MASS PROPERTIES FROM A FINITE ELEMENT MODEL

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Introduction

Assume a finite element model with six degrees-of-freedom per node.

Two methods are presented for finding the mass properties. The goal of each method is to find a rigid-body mass matrix of the form:

$$\left[M_{\text{rigid}}\right] = \begin{bmatrix} m & 0 & 0 & 0 & C & -B \\ 0 & m & 0 & -C & 0 & A \\ 0 & 0 & m & B & -A & 0 \\ 0 & -C & B & I_{xx} & I_{xy} & I_{xz} \\ C & 0 & -A & I_{yx} & I_{yy} & I_{yz} \\ -B & A & 0 & I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \quad (1)$$

where

m is the mass of the structure;

I_{xx}, I_{yy}, I_{zz} are inertia moments around x-axis, y-axis and z-axis respectively;

I_{yz}, I_{zx}, I_{zy} are products of the moments of inertia;

A, B, C are parameters based on the relations:

$A = m\,x_g$, $B = m\,y_g$, $C = m\,z_g$.

(x_g, y_g, z_g) are the coordinates of the center of gravity.

Note that the inertia terms are referenced to a specified point which depends on the method as discussed in the follow sections. The parallel-axis theorem can then be used to determine the inertia about the center or gravity.

Method 1

The first method requires a mass matrix, a nodal location vector, and geometric rigid-body mode matrix.

The geometric rigid-body mode matrix Φ_{GRB} is easily formed, without any partitioning or transformation.

$$\Phi_{GRB} = \begin{bmatrix} \begin{pmatrix}1\\0\\0\\0\\0\\0\end{pmatrix}_1 & \begin{pmatrix}0\\1\\0\\0\\0\\0\end{pmatrix}_1 & \begin{pmatrix}0\\0\\1\\0\\0\\0\end{pmatrix}_1 & \begin{pmatrix}0\\-\Delta z_i\\\Delta y_i\\1\\0\\0\end{pmatrix}_1 & \begin{pmatrix}\Delta z_i\\0\\\Delta x_i\\0\\1\\0\end{pmatrix}_1 & \begin{pmatrix}-\Delta y_i\\\Delta x_i\\0\\0\\0\\1\end{pmatrix}_1 \\ \begin{pmatrix}1\\0\\0\\0\\0\\0\end{pmatrix}_2 & \begin{pmatrix}0\\1\\0\\0\\0\\0\end{pmatrix}_2 & \begin{pmatrix}0\\0\\1\\0\\0\\0\end{pmatrix}_2 & \begin{pmatrix}0\\-\Delta z_i\\\Delta y_i\\1\\0\\0\end{pmatrix}_2 & \begin{pmatrix}\Delta z_i\\0\\\Delta x_i\\0\\1\\0\end{pmatrix}_2 & \begin{pmatrix}-\Delta y_i\\\Delta x_i\\0\\0\\0\\1\end{pmatrix}_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \begin{pmatrix}1\\0\\0\\0\\0\\0\end{pmatrix}_p & \begin{pmatrix}0\\1\\0\\0\\0\\0\end{pmatrix}_p & \begin{pmatrix}0\\0\\1\\0\\0\\0\end{pmatrix}_p & \begin{pmatrix}0\\-\Delta z_i\\\Delta y_i\\1\\0\\0\end{pmatrix}_p & \begin{pmatrix}\Delta z_i\\0\\\Delta x_i\\0\\1\\0\end{pmatrix}_p & \begin{pmatrix}-\Delta y_i\\\Delta x_i\\0\\0\\0\\1\end{pmatrix}_p \end{bmatrix} \tag{1}$$

where

$$\Delta x_i = x_i - x_q \tag{2}$$

$$\Delta y_i = y_i - y_q \tag{3}$$

$$\Delta z_i = z_i - z_q \tag{4}$$

and

i is the node index,

p is the total number of nodes,

(x_g , y_g , z_g) are the coordinates of an arbitrary reference point.

The next step is to form the rigid-body matrix.

$$\left[M_{\text{rigid}}\right] = \Phi_{GRB}{}^T \; M \; \Phi_{GRB} \tag{5}$$

where M is the mass matrix.

The mass properties can then be extracted using equations (1) and (5).

<u>Method 2</u>

The second method requires mass and stiffness matrices. The coordinates of a reference node are also needed.

Component mode synthesis, or Guyan reduction, is performed as follows.

One node is chosen as a reference node. The six degrees-of-freedom at the reference node are retained as primary dof. All other dofs are reclassified as secondary dof.

The Mass matrix is partitioned as follows.

$$\tilde{M} = \begin{bmatrix} \tilde{M}_{II} & \tilde{M}_{IN} \\ \tilde{M}_{IN}{}^T & \tilde{M}_{NN} \end{bmatrix} \tag{6}$$

The mass values for the six primary dofs are partitioned in the $\tilde{M}_{II}$ submatrix.
The mass values for the secondary dofs are partitioned in the $\tilde{M}_{NN}$ submatrix.

The stiffness matrix is partitioned as follows.

$$\tilde{K} = \begin{bmatrix} \tilde{K}_{II} & \tilde{K}_{IN} \\ \tilde{K}_{IN}{}^{T} & \tilde{K}_{NN} \end{bmatrix} \tag{7}$$

The constraint mode submatrix is formed.

$$\phi_{CN} = -K_{NN}{}^{-1} K_{IN}{}^{T} \tag{8}$$

$$\left[M_{\,rigid} \right] = \left[I \quad \phi_{CN} \right] \begin{bmatrix} \tilde{M}_{II} & \tilde{M}_{IN} \\ \tilde{M}_{IN}{}^{T} & \tilde{M}_{NN} \end{bmatrix} \begin{bmatrix} I \\ \phi_{CN} \end{bmatrix} \tag{9}$$

The mass properties can then be extracted using equations (1) and (9).

Note that the center of gravity terms in equation (9) are relative to the coordinates of the reference node.

Further information regarding component mode synthesis is given in Reference 2.

Conclusion

Note that the first method is simpler and has better numerical accuracy.

The second method requires taking the inverse of a submatrix. The challenge is that the stiffness matrix has both translational and rotational dofs. The corresponding stiffness values may thus vary significantly within the submatrix. This causes potential numerical error due to round-off. Experience has shown that the input stiffness matrix must be given in full double precision format to mitigate potential numerical error.

References

1. Okuma, Heylen, Sas; Identification of Rigid Body Properties of 3-D Frame Structure by MCK Identification Method.

2. T. Irvine, Component Mode Synthesis, Fixed-Interface Model, Revision A, Vibrationdata, 2010.