write a more general expression for the response to a sequence of impulses. These expressions will then be used to estimate the primary response of waveforms where the velocity does not increase monotonically. Let the input be given by

$$\ddot{x}(t) = \sum_{i=1}^{L} A_i \delta\ (t - i\tau) \quad (20)$$

where δ is the unit impulse function, and A_i is the velocity change of each impulsive input. If the total pulse duration is short compared to the period of the SDOF system (i.e. $\tau\omega_n L$ <<1) and the damping is small, the response of the system can be approximated by a sum of responses like Eq (17)

$$y(t) = \sum_{i=1}^{L} A_i\omega_n[\omega_n(t- i\tau) + 2\zeta]U(t-i\tau) \quad \text{for } t\leq L \quad (21)$$

where

U(t) = the unit step function.

This response is just a sequence of straight lines whose slope changes at each impulsive input. The maximum response will occur at one of these changes in slope. The response at the impulsive input at time $m\tau$ can be evaluated as

$$\ddot{y}(m\tau) = \sum_{i=1}^{L} A_i\omega_n[\tau_n(m-i)\ \tau + 2\zeta]U(m-i). \quad (22)$$

The primary response can be approximated by the largest of this set of values, $y_{max}(m\tau)$, for some value of m between 1 and L. Thus for the damped case the maximum response will be of the form

$$\ddot{y}_{max}(m\tau) = C_1\omega_n^2 + C_2\omega_n. \quad (23)$$

The undamped case will always be of the form

$$\ddot{y}_{max}(m\tau) = C_3\omega_n^2. \quad (24)$$

The relative displacement model will give a maximum primary response of the form

$$\ddot{y}_{eq\ max}(m\tau) = C_4\omega_n^2 \quad (25)$$

We can now see that the primary response for the relative displacement model, and for the undamped acceleration model will always have a slope of 2 or 12 dB/octave at low frequencies. If the slope of the residual response is greater than 12 dB/octave the residual response must be less than the primary response for a very low frequency. The final slope of the shock response spectrum will be 12 dB/octave.

The damped acceleration model will have two regions of interest. For the intermediate frequencies where the first term of Eq (23) dominates, the primary spectrum will have a slope of 12 dB/octave. At the very low frequencies where the second term of Eq (23) dominates the slope will be 6 dB/octave. Thus the primary response must be greater than or equal to the residual response which rolls of with a slope of at least 6 dB/octave. The conclusion is that for small damping, the maximax shock spectrum for the acceleration model will will always have a low frequency slope of 6 dB/octave.

The usual assumption is that the residual spectrum is larger than the primary spectrum at low natural frequencies. We see that this is the case for single sided waveforms, but is not true for double sided acceleration pulses.

Two examples will illustrate this important result. For the first example, consider an acceleration input as given by Figure 1. The velocity and displacement waveforms for this acceleration are shown in Figures 2 and 3. The input can be approximated by three impulses

$$\ddot{x}(t) = \delta(t-0.001) - 2\delta(t-0.023) + \delta(t-0.045) \qquad (26)$$

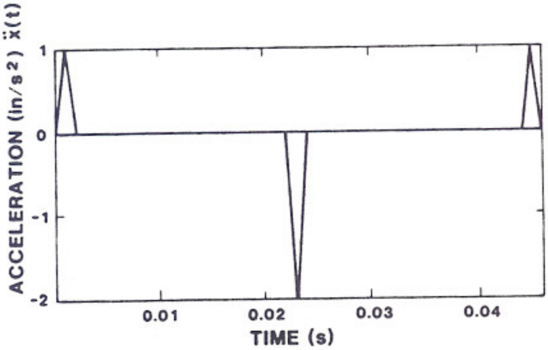

Figure 1 Base Input Acceleration, Basically Three Impulses

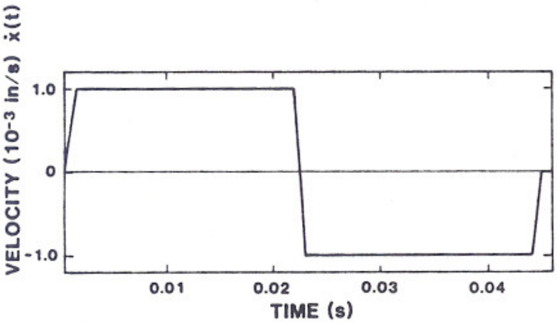

Figure 2 Base Input Velocity

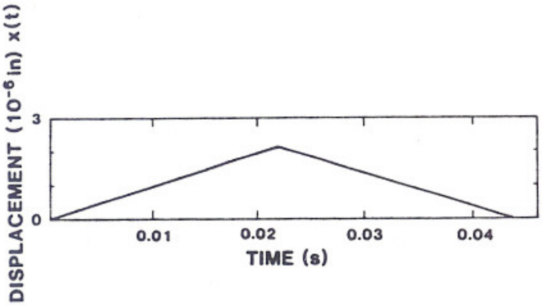

Figure 3 Base Input Displacement

The approximate response given for a time step of 1 ms is given by Eq (21) as

$$\ddot{y}(10^{-3}m) = (10^{-3})\,\omega_n\{[10^{-3}\omega_n(m-1) + 2\zeta]U(m-1) - 2\,[10^{-3}\omega_n(m-23) + 2\zeta]U(m-23) + [10^{-3}\omega_n(m-45) + 2\zeta]U(m-45)\}. \qquad (27)$$

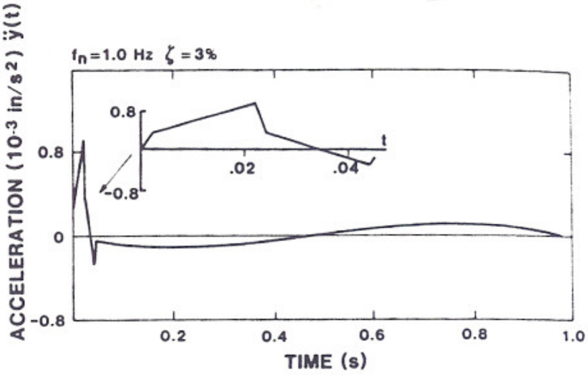

Figure 4 Acceleration Response of 1-Hz, 3% Damped System to the Input of Figure 1.

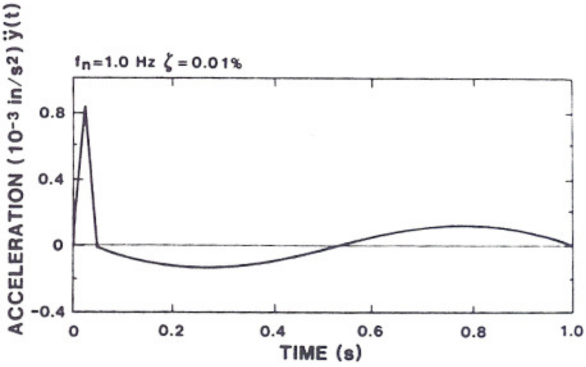

Figure 5 Acceleration Response of 1-Hz, 0.01% Damped System to the Input of Figure 1.

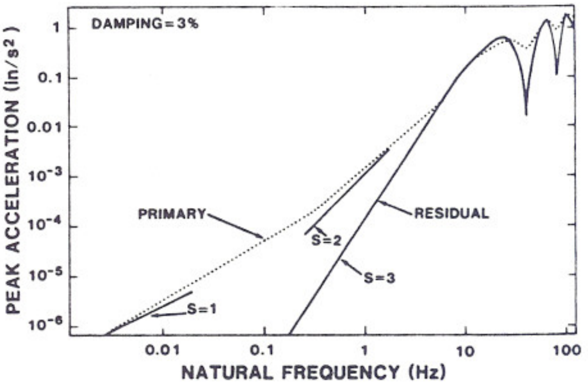

Figure 6 Acceleration Shock Response Spectrum of the Input of Figure 1.

The acceleration response for a natural frequency of 1 Hz and 3% damping is shown as Figure 4. The inset in Figure 4 is the acceleration expanded for the early portion of the period. The primary response is close to the straight line approximation predicted by Eq (21). Equation (19) predicts that the primary response at low frequencies will be proportional to the base displacement if the system is undamped. Figure 5 shows the response for a damping of 0.01%. A comparison of Figures 3 and 5 confirms the above prediction. In both Figures 4 and 5, the primary response is larger than the residual response.

The shock spectrum for this input is given in Figure 6. The shock spectrum at the low frequencies can be predicted using Eq (27). For example, the spectrum at 0.1 Hz can be estimated by noting that the maximum response will occur near t = 22 ms

$$\ddot{y}_{max} = 2\pi(.1)(10^{-3})[10^{-3}(2\pi)(.1)(21) + 2(.03)] \quad (28)$$

$$\ddot{y}_{max} = 4 \times 10^{-5} \text{ in/s}^2,$$

which agrees with Figure 6.

A second example is given by the WAVSYN in Figure 7. The WAVSYN pulse is defined by

$$x(t) = A \sin(2\ bt) \sin(2\ ft) \quad \text{for } 0 < t < T \quad (29)$$
$$= 0 \quad \text{elsewhere}$$

where

A = the amplitude
f = the frequency of the pulse
b = the frequency of a half sine window, f/N
T = the pulse duration, N/(2f)
N = an odd integer greater than 1.

In Figure 7, A = 1 g, N = 39, and f = 1 Hz. The velocity and displacement changes for this waveform are zero. The above discussion shows the residual spectrum should have a slope of 12 dB/octave, and the primary spectrum should have a final slope of 6 dB/octave. Figure 8 is the acceleration shock spectrum of this waveform. We can see the region just below 1 Hz where the residual response is larger, the area at about 0.5 Hz where the response is dominated by the primary response damping forces, and at the very low frequencies the charactistic final slope of 6 dB/octave.

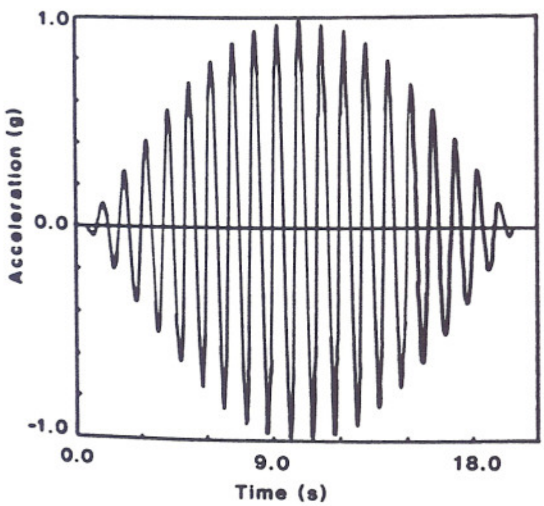

Figure 7 A 1-Hz, 1-g, 39-Half Cycle, WAVSYN Pulse

Errors Caused by Zero Offsets

A small error in the zero line of an acceleration waveform can cause large errors in the low frequency end of the shock response spectrum. The error occurs because the zero offset will cause an error in the final velocity of the waveform. A small error integrated over the entire length of the time history can result in a substantial velocity error. The offset will appear to the shock spectrum calculations as a square wave of a duration equal to the pulse duration. An example is shown in Figure 9, and Figure 10 (curves a and c). Figure 9 is an acceleration input composed of the sum of two exponentially decaying sinusoids. One component is at 100 Hz with an amplitude of 1 g. The second is a highly damped component at a much lower frequency whose amplitude and delay were chosen to force the velocity and displacement changes to be zero. Figure 10, curve a, is the shock spectrum of this waveform. A 0.05 g zero offset was added to the waveform and the shock spectrum was recomputed (curve c on Figure 10). The offset looks like an added square wave with an amplitude of 0.05 g. The shock spectrum of this square wave has a peak amplitude of 0.1 g. Since the duration of this square wave is 0.2048 s the first peak in the shock spectrum of this component will be at about 2.4 Hz. Curve c of Figure 10 confirms these predictions.

One of the dangers of these errors is that the errors can propagate into specifications derived from the measured shock spectrum, which in turn leads to tests with unreasonable velocity and displacement requirements.

Errors Caused by Waveform Truncation

Truncation of an acceleration waveform can also cause errors in the low frequency end of the shock response spectrum. As for the offset errors, these errors are caused by an incorrect final velocity. The error in the final velocity divided by the pulse duration and converted to acceleration units will give an approximation of

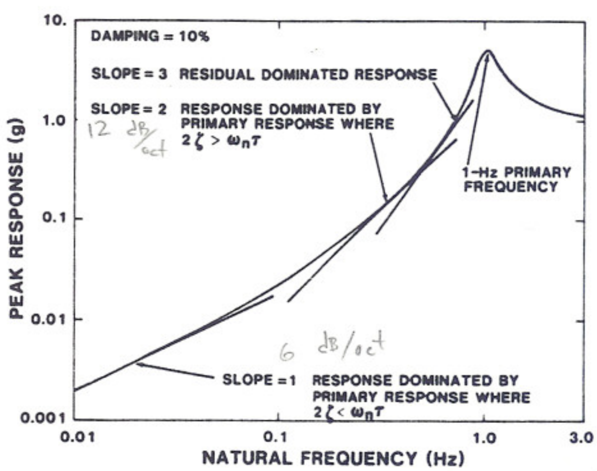

Figure 8 Shock Response Spectrum of a 1-Hz, 1-g, 39-Half Cycle, WAVSYN Pulse

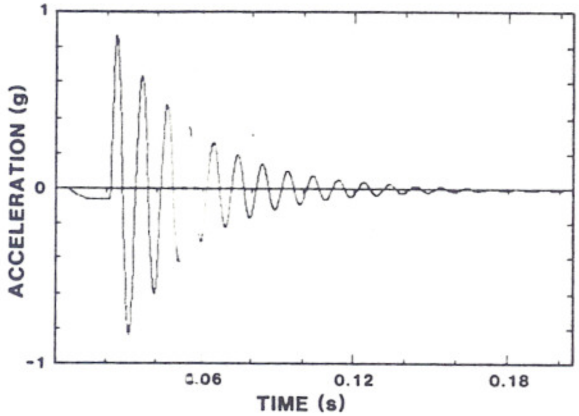

Figure 9 Acceleration of the Sum of Two Exponentially Decaying Sinusoids

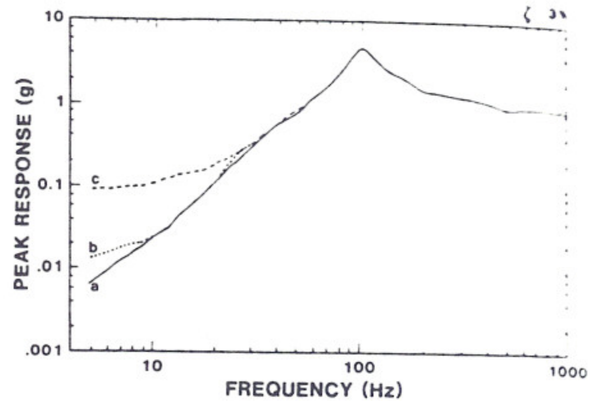

Figure 10 Shock Response Spectrum of the Acceleration Waveform Shown in Figure 9

the error. To illustrate, the waveform in Figure 9 was truncated at 0.0564 s. The resulting velocity error was about 0.18 in/s. This translates into an acceleration error of about 0.008 g. We would then expect an error in the shock spectrum of about twice this value (0.016 g) with the lowest frequency peak at about 8.9 Hz. Curve b, Figure 10 confirms this error.

As in the case of offset errors, the biggest danger of this error is the propagation of the error into specifications.

Incorrect Algorithms

This source of error cannot be discussed in detail without a thorough knowledge of the algorithm in question. Recursive digital filters are a popular method to calculate the shock response spectrum, and it is known that the filter weights are subject to serious roundoff problems at low natural frequencies. Careful attention must be paid to the calculation of these weights and to the details of the implementation at low natural frequencies. In some methods the data are filtered with a low pass digital filter and the data are then decimated before the spectrum is calculated at the low frequencies. This could lead to errors in calculating the primary spectrum, which has been demonstated to be important at the low frequencies. More investigation is required in this area. A good test for any algorithm is to reproduce Figure 8.

Application to Pyrotechnic Shock Data

We can now use the above results to establish guidelines when viewing pyrotechnic and similar shock data when presented in the form of shock response data. First one must establish whether the acceleration model or the relative displacement model was used to calculate the shock response. This determines the final slope of the data. If the velocity change is zero, the shock response should have a final slope of 12 dB/octave or change from a slope of 12 dB/octave to 6 dB/octave (depending on the model used) in a smooth manner as the frequency decreases. The curve will be concave. If a convex "bump" is observed at the low frequencies, the bump likely represents a velocity change which appears to the computations as a acceleration square wave added to the waveform.

Remember the shock spectrum of a square wave has an intial slope of 6 dB/octave, reaches a peak of twice its amplitude at a frequency equal to the inverse of twice its duration, has some ripples with each peak of the ripples reaching a magnitude of twice the amplitude, and continues along a line with a slope of zero at an amplitude of twice its amplitude as the frequency increases. Because of this characteristic shape of the shock spectrum of a square wave, the shock spectrum with an added square wave can appear to have a slope of zero at low frequencies. The amplitude of this area of zero slope will be a function of both the observed velocity change and the duration of the digitized waveform. If the velocity change is a constant and not an error caused by a zero offset or truncation, the amplitude of this area will decrease as the duration of the digitized waveform is increased. The magnitude of the velocity change can be estimated by dividing the shock spectrum amplitude in the region of the zero slope by two, multiplying by the pulse duration and converting to velocity units. If this velocity change is representative of the environment the data are probably valid. But if the velocity change is unreasonably large, the low end of the shock spectrum is in error. The source of the error can then be investigated.

The natural frequency where the shock spectrum should again start to decline is given by the inverse of twice the duration. The final slope at frequencies much less than the inverse of twice the duration will be 6 dB/octave.

Shock Spectrum at High Natural Frequencies

This is not the main topic of this paper, but I would like to make some comments on the subject. A common rule of thumb in transient data reduction is that you should sample the data with a minimum of 10 times the highest frequency of interest. Some have interpreted this rule to mean the shock response spectrum should not be calculated above a natural frequency which is greater than 1/10 th of the sampling frequency. Often this is overly conservative. The frequency content of the data is of primary concern, not the natural frequency of the SDOF system used in the shock spectrum calculations. The data must be sampled frequently enough to avoid large errors in the detection of the peak of the transient. Some authors [3] have suggested 6 to 10 samples/cycle for a 5% error bound. This is often conservative because it was assumed that the input was a sine wave at the highest frequency. Three or four samples of the highest frequency may be adequate for peak detection if the high frequency content is a small part of the total energy in the waveform. The important point is that the sample rate should be picked with only the charactistics of the input waveform in mind.

The next question is, will the algorithm used to calculate the shock response spectrum calculate the correct values for the range of natural frequencies desired. It is known that the oldest and simplest form of the recursive filter algorithm has serious errors as the natural frequency approaches half the sampling frequency [4]. Direct integration methods have similar problems. Using these algorithms the rule of 1/10 should be followed. But an improved algorithm [4] avoids this problem. If the improved algorithm is used the natural frequency can even be above the sampling frequency if the input transient peak has been adequately detected. The new algorithm assumes the input waveform can be adequately described by a series of straight lines connecting the sample points. The discontinuties in slope caused by the straight line segment approximation will generally introduce high frequency energy into the waveform and the shock spectrum will be slightly higher than the true value at the high frequencies. The errors of peak detection will always bias the results in the negative direction. The peak detection errors are usually the largest. The value approached for the shock spectrum as the natural frequency increases is the value of the largest sample in the set of data samples, and is as accurate as the detected peak value.

References

1. Y. Matsuzaki, and S. Kibe, "Shock and Seismic Response Spectra in Design Problems", Shock and Vibration Digest, 15(10)(Washington, DC: Shock and Vibration Information Center, Naval Research Laboratory, 83).

2. C.M. Harris and C.E. Crede, Shock and Vibration Handbook (New York: McGraw Hill, 1961),vol2,p23.

3. Kelly and Richman, Principles and Techniques of Shock Data Analysis, pp 152-153, SVM-5 (Washington, DC: Shock and Vibration Center, Naval Research Laboratory, 1969).

4. Smallwood, D.O., "An Improved Recursive Formula for Calculating Shock Response Spectra", The Shock and Vibration Bulletin 51(2), pp 211-217 (Washington, DC: Shock and Vibration Information Center, Naval Research Laboratory, May 1981).

Appendix A

Listing of Subroutine for a Ramp Invarient Simulation of a Single-Degree-of-Freedom System for the Calculation of the Shock Response Spectrum

REFERENCE: Smallwood D. O., "An Improved Recursive Formula for Calculating the Shock Response Spectra," Shock and Vibration Bulletin, No. 51, part 2, pp 211-217, May 1981.

```
      SUBROUTINE FILMAX(Y,XX,FN,SR,Z,XM1,XM2,
     &                  YM1,YM2,IFLAG)
C
C     D. SMALLWOOD 4-14-80
C     MODIFIED     8-15-80
C     MODIFIED     12-17-84 MADE CALCULATION OF
C                           FILTER WEIGHTS A
C                           A SEPARATE SUBROUTINE
C
C     APPROXIMATES A ONE-ZERO-TWO-POLE SINGLE
C     DEGREE OF FREEDOM RESONATOR USING A RAMP
C     INVARIENT SIMULATION AND RETURNS THE
C     RESPONSE.
C
C     Y=FILTER OUTPUT
C     XX=FILTER INPUT
C     Z=FRACTION OF CRITICAL DAMPING
C     SR=SAMPLE RATE (SAMPLES/SEC)
C     FN=NATURAL FREQUENCY OF RESONATOR (HZ)
C     IFLAG.NE.0 FOR FIRST CALL TO ROUTINE
C               SETUP FILTER COEFF.
C          =0 USES FILTER COEFF. DETERMINED
C             FROM A PREVIOUS CALL.
C
C          NOTE: IFLAG CHANGED TO 0 AFTER 1ST
C                CALL, MUST BE RESET BY
C                USER FOR NEW FILTER.
C     IFLAG=1 SETS UP AN BASE ACCEL. INPUT,
C             ACCELERATION RESPONSE
C             SHOCK SPECTRUM
C     IFLAG=-1 SETS UP A BASE ACCELERATION,
C              RELATIVE DISPLACEMENT
C              (EXPRESSED IN EQUIVALENT STATIC
C              ACCEL UNITS) SHOCK SPECTRUM.
C     XM1=1ST PAST VALUE OF INTIAL INPUT
C     XM2=2ND PAST VALUE OF INTIAL INPUT
C     YM1=1ST PAST VALUE OF INTIAL RESPONSE
C     YM2=2ND PAST VALUE OF INTIAL RESPONSE
C
```

```
      IF(FN) 4,4,5
    4 Y=0.
      RETURN
C
C     1ST CALL SET UP FILTER COEFFICIENTS
C
    5 IF(IFLAG) 3,10,3
    3 CALL WGHT(FN,SR,Z,IFLAG,B0,B1,B2,A1P2,
     &           A2M1)
      IFLAG=0

   10 Y=B0*XX +B1*XM1 +B2*XM2 +YM1-(YM1-YM2)
     &  -A1P2*YM1-A2M1*YM2
      YM2=YM1
      YM1=Y
      XM2=XM1
      XM1=XX
      RETURN
      END

      SUBROUTINE FILTR(X,FN,SR,Z,ITYPE,ISIZE,Y)
C
C  SUBROUTINE TO FILTER A TIME HISTORY WITH A
C  SDOF FILTER USING A
C  RAMP INVARIENT FILTER SIMULATION
C
C  INPUT: X= INPUT DATA ARRAY
C         FN= NATURAL FREQUENCY (HZ)
C         SR= SAMPLE RATE OF INPUT DATA ARRAY
C             (SAMPLES/SEC)
C         Z= FRACTION OF CRITICAL DAMPING
C         ITYPE=1--ABSOLUTE ACCELERATION MODEL
C               -1--RELATIVE DISPLACEMENT MODEL
C         ISIZE= THE NUMBER OF POINTS IN THE
C                X AND Y ARRAYS
C
C  OUTPUT: Y= OUTPUT DATA ARRAY
C
C  DO SMALLWOOD SANDIA NATIONAL LABS
C  ALBUQUERQUE NM 12-17-84
C
      DIMENSION X(1),Y(1)
C
C  FIND FILTER WEIGHTS
C
      CALL WGHT(FN,SR,Z,ITYPE,B0,B1,B2,A1P2,
     &           A2M1)
C
C  FILTER
C
      YM2=0.
      YM1=0.
      XM1=0.
      XM2=0.

      DO 10 I=1,ISIZE
      Y(I)=B0*X(I) +B1*XM1 +B2*XM2 -YM1
     &    +(YM1-YM2)-A1P2*YM1-A2M1*YM2
      YM2=YM1
      YM1=Y(I)
      XM2=XM1
   10 XM1=X(I)

      RETURN
      END
```

```
      SUBROUTINE WGHT(FN,SR,Z,IFLAG,B0,B1,B2,
     &                 A1P2,A2M1)
C
C     D. SMALLWOOD 12-17-84
C
C     FINDS THE WEIGHTS FOR A ONE-ZERO-TWO-POLE
C     SINGLE DEGREE OF FREEDOM RESONATOR
C     WITH A RAMP INVARIENT SIMULATION.
C
C     INPUTS:
C            FN= NATURAL FREQUENCY (HZ)
C            SR= SAMPLE RATE (SAMPLES/SEC)
C             Z=FRACTION OF CRITICAL DAMPING
C            IFLAG=1 SETS UP AN BASE ACCEL INPUT
C                    ACCEL RESPONSE
C                    SHOCK SPECTRUM
C            IFLAG=-1 SETS UP A BASE ACCEL.
C                    RELATIVE DISPLACEMENT
C                   (EXPRESSED IN EQUIVALENT
C                   STATIC ACCEL UNITS) SHOCK
C                   SPECTRUM.
C  OUTPUT:
C          B0,B1,B2,A1P2,A2M1  THE FILTER WEIGHTS
C
C

      DOUBLE PRECISION PI,W,WD,E,SP,DY,SQ,FACT,
     &                 C,DZ
      DATA PI/3.1415926535D0/
C
    3 W=2.0D0*PI*DBLE(FN)/DBLE(SR)
      IF(W-1.0D-3) 1,2,2
    1 X=SNGL(W)
C
C     USE THESE COEFFICIENTS WHEN W IS SMALL,
C     FOR BOTH MODELS
C
      A1P2= 2.0*Z*X +X*X*(1.0-2.0*Z*Z)
      A2M1=-2.0*Z*X +2.0*Z*Z*X*X
      IF(IFLAG) 35,10,20
    2 DZ=DBLE(Z)
C
C     THESE ARE EXACT EXPRESSIONS,
C     USED WHEN W IS LARGE
C
C    USE THESE EXACT EXPRESSIONS WHEN W IS LARGE
C
      SQ=DSQRT(1.0D0-DZ*DZ)
      E=DEXP(-DZ*W)
      WD=W*SQ
      SP=E*DSIN(WD)
      FACT=(2.0D0*DZ*DZ -1.0D0)*SP/SQ
      C=E*DCOS(WD)
C
C     A1P2 AND A2M1 ARE THE SAME FOR BOTH MODELS
C     A1P2=A1+2  A2M1=A2-1
C
      A1P2=SNGL(2.0D0-2.0D0*C)
      A2M1=SNGL(-1.0D0+E*E)
      IF(IFLAG) 6,10,30
C
C     EXACT EXPRESSIONS, W LARGE,
C     RELATIVE DISPLACEMENT MODEL
C
    6 B0=SNGL((2.0D0*DZ*(C-1.0D0) +FACT +W)/W)
      B1=SNGL((-2.0D0*C*W +2.0D0*DZ*(1.0D0-E*E)
     &   -2.0D0*FACT)/W)
      B2=SNGL((E*E*(W+2.0D0*DZ) -2.0D0*DZ*C
     &   +FACT)/W)
```

```
      GO TO 10
C
C     USE THESE COEF. FOR SMALL W,
C     RELATIVE DISPLACEMENT MODEL
C
   35 B0=X*X/6.
      B1=2.0*X*X/3.
      B2=X*X/6.

      GO TO 10
C
C     USE THE COEFF FOR ACCEL INPUT,
C     ACCEL OUTPUT MODEL
C
C
C     EXACT EXPRESSIONS FOR W LARGE.
C     ACCEL OUTPUT MODEL
C
   30 SP=SP/WD
      B0=SNGL(1.0D0-SP)
      B1=SNGL(2.0D0*(SP-E*DCOS(WD)))
      B2=SNGL(E*E-SP)

      GO TO 10
C
C     USE THESE COEFF FOR SMALL W,
C     ACCEL OUTPUT MODEL
C
   20 B0=Z*X+(X*X)*((1.0/6.0)-2.0*Z*Z/3.0)
      B1=2.0*X*X*(1.0-Z*Z)/3.0
      B2=-Z*X+ X*X*((1.0/6.0)-4.0*Z*Z/3.0)
  10  RETURN
      END
```

Discussion

Mr. Galef (TRW): I have been investigating some of these things in considerable detail over recent years, and I have come to a different conclusion than you have. My conclusion has been that damping is a second order effect, or a considerably higher order effect, at low frequencies and high frequencies; damping is also a considerably higher order effect for many pulses, for the rest of the frequencies as well. The only time damping is really important is in an oscillatory function, such as the damped sine wave that you were using, and then damping is only important at frequencies near where the Fourier transform peaks. I believe your different conclusion may have resulted from using physically invalid pulses. That particular sine wave where you put a compensating acceleration at a low value for a long time prior to the thing, that does not happen in the real world. In the real world we have a very large, very short duration, compensating pulse to give us a net velocity of zero and a net displacement of zero. For that case, my results very clearly show the 12 dB per octave. Until I can perhaps clarify this with you, I think I will continue to reject data that shows 6 dB per octave with the same enthusiasm that I reject data that shows zero shift.

Mr. Smallwood: I encourage you to read the written version of the paper, because I think you will see my mathematics is fairly straight forward and indicates a problem. I agree with your conclusions on single-sided wave forms. They roll off at 6 dB per octave anyway. But I think you will see damping is important for the double-sided wave forms at the very low natural frequencies. The net result is you cannot represent these complicated wave forms as simple impulses, because you have to represent them by multiple impulses. When you do that, the primary response becomes dominant.

Mr. Rehard (National Technical Systems): If we are looking at frequencies of one Hz, when we calculate the response spectra from that low frequency, what kind of error would come in between one Hz and DC? How do you know that there isn't a zero shift only by looking at the time history? You would have to look at the time history, because it will try to look flat the closer you get to zero.

Mr. Smallwood: When the natural period of the single degree-of-freedom system gets long, compared to the complete data window through which you look at the data, then I think you will ultimately see the shock spectrum start to roll off. It is flat only to those natural frequencies whose period is comparable to the period of the data window that you use to look at the data. If the period of the window you use to look at the data is one second long, I would expect frequencies a decade below one Hz will start to show a slope again of 6 dB per octave. That flatness does not go on forever down to DC. Eventually, it will turn around and roll off. Often, it is so far down, 40 dB or 60 dB down, that people really do not worry about it; they are not concerned about it. So, you never even plot the shock spectrum, but you use very low frequencies.

Mr. Rehard: It is a tough question for me because I do not know where the two would end. I do not know if I could ever prove it, or not, that it really turns around and does 6 dB.

Mr. Smallwood: The only thing that you can do is to extend the natural frequency down lower and lower. That gets to be computationally expensive. That is the reason people do not normally do it.