MASS LOADING EFFECTS FOR HEAVY EQUIPMENT AND PAYLOADS
Revision F

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Introduction

Consider a launch vehicle with a payload.

Intuitively, a realistic payload which is more massive than its mounting structure will tend to have a lower response to shock and vibration than the base structure's own response. This hypothesis assumes that the excitation flows from some external source through the base into the payload.

MIL-STD-810F gives a reasonable explanation for this mass-loading effect. An excerpt is given in Appendix A.

Furthermore, MIL-STD-810F, TABLE 514.5C-III, Jet Aircraft Vibration Exposure, allows the random vibration PSD (G^2/Hz) test level to be scaled by 0.25 due to mass loading for equipment 72 kg (160 lbm) or greater. This is equivalent to a 6 dB reduction. This table is also included in Appendix A.

The purpose of this analysis is to explore and expand upon the MIL-STD-810F recommendation, using a "very simplified" approach.

The analysis develops examples to derive a mass-loading reduction formula, particularly for launch vehicle equipment and payloads.

The derived method is suitable for liftoff and aerodynamic vibroacoustics, as well as stage separation shock. It may or may not be appropriate for structural-borne random or sine vibration from motors where the motor mass is greater than the payload mass.

Assumptions

1. The payload and its mounting base can be modeled as a two-degree-of-freedom system.
2. The mass of the payload is 40 lbm or greater.
3. The natural frequency of the payload by itself is 50 Hz or less.
4. The natural frequency of the base by itself is approximately 100 Hz, thus satisfying the octave rule for frequency separation.
5. The damping of each mode is 5%.
6. The random vibration source can be characterized as a harmonic force applied to a base mass.
7. Any significant flight excitation would be at frequencies above the second modal frequency, which is approximately 100 Hz.

8. The reduction factor is calculated by dividing the response of the payload mass by the response of the base mass at the second modal frequency.
9. The resulting reduction factor is then taken to be constant for all excitation frequencies for the given payload mass.

Furthermore, the acceleration response of the payload mass in the two-degree-of-freedom model may be considered as the base input acceleration applied to the payload for design or test purposes. This is explained more clearly in the next section.

Modeling Example

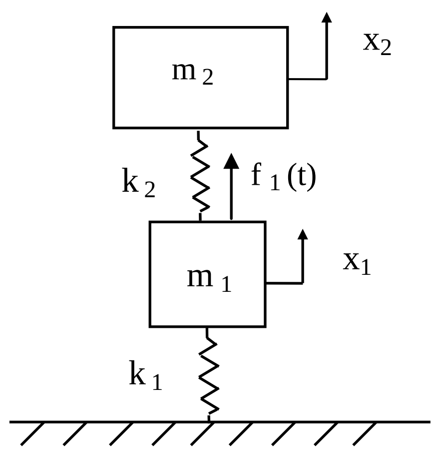

Figure 1. Two-degree-of-freedom System

Modeling Approach

Mass m_2 represents a payload. The payload may contain circuit boards, stowed solar arrays and various instruments, but it is modeled as a discrete mass.

The spring k_2 could represent a payload mounting cone.

The mass m_1 and the spring k_1 could represent an avionics module of some sort.

The analysis assumes that the payload mass m_2 would eventually be hardmounted directly to a shaker table without spring k_2. The payload would then be subjected to base excitation. In reality, analysis might be substituted for actual testing.

Base Values for the Sample Calculations

The base mass m_1=100 lbm. The spring stiffness k_1= 1.0e+05 lbf/in.

The single-degree-of-freedom system consisting of base mass m_1 and spring k_1 has a natural frequency of 100 Hz.

Payload Values for the Sample Calculations

The payload mass m_2 is an independent variable. The spring stiffness k_2 is varied accordingly so that the single-degree-of-freedom system consisting of payload mass m_2 and spring k_2 has a natural frequency of either 20 or 50 Hz.

Modal Damping

The modal damping is 5% for each mode.

Force Input

The forcing frequency is varied from 1 to 10,000 Hz.

The force amplitude is 1000 lbf, but this is unimportant because the goal is simply a ratio of the payload acceleration relative to the base acceleration.

Typical FRF

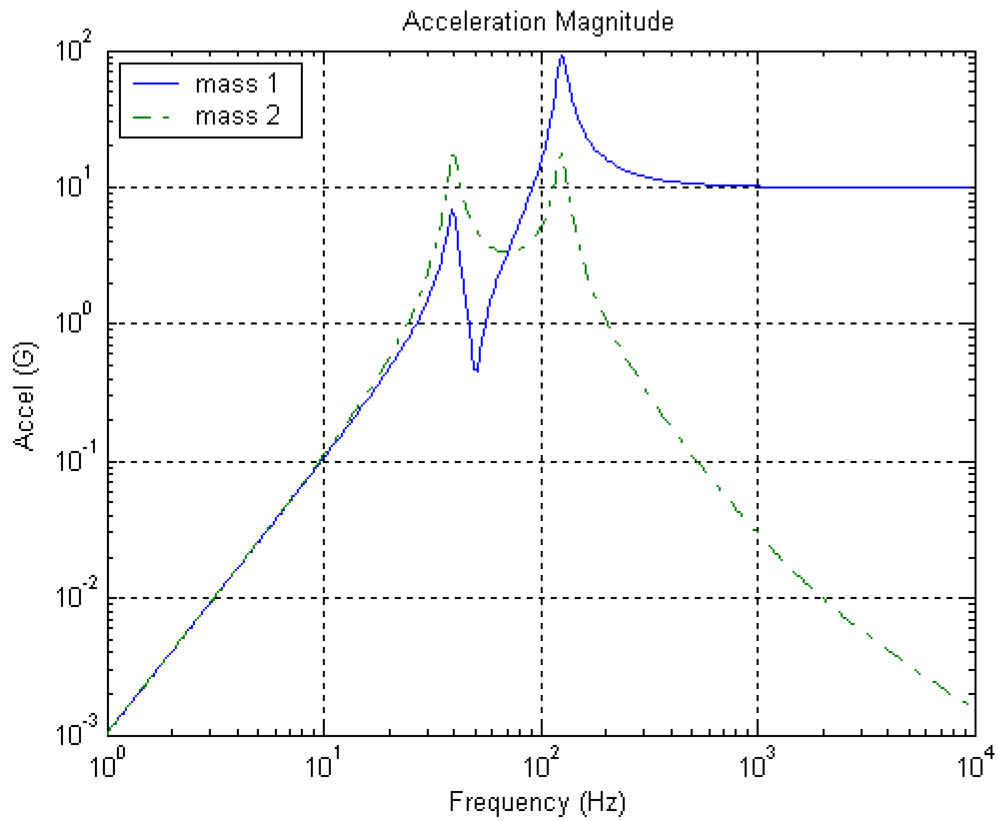

Figure 2. Case with Payload Values: m2 = 200 lbm, k2 = 5.11e+04 lbf/in, (payload fn=50 Hz)

Mass 1: Base Mass, local peaks

39.17 Hz	6.806 G
124.4 Hz	91.29 G

Mass 2: Payload Mass, local peaks

39.17 Hz	17.35 G
124.4 Hz	17.62 G

The payload mass response is 14.3 dB lower than that of the base mass at 124.4 Hz. This value is taken as the reduction factor even though the attenuation is much greater at higher frequencies.

Results Summary

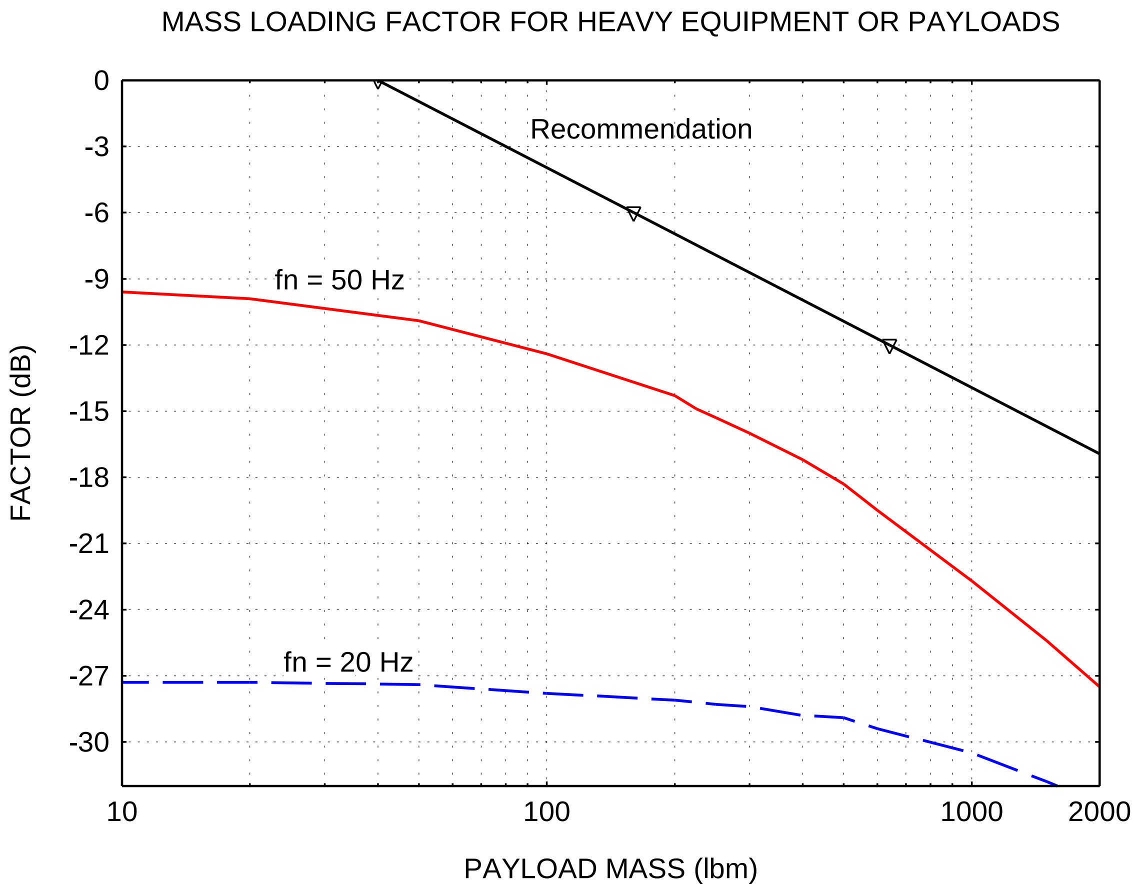

Figure 3.

The two frequency curves in Figure 3 were generated by applying the previous values to the model in Figure 1 via the method in Reference 2.

The natural frequency is the natural frequency of the payload and its own mounting spring behaving as a single-degree-of-freedom system.

The curves show that a greater reduction occurs as either the payload mass is increased or as the payload natural frequency is decreased.

Recommendations

The recommended attenuation factor for payload mass loading is

$$\text{Factor} = \left\{ -3 \, \frac{\ln\left[\dfrac{\text{mass}}{40\,\text{lbm}}\right]}{\ln[2]} \right\} \text{dB}, \quad \text{mass} \geq 40\,\text{lbm} \tag{1}$$

Again, the mass term in equation (1) is the payload mass.

The slope of equation (1) is -3 dB / (doubling of mass), where payload mass is the independent variable.

The recommendation is conservative within the assumptions of this analysis.

Note that the recommendation matches the MIL-STD-810F value of -6 dB for 160 lbm.

Further conservatism can be added by limiting the factor to $\geq$ -12 dB for mass values $\geq$ 640 lbm.

Obviously, a better choice would be to perform an analysis on the given system under consideration, rather than to rely on equation (1). The recommended formula can be used, however, as a preliminary estimate.

Equation (1) is expressed in metric units as

$$\text{Factor} = \left\{ -3 \, \frac{\ln\left[\dfrac{\text{mass}}{18\,\text{kg}}\right]}{\ln[2]} \right\} \text{dB}, \quad \text{mass} \geq 18\,\text{kg} \tag{2}$$

Future Work

The approach presented in this analysis is definitely a simplification.

Additional cases need to be analyzed by varying the mass and stiffness parameters in the two-degree-of-freedom model.

Furthermore, the analysis needs to be expanded by considering systems with additional degrees of freedom.

The reduction factor should be defined as a function of both frequency and mass.

Base excitation should also be modeled.

Mass Acceleration Curves

As an aside, Mass Acceleration Curves (MAC) are used to represent the combined effects of rigid-body launch vehicle acceleration, transient loading, and random vibration.

Examples of Mass Acceleration Curves are given in Appendix B. The ending slope of each curve is about -2.8 dB / (doubling of mass).

Force Limited Testing

Force limited testing is a related concern. See NASA-HBBK-7004B for further details.

References

1. MIL-STD-810F, Department of Defense Test Method Standard for Environmental Engineering Considerations and Laboratory Tests, 2000.

2. T. Irvine, The Steady-State Frequency Response Function of a Multi-degree-of-freedom System to Harmonic Force Excitation, Vibrationdata, 2007.

3. NASA-HBBK-7005, Dynamic Environmental Criteria, 2001.

4. NASA-STD-5002, Load Analysis of Spacecraft and Payloads, 1996.

5. NASA, Combination Methods for Deriving Structural Design Loads Considering Vibro-Acoustic, etc., Responses, Reliability Preferred Practice No. PD-ED-1211.

6. Vibration, Shock, and Acoustics; McDonnell Douglas, Western Division, 1971.

7. T. Irvine, Radiation & Driving Point Impedance of a Thin, Isotropic Plate, Rev B, Vibrationdata, 2008.

8. NASA-HBBK-7004B, Force Limited Vibration Testing, 2003.

9. NASA CR-1302, Summary of Random Vibration Prediction Procedures, 1969.

10. NASA TN D-1836, Techniques for Predicting Localized Vibratory Environments of Rocket Vehicles, 1963.

11. T. Irvine, Mass-loading Parametric Study via a Two-degree-of-freedom System, Vibrationdata, 2011.

APPENDIX A

Excerpt from MIL-STD-810E, Method 514.5, Annex B

2.4 Platform/Materiel and Fixture/Test Item Interaction.

Generally, it is assumed that the vibration environment of the materiel is not affected by the materiel itself. That is, the vibration of the platform at the materiel attachment point would be the same whether or not the materiel is attached. Since the entire platform, including all materiel, vibrates as a system, this is not strictly correct. However, when the materiel does not add significantly to the mass or stiffness of the platform, the assumption is correct within reasonable accuracy. The following sections discuss the limitations of this assumption. Note that these effects also apply to sub-elements within materiel and to the interactions of materiel with vibration excitation devices (shakers, slip tables, fixtures, etc.).

2.4.1 Mechanical impedance.

a. Large mass items. At platform natural frequencies where structural response of the platform is high, the materiel will load the supporting structures. That is, the mass of the materiel is added to the mass of the structure, and it inertially resists structural motions. If the materiel mass is large compared to the platform mass, it causes the entire system to vibrate differently by lowering natural frequencies and changing mode shapes. If the materiel inertia is large compared to the stiffness of the local support structure, it causes the local support to flex, introducing new low frequency local resonances. These new local resonances may act as vibration isolators (see Annex B, paragraph 2.4.2 below).

b. Items acting as structural members. When materiel is installed such that it acts as a structural member of the platform, it will affect vibrations and it will be structurally loaded. This is particularly important for relatively large materiel items but it applies to materiel of any size. In these cases, the materiel structure adds to the stiffness of the platform and may significantly affect vibration modes and frequencies.

Further, the materiel will be subjected to structural loads for which it may not have been designed. An example is a beam tied down to the cargo deck of a truck, aircraft, or ship. If the tie-downs are not designed to slip at appropriate points, the beam becomes a structural part of the deck. When the deck bends or twists, the beam is loaded and it changes the load paths of the platform structure. This may be catastrophic for the beam, the platform, or both. Be careful in the design of structural attachments to assure that the materiel does not act as a structural member.

c. Large item mass relative to supporting structures. When materiel items are small relative to the overall platform but large relative to supporting structures, account for the change in local vibration levels, if practical. This effect is discussed in Annex A, paragraph 2.3.1 for materiel mounted in jet aircraft. Note that due to differences in environments, relative sizes, and structural methods, the factor defined in Annex C, table 514.5C-III is only applicable to materiel mounted in full sized jet aircraft.

d. Large item size relative to platform. When materiel is large in size or mass relative to the platform, always consider these effects. This is imperative for aircraft and aircraft stores. Catastrophic failure of the aircraft is possible. It is also imperative to consider these effects in design of vibration test fixtures. Otherwise, the vibration transmitted to the test materiel may be greatly different than intended.

TABLE 514.5C-III. Jet aircraft vibration exposure.

$W_0 = W_A + \sum_1^n(W_J)$ W_0, W_A, W_J - Exposure levels in acceleration spectral density (g^2/Hz).					
Aerodynamically induced vibration $W_A = a \times b \times c \times (q)^2$					
Jet engine noise induced vibration $W_J = \{[048 \times a \times d \times \cos^2(\theta)/R] \times [D_c \times (V_c / V_r)^3 + D_f \times (V_f / V_r)^3]\}$					
a	-	Platform / Materiel interaction factor (see Annex B, paragraph 2.4). Note that this factor applies to W_0 and not to the low frequency portion (15 Hz to break) of figure 514.5C-14.			
	=	1.0 for materiel mounted on vibration isolators (shock mounts) and materiel weighing less than 36 kg.			
	=	$1.0 \times 10^{(0.6 - W/60)}$ for materiel weighing between 36 and 72.12 kg.(w = weight in kg)			
	=	0.25 for materiel weighing 72.12 kg or more.			
b	-	Proportionality factor between vibration level and dynamic pressure (SI units).	$\sum_1^n$	-	Jet noise contribution is the sum of the W_J values for each engine.
	=	2.96×10^{-6} for materiel mounted on cockpit instrument panels.	d	-	Afterburner factor.
				=	1.0 for conditions where afterburner is not used or is not present.
	=	117×10^{-5} for cockpit materiel and materiel in compartments adjacent to external surfaces that are smooth and free from discontinuities.		=	4.0 for conditions where afterburner is used.
	=	611×10^{-5} for materiel in compartments adjacent to or immediately aft of external surface discontinuities	R	-	Vector distance from center of engine exhaust plane to materiel center of gravity, m (ft).
		(cavities, chines, blade antennae, speed brakes, etc.), fuselage aft of wing trailing edge, wing, empennage, and pylons.	θ	-	Angle between R vector and engine exhaust vector (aft along engine exhaust centerline), degrees For $70° < \theta \le 180°$ use $70°$.
c	-	Mach number correction. Note that this factor applies to W_0 and not to the low frequency portion (15 to TBD Hz at 0.04 g^2/Hz) of figure 514.5C-8. (Annex A, paragraph. 2.3.1)	D_c	-	Engine core exhaust diameter, m (ft).
			D_f	-	Engine fan exhaust diameter, m (ft).
			V_r	-	Reference exhaust velocity, m/sec (ft/sec).
				=	564 m/sec
	=	1.0 for $0 \le$ Mach ≤ 0.9	V_c	-	Engine core exhaust velocity Engine core exhaust velocity (without afterburner), m/sec (ft/sec).
	=	(-4.8M + 532) for $0.9 \le$ Mach ≤ 1.0 (where M = Mach number)			
	=	0.52 for Mach number greater than 1.0	V_f	-	Engine fan exhaust velocity (without afterburner), m/sec (ft/sec).
q	-	Flight dynamic pressure, kN / m^2 (lb/ft^2). (See Annex B, para. 2.6.1 and table 5145C-VI)			
If Dimensions are in feet and pounds then:					
a	=	1.0 for materiel mounted on vibration isolators (shock mounts) and materiel weighing less than 80 lb.			
	=	$1.0 \times 10^{(0.60 - 0.0075 W)}$ for materiel weighing between 80 and 160 lb.			
	=	0.25 for materiel weighing 160 lb. or more.			
b	=	6.78×10^{-9}, 2.70×10^{-8}, or 1.40×10^{-7} in the order listed above.			
V_r	=	1850 feet / second			

APPENDIX B

Mass Acceleration Curves

NASA-HDBK-7005
March 13, 2001

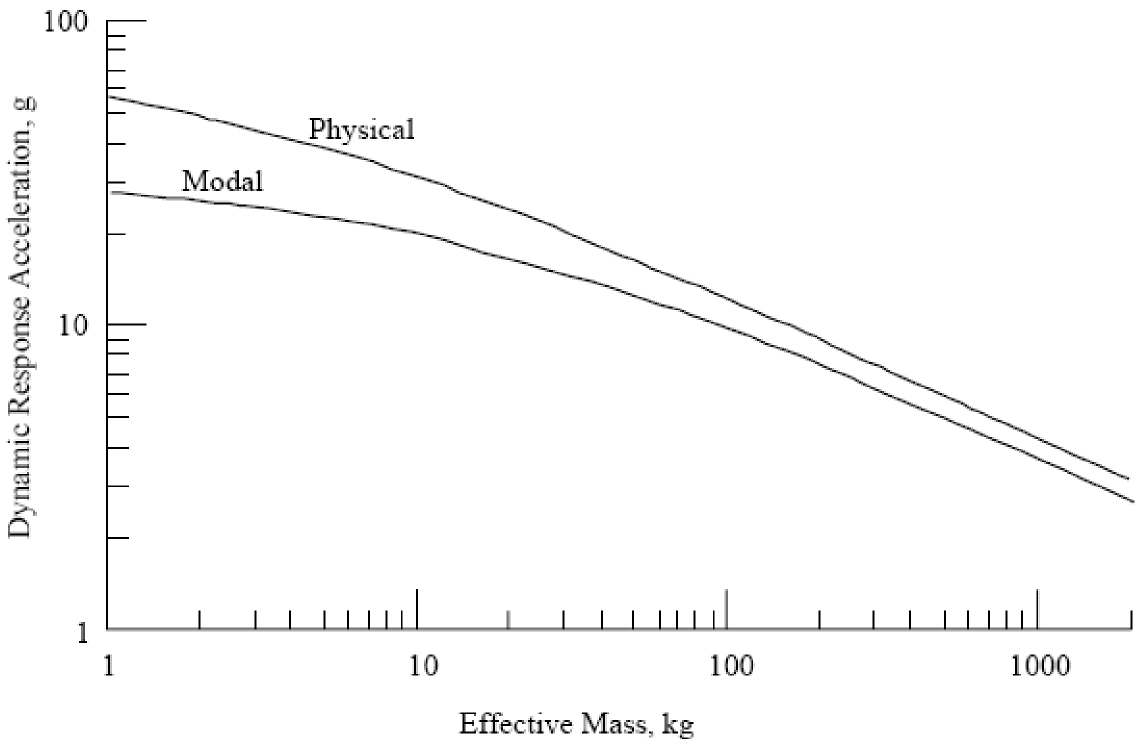

FIGURE 8.2. Physical and Modal MAC for the Galileo Spacecraft Launched on Shuttle and Inertial Upper Stage.

Specific requirements for payload math models at lower frequencies include the following [8.2]:

a. Finite elements must be used to model the structure and/or hardware.

b. Traditionally, a loads model must first be developed that has adequate fidelity to describe the payload dynamic behavior in a frequency range specified by the launch vehicle organization. Usually this range extends to 50 Hz, although a lower or higher maximum or cutoff frequency is sometimes specified for larger or smaller launch vehicles, respectively. For example, the cutoff frequency for the Shuttle is 35 Hz. Overall payload and subsystem modes must be accurately modeled up to an upper bound frequency, which must exceed 1.4 times the cutoff frequency of the loads analysis.

APPENDIX C

FOUR-DOF EXAMPLE

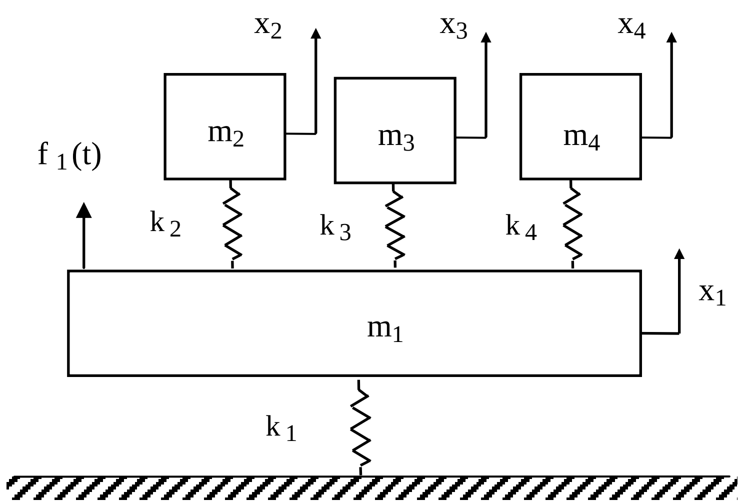

Figure C-1.

Four-DOF Example

This is a repeat of the example in the main text except that the payload mass is divided into three equal parts.

Base Values for the Sample Calculations

The base mass m_1=100 lbm. The spring stiffness k_1= 1.0e+05 lbf/in.

The single-degree-of-freedom system consisting of base mass m_1 and spring k_1 has a natural frequency of 100 Hz.

Payload Values for the Sample Calculations

The payload masses m_2, m_3 and m_4 are independent variable, but they are equal to the same value for each case.

$$m_2 = m_3 = m_4 = m_p$$

The spring stiffnesses k_2, k_3 and k_4 are also equal to the same value for each case.

$$k_2 = k_3 = k_4 = k_p$$

The stiffness values are sent such that

$$\frac{1}{2\pi}\sqrt{\frac{k_p}{m_p}} = \begin{cases} 20 \text{ Hz, for case } 1 \\ 50 \text{ Hz, for case } 2 \end{cases}$$

Modal Damping

The modal damping is 5% for each mode.

Force Input

The forcing frequency is varied from 1 to 10,000 Hz.

The force amplitude is 1000 lbf, but this is unimportant because the goal is simply a ratio of the payload acceleration relative to the base acceleration.

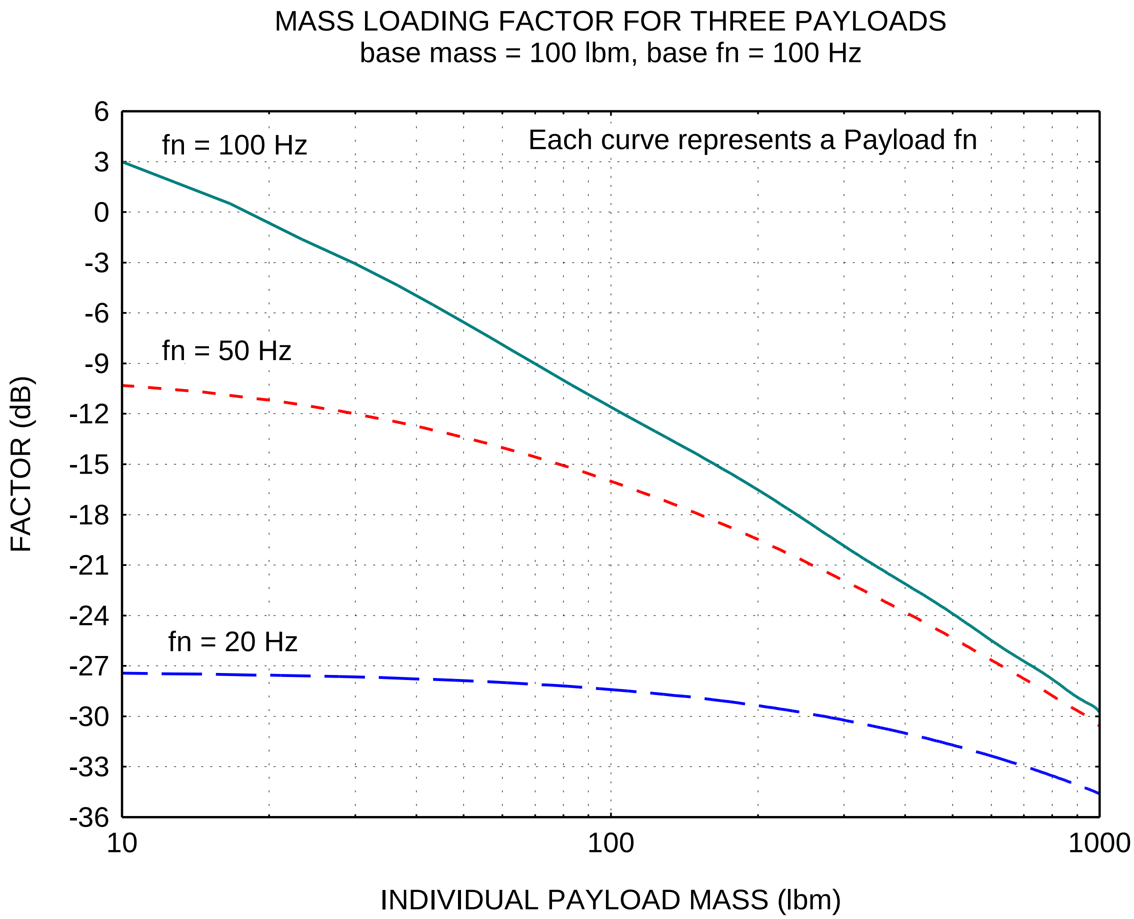
MASS LOADING FACTOR FOR THREE PAYLOADS
base mass = 100 lbm, base fn = 100 Hz
fn = 100 Hz
Each curve represents a Payload fn
fn = 50 Hz
fn = 20 Hz
FACTOR (dB)
6
3
0
-3
-6
-9
-12
-15
-18
-21
-24
-27
-30
-33
-36
10
100
1000
INDIVIDUAL PAYLOAD MASS (lbm)

Figure C-2.

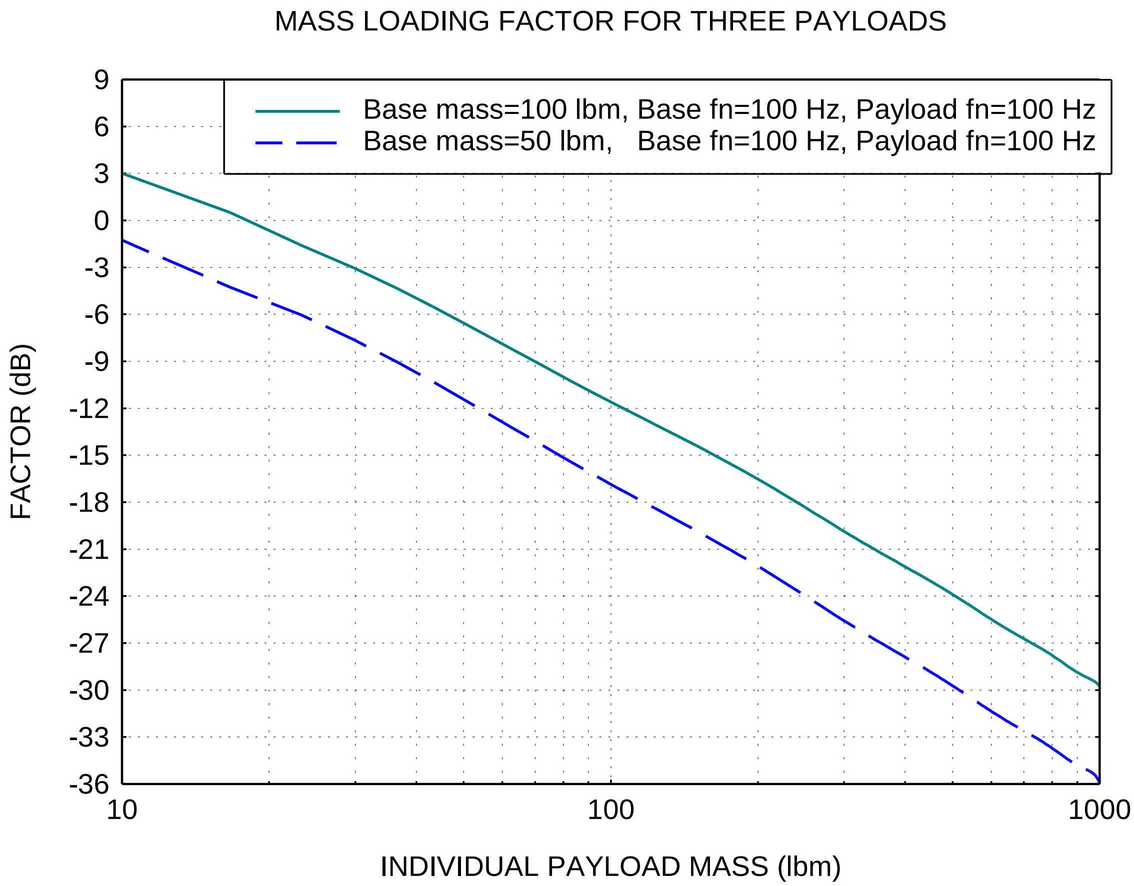

Figure C-3.

Greater attenuation is achieved for lower base mass.

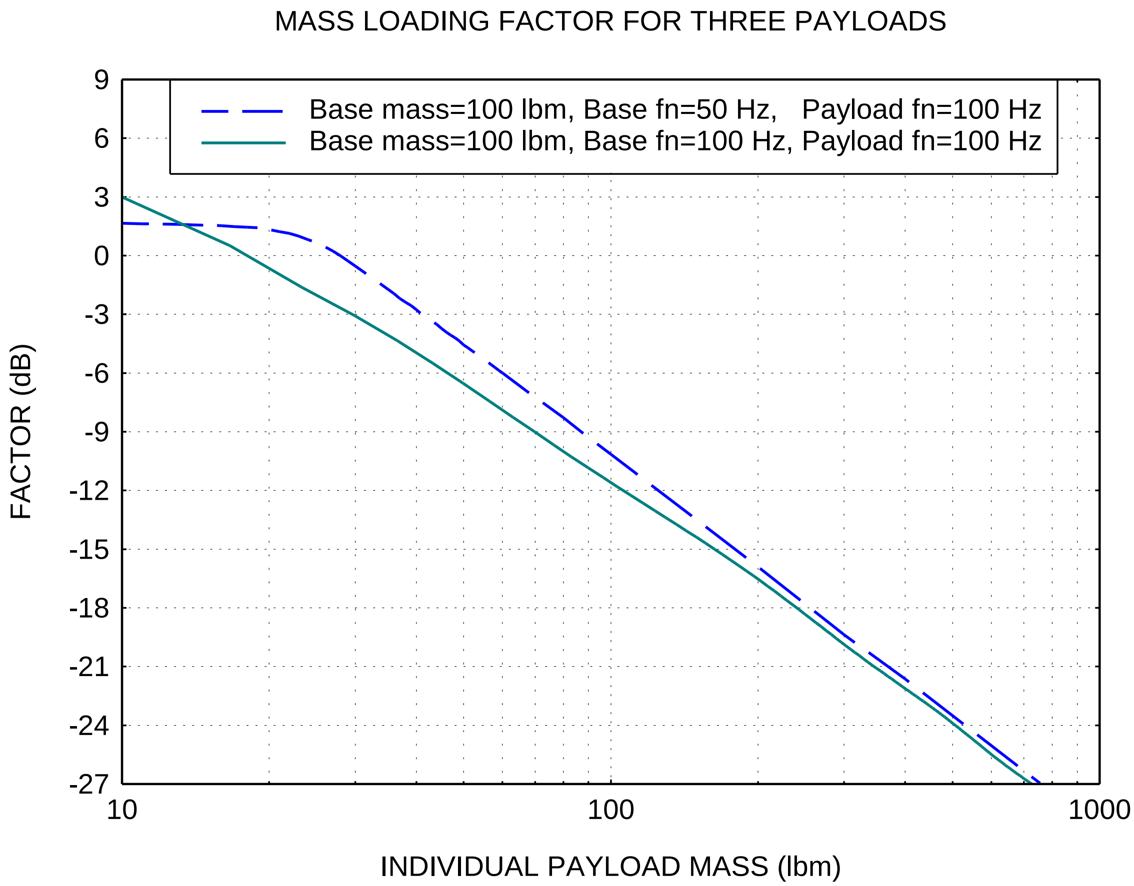

Figure C-4.

A higher natural frequency yields greater attenuation for payload mass values above 12 lbm.

A factor of -12 dB is achieved if all of the following conditions are met:

1. The base mass is $\leq$ 100 lbm
2. The base fn is $\geq$ 50 Hz
3. The individual payload mass is $\geq$ 126 lbm
4. The individual payload fn is $\leq$ 100 Hz

Sample Output File

```
Base mass=100 lbm   Payload mass=126 lbm (individual)
Base fn=50 Hz       Payload fn=100 Hz


The mass matrix is

m =

    0.2591         0         0         0
         0    0.3264         0         0
         0         0    0.3264         0
         0         0         0    0.3264


 The stiffness matrix is

k =

      411960     -128800     -128800     -128800
     -128800      128800           0           0
     -128800           0      128800           0
     -128800           0           0      128800


  Natural Frequencies =

      22.4 Hz
     99.97 Hz
     99.97 Hz
     223.1 Hz

  Modes Shapes (column format) =

       -0.862             0             0        -1.765
       -0.908         0.028         1.429        0.4436
       -0.908        -1.251         -0.69        0.4436
       -0.908         1.223        -0.739        0.4436

 Total Modal Mass =     478.0000 lbm


 Mass 1:
    22.63 Hz    19.07 G
    221.6 Hz    79.22 G
```

Mass 2:

22.63 Hz	20.09 G
221.6 Hz	20.32 G

Mass 3:

22.63 Hz	20.09 G
221.6 Hz	20.32 G

Mass 4:

22.63 Hz	20.09 G
221.6 Hz	20.32 G

20*log10(20.32/79.22) = -11.8 dB

APPENDIX D

MASS LOADING VIA MECHANICAL IMPEDANCE

The following approach applies to a panel excited by random vibration. A mass is to be added to the panel. The method is taken from Reference 6.

Variables

C_L	=	Longitudinal wave speed
G_L	=	Power spectral density for mass-loaded panel
G_U	=	Power spectral density for unloaded panels
h	=	Thickness
M	=	Added local mass
Z_S	=	Panel impedance
ω	=	Angular frequency (rad/sec)
ρ	=	mass/volume

The power spectral density ratio is

$$\frac{G_L}{G_U} = \frac{Z_S{}^2}{Z_S{}^2 + (\omega M)^2} \tag{D-1}$$

The mechanical impedance for an infinite panel is

$$Z_S \approx 2.3 C_L \rho h^2 \tag{D-2}$$

Equation (D-2) is taken from Reference 7.

APPENDIX E

Mass Loading

Consider the case where vibration predictions are desired at points where heavy components will be mounted.

Assume that the acceleration PSD for the new vehicle has been predicted, but that component weight was omitted in the calculation.

NASA CR-1302, section 5.7.1, suggests the following correction factor:

$$G_{nc}(f) = \frac{W_n}{W_n + W_c} G_n(f) \quad \text{(E-1)}$$

where

$G_n(f)$	Acceleration PSD of the new vehicle structure without components
W_n	Weight of new vehicle in general region of interest without components
W_c	Weight of all attached component in general region of interest
$G_{nc}(f)$	Acceleration PSD of the new vehicle structure with components attached

Equation (E-1) is referred to as the Barrett method, as given in NASA TN D-1836, equation (22).

Substituting the square of the weight ratio would seem to be more consistent with the laws of physics.

Nevertheless, empirical data has shown a better match with the form in equation (E-1). The analytical response of a multi-degree-of-freedom system can be used to evaluate this claim, as shown in Reference 11.