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MODAL VELOCITY AS A CRITERION OF SHOCK SEVERITY

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An examination of reported spectral shock response data shows the dynamic range of accelerometer data to cover several orders of magnitude; very often the acceleration "g" levels remain the same order of magnitude as the frequency in Hz. No one has explained the damaging effects of high-frequency structural accelerations -- for example, 30, 000 g's at 20, 000 Hz. To shed light on both problems, this work considers the simple stress mechanisms of longitudinal waves in rods and transverse waves in beams and proves that modal stress is only a function of velocity and independent of frequency. Thus modal velocity singly predicts stress. This analysis cannot take into account failures other than those due to high stress.

The paper in essence urges the development of a more adequate velocity transducer, the use of modal velocity as a severity criterion, and the use of velocity as prime shock-measurement parameter.

INTRODUCTION

Characteristics common to a variety of reported shock spectra, and certain other factors, have led to the conclusion that velocity, suitably interpreted, may be the unifying thread throughout dynamic analysis. The most important considerations are:

1. Existence of a heuristic relationship between shock-induced velocity and damage.

2. Constant velocity tendency of most reported shock spectra.

3. Analytically derived, direct relationship between stress and modal velocity.

Collectively, these factors indicate strongly that shock measurements should be made in terms of velocity, that the much desired correlation between shock-induced motion and structural damage will be found in the velocity spectrum of the shock, and normal mode theory should be used in shock-resistant design.

SHOCK IN TERMS OF ACCELERATION

Current practice in specification of shock tests is to specify the type of shock machine, the shock spectrum, the acceleration-time history, or a combination of these.[1,2] Regardless of how it was specified, the test is nearly always reported by means of acceleration spectrum or acceleration-time history.

Why has acceleration become the predominant shock motion parameter? Does it offer advantages over velocity or displacement?

Yes! Acceleration transducers are inherently smaller and lighter than velocity transducers and, because only small operational motions are required within them, are free of bottoming which is so often incurred in velocity transducers in shock.[3] Contrasted to strain gages, acceleration transducers are easier to install, and they can be removed and used again.

Further, Newton's Second Law of Motion tells us that force equals mass times acceleration, and all engineers have trained awareness that damage to a structure is dependent on forces borne by it.[1]

As a measure of shock severity, however, acceleration levels (without regard to frequency) do not exhibit a straightforward correlation with shock-induced damage. Shock literature contains many discussions on this lack of correlation and on arbitrary measures taken in attempting to establish better correlation.[1, 2, 4, 5] So long as acceleration remains the predominantly used shock parameter, the correlation between shock level and shock-induced damage will remain elusive, as shock-induced accelerations have too wide a dynamic range to allow resolution of all damage-causing accelerations.

J. P. Walsh of the Naval Research Laboratory has pinpointed the problem regarding dynamic range, resolution, and damage correlation. He states:[6]

"On the acceleration-time record the high-frequency component obscured the low-frequency components. The maximum velocity and time to maximum velocity could not be determined because reliable integration was not possible. No information about the displacement-time curve could be found.

"In order to determine the range of the instruments which would be required to record displacement, velocity, and acceleration under shipboard shock conditions, a simple apparatus was studied. It was composed of different parts having high and low natural frequencies and made of brittle and ductile materials. It was shown that a variation in acceleration between 2.45 g and 9×10^3 g combined with displacements varying between 1.9 and 7.6×10^{-4} inches was necessary to produce damage. The extremes of each would damage one part but not affect the others. The extremes of velocity associated with the extremes of displacement and acceleration were 2.5 feet per second and 20 feet per second. This is a narrow range compared with the ranges of displacement and acceleration."

In the case just cited, the dynamic range for damage-causing motions was 3700 to 1, or 71 dB, for acceleration; 2500 to 1, or 68 dB, for displacement; and 8 to 1, or 18 dB, for velocity. Compared to the 40-dB resolution available with current analog tape recorders or oscillographs, it is easily seen that the lower-level damage-causing accelerations (or displacements, for that matter) would not be resolvable with today's instruments and techniques. Note well that damage-causing motions described in terms of velocity spanned only an 18-dB range. Therefore, only the velocity parameter would have permitted recording this shock with inclusion of all of the damaging components.

HEURISTIC VELOCITY - DAMAGE CORRELATION

The above experiment indicates the heuristic relation between damage and velocity; in fact, it is one of the few reported studies of motion parameter and damage. Shipboard shock studies have also indicated this relation. Oleson[7] specifically cites the "empirical correlation" between damage and velocity. Shaw,[8] of the Royal Navy, in explaining choice of velocity transducers, states that they "... could obtain more readily from velocity-time records information on the damaging characteristics of shock..." And in discussing explosively generated ground motions, Hudson[9] has referenced several studies in which "...velocity shows good correlation with damage over a wide range of frequencies."

Thus a great deal of experience leads one to expect a strong correlation between shock-induced velocity and damage.

SHOCK CHARACTERIZED BY CONSTANT VELOCITY SPECTRUM

In addition to the work reported by J. P. Walsh,[6] many other reported shock data reveal a tendency toward a constant velocity spectrum. Figures 1, 2 and 3 are acceleration spectra of response to gunfire shock. Figure 4 presents acceleration spectra from railroad coupling shock, while Fig. 5B is the response of the anvil table of the Navy Medium Weight Shock Machine upon hammer impact.

Even though these spectra are collected from a variety of types of shock, they all exhibit a strong constant velocity tendency. (The single line with a positive 45° slope in Fig. 4 represents a constant 61.4 inches per second.) And the examples included here represent only a few of the shock spectra exhibiting the constant-velocity characteristic. To gain full appreciation of this fact, one should sketch constant velocity lines in any acceleration spectrum he finds. When both acceleration and frequency are plotted logarithmically, drawing the $V=k$ lines is easy. Drawing a straight line through the points where

FROM NOLTR 69-64, FIG. 24

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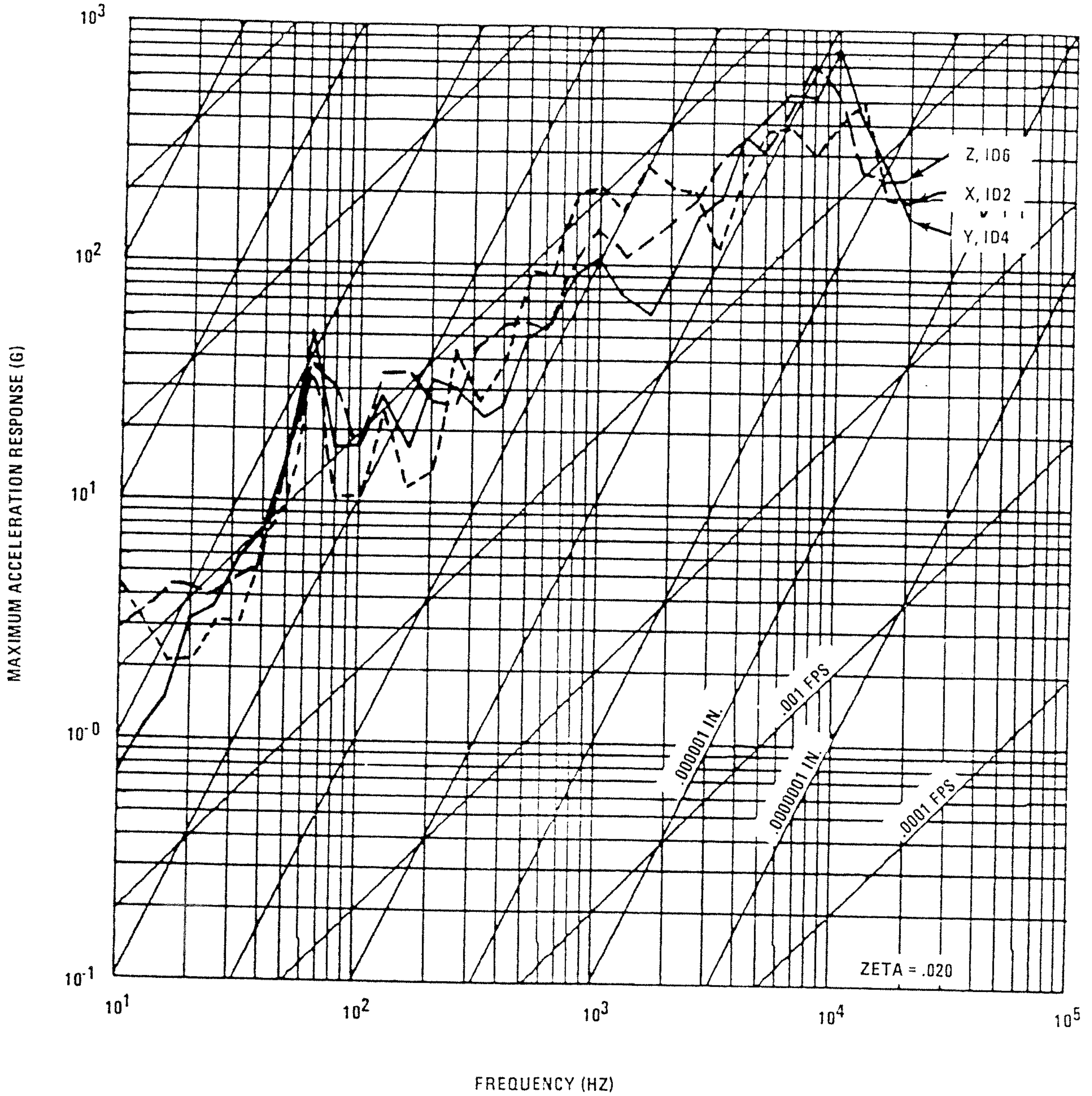

FIGURE 1. SHOCK SPECTRA

FROM NOLTR 69-64, FIG. 25

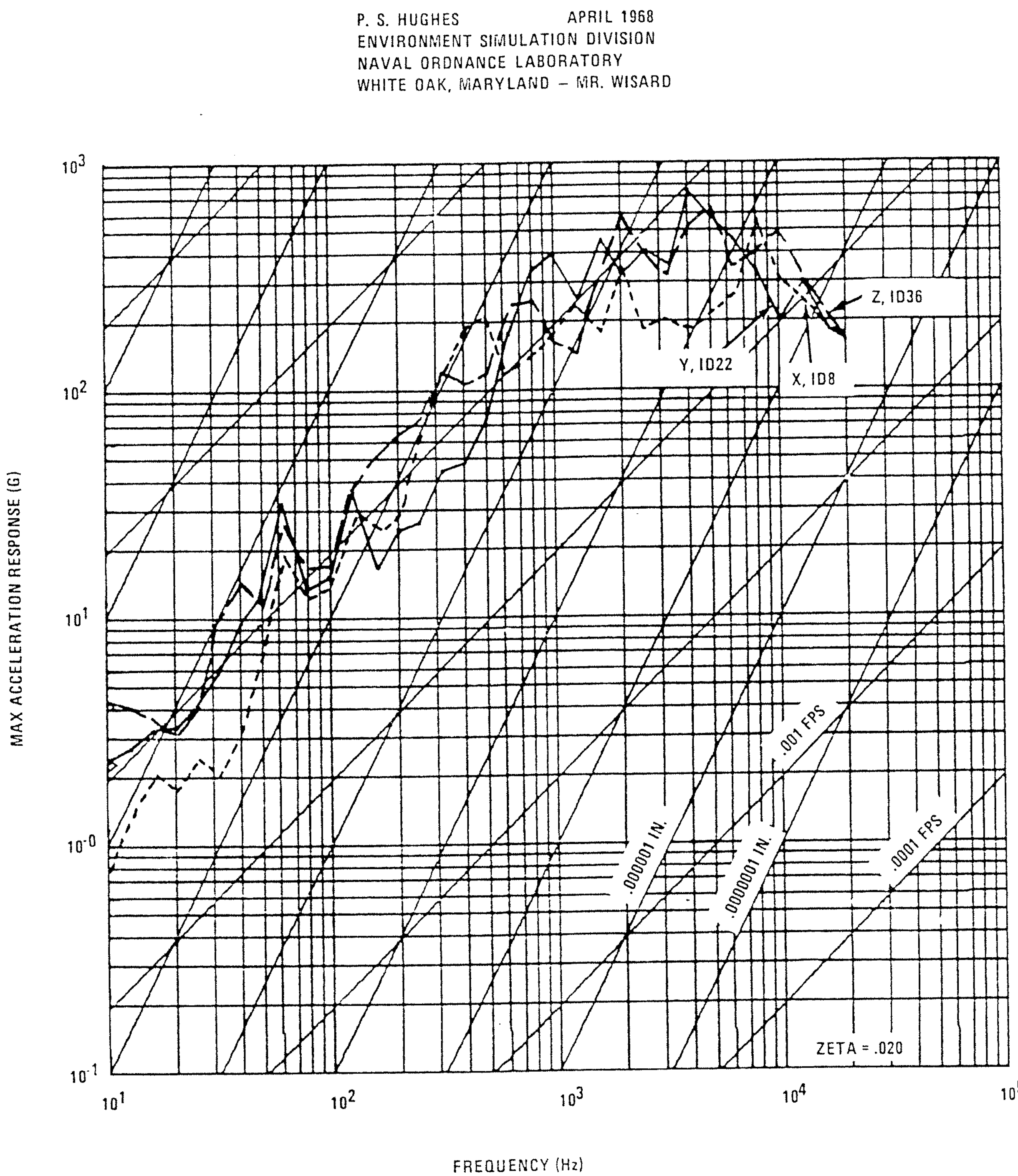

FIGURE 2. SHOCK SPECTRA

FROM NOLTR 69-64, FIG. 26

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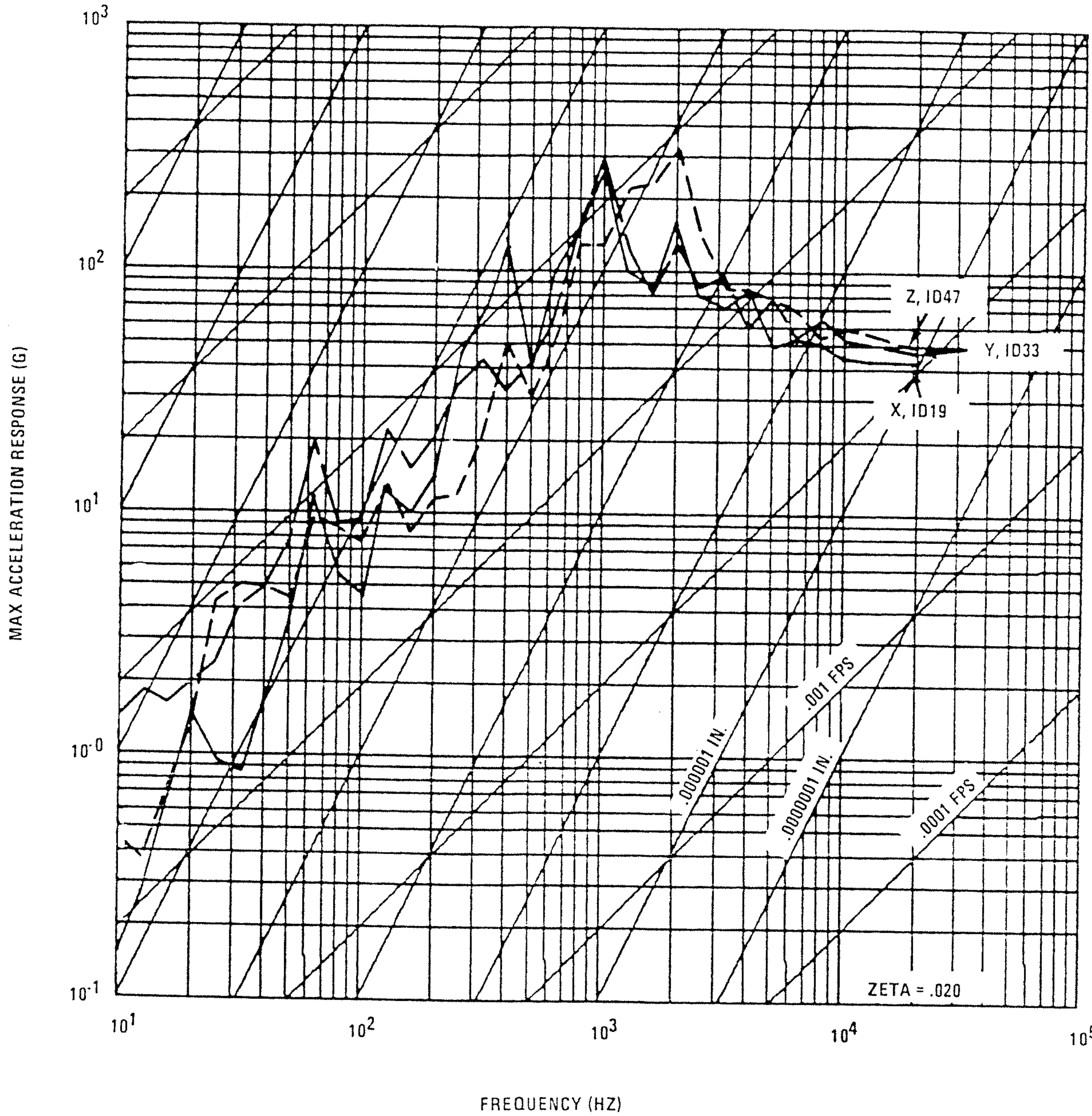
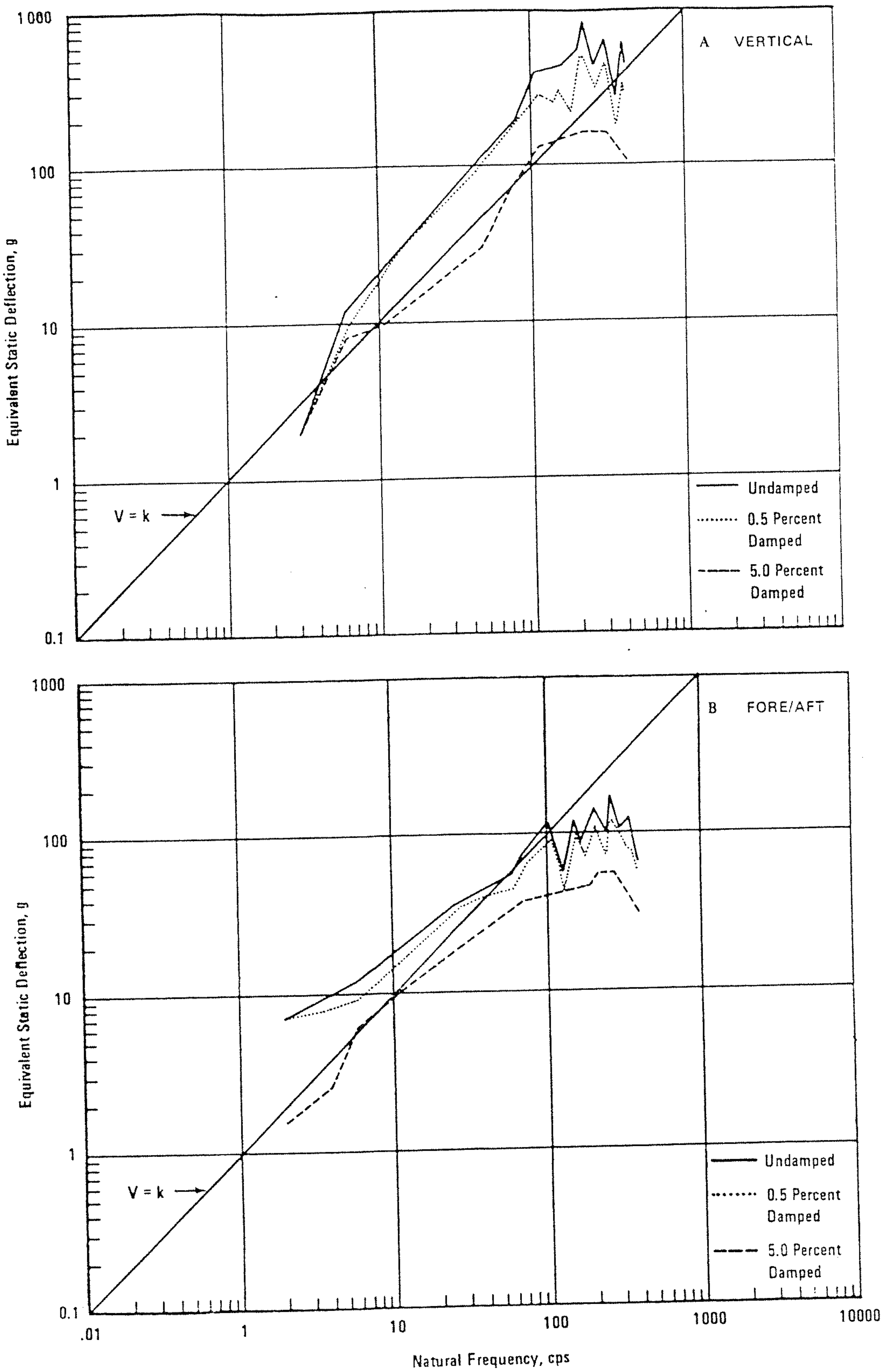


FIGURE 3. SHOCK SPECTRA



IGURE 4. RAILROAD COUPLING SHOCK SPECTRUM—STANDARD DRAFT GEAR (6.0 MPH) (FROM NAS REPORT 8-11451)

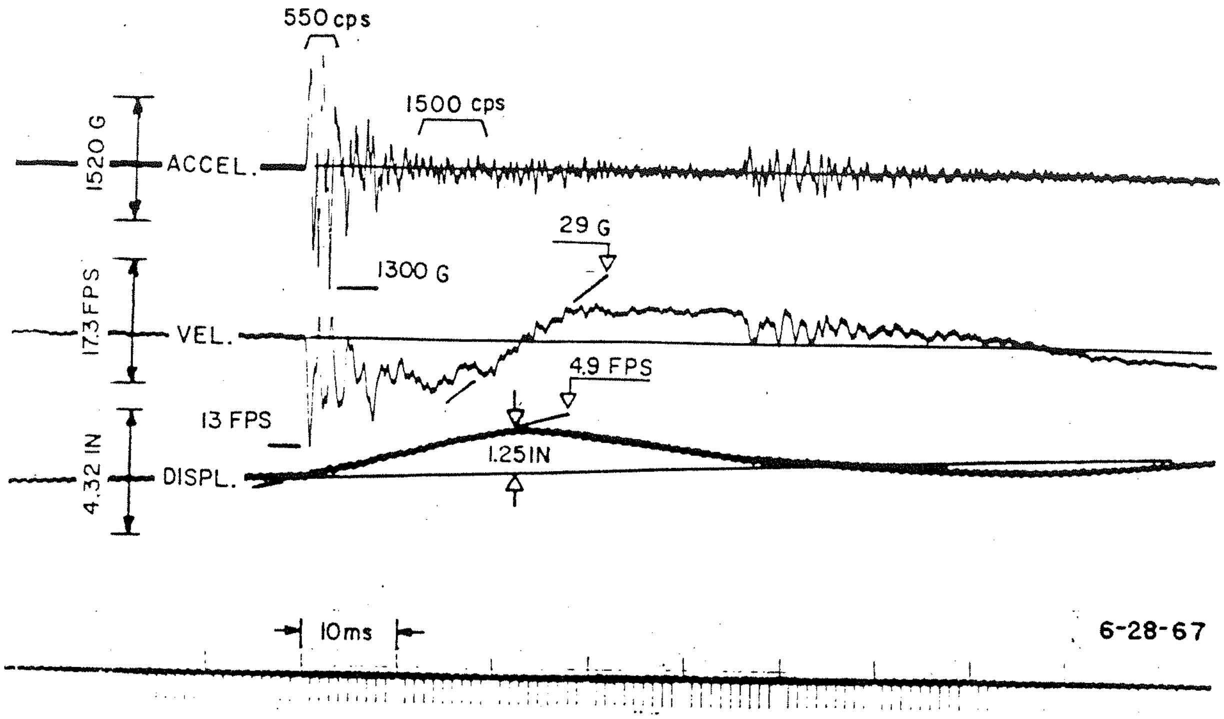


FIGURE 5A. A TAPE RECORDER ACCELEROMETER RECORD, REPRODUCED THROUGH THE SHOCK SIGNAL INTEGRATOR FOR OSCILLOGRAPHIC PRESENTATION. THIS RECORD WAS OBTAINED FROM A SPECIALLY MOUNTED PIEZO-RESISTIVE ACCELEROMETER ATTACHED TO A STANDARD NAVY "LIGHT WEIGHT SHOCK MACHINE." (FROM NRL MEMORANDUM REPORT 1903)

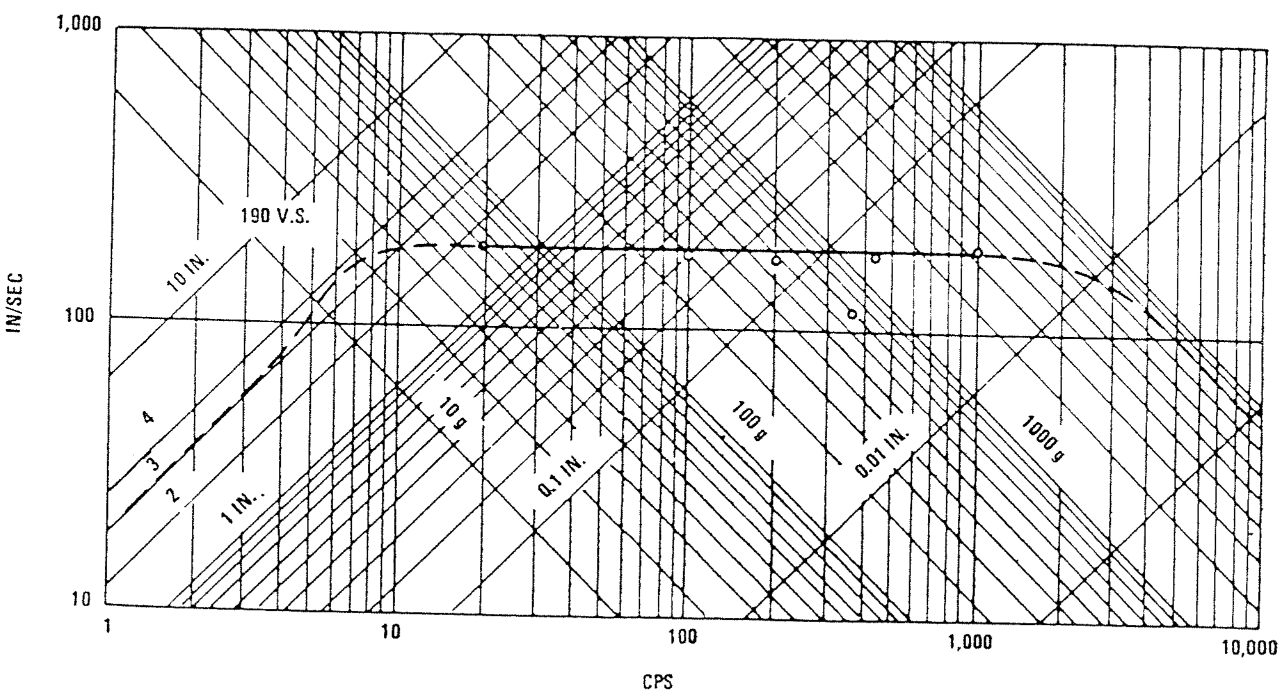


FIGURE 5B. SHOCK SPECTRUM FOR MOTION OF ANVIL TABLE OF HI SHOCK MACHINE FOR MEDIUMWEIGHT EQUIPMENT; 3-FT HAMMER DROP, 3-IN. TABLE TRAVEL, 1115-LB LOAD, 1858 LB ON TABLE, DROP-HEIGHT 150 PERCENT GREATER THAN SPECIFIED (5) FOR CLASS A SCHOCK. (FROM NRL REPORT 5618) acceleration in g's numerically equals the frequency in Hz, one constructs a line where the velocity at all points is 61.4 inches per second. All other parallel lines also represent $V=k$ and their magnitudes can be easily determined. For instance, when the number g's is three times the frequency, velocity is 3 x 61.4 inches per second.

While all shock spectra may not exhibit the strong constant-velocity tendency of the examples cited here, it will be evident that the dynamic range of the velocity spectrum is much less than either acceleration or displacement. It follows that the total vibrational velocity will tend toward constancy more than either acceleration or displacement.

STRESS-VELOCITY CORRELATION

Modal analysis[10] has shown that dynamic structural response of linear undamped systems can be treated as though composed of separate responses of the normal or free vibration modes of the structure. Motion of any point of a structure, where a suitable motion transducer might be located, will yield a time response that is built up of these separate modal motions.

Ideally then, spectral analyses of the motion at this point ought to show distinct, significant amplitudes at the frequencies of all modes which are somewhat antinodal at this point. Thus a shock or Fourier spectrum of a motion, for a linear system at least, indicates the modal makeup of the motion. A spectrum peak at some particular frequency indicates that some mode with that frequency was responding with an amplitude at least that great. If we fortuitously placed the transducer at the maximum amplitude station for that mode, we would know (if we knew the mode shape) the stresses induced throughout the structure for that portion of the response.

However, there is no single antinodal position at which a motion transducer could be placed. This or that convenient mounting position may be antinodal, nodal, or more probably intermediate to the bulk of the modes responding. Thus analysis of motion histories cannot be expected to yield the amplitude of all or even most of the structural modes. The response however will be an indication of the lower limit of the actual modal response. We shall therefore consider the modal response characteristics of some simple structures and theoretically show that maximum modal velocity is a valid indication of maximum modal stress, independent of the frequency and mode shape. Although no generalization for all structures can be made, the results certainly lead one to expect that velocity is the single, most directly damage-related dynamic motion property. Naturally this analysis will in no way explain shock or vibration failures that are caused by effects other than high stress.

Actually, there are only a relatively few classifications of load-carrying mechanisms. There is uniform stress (tension or compression of a member), beam bending, torsion, shear, membrane stresses in plates and shells, bending stresses in plates and shells, and a few others. We shall consider uniform and bending stresses in uniform slender longitudinal members, all with an infinitude of modes, and show that without detailed information one can predict the severity of the resulting stresses.

LONGITUDINAL VIBRATIONS IN RODS

We shall begin by considering the longitudinal free vibrations of a long thin rod. These free vibration shapes are the modes that are excited by a shock input. We confine ourselves to the easiest situation where the longitudinal wavelength is long in comparison with the bar cross-sectional dimensions. For this case the cross sections remain plane with uniform stress and the lateral or "Poisson" deformation has insignificant effects. (See Ref. 11, pp. 297-298.) Let us consider a semi-infinite rod being sinusoidally excited at its end. Since it is semi-infinite no reflections can occur and hence it can (within its elastic limit) vibrate at any amplitude and frequency, thus accepting all inputs. Timoshenko,[11] on page 299 shows that transverse planes in such a rod have the motion

$$u = \left(C \cos \frac{px}{a} + D \sin \frac{px}{a}\right) \left(A \cos pt + B \sin pt\right) \qquad (1)$$

where: x = distance down rod

u = displacement of a plane located at x

p = circular frequency

a = wave speed $\equiv \sqrt{E/\rho}$

E = Young's modulus

ρ = density

t = time.

Without loss of generality we may select the rod end as an antinode and begin time such that Eq. (1) becomes

$$u = A \cos \frac{px}{a} \cos pt \tag{2}$$

From elasticity[12] the strain and hence stress is given by

$$\sigma = E\epsilon = E\frac{\partial u}{\partial x} = -EA\frac{p}{a} \sin \frac{px}{a} \cos pt \tag{3}$$

the maximum value of which is

$$\sigma_{max} = \frac{EAp}{a} \tag{4}$$

Note now that the maximum stress for a constant displacement amplitude, A, depends upon the frequency, p. From Eq. 2 the maximum displacement, velocity, and acceleration are

$$\left.\begin{aligned} u_{max} &= A \\ \dot{u}_{max} &= Ap \\ \ddot{u}_{max} &= Ap^2 \end{aligned}\right\} \tag{5}$$

By using V for the maximum modal velocity, Eq. 5 yields

$$V = Ap. \tag{6}$$

The substitution of Eq. 6 into Eq. 4 yields

$$\sigma_{max} = \frac{E}{a} V_{max} \tag{7}$$

Using the value of the wave speed from Eq. 1, Eq. 7 may be conveniently expressed as

$$\sigma_{max} = V_{max} \sqrt{E\rho}. \tag{8}$$

Thus in all semi-infinite rods vibrating longitudinally at any frequency or amplitude within the restrictions set forth above, the maximum modal velocity alone determines the maximum modal stress.

It is significant to note that maximum acceleration does not so simply relate to stress. In fact, a formulation of the expression analogous to Eq. 8 in terms of maximum acceleration from Eqs. 5 and 8 yields

$$\sigma_{max} = \frac{\ddot{u}_{max}}{p} \sqrt{E\rho}. \tag{9}$$

Eq. 9 shows that for modes with constant maximum acceleration, the stress is inversely proportional to frequency. Thus, high acceleration at high frequencies does not necessarily indicate high stress. Alternatively, as indicated in Eq. 8, high velocities do indicate high stresses.

We might expect that Eq. 8 would also apply to finite rods and indeed it does. Following Timoshenko (see Ref. 11, pg 299) we consider a rod of length ℓ, with free ends. This rod has the natural frequencies

$$p_i = \frac{ia\pi}{\ell} \tag{10}$$

where $i = 1, 2, 3, \ldots$ and designates the mode.

The complete free (or ringing) solution is

$$u = \sum_{i=1}^{\infty} \cos \frac{i\pi x}{\ell} \left[A_i \cos \frac{i\pi a t}{\ell} + B_i \sin \frac{i\pi a t}{\ell}\right]. \tag{11}$$

B_i can be made zero by starting time appropriately, hence the modal displacement may be written as

$$u_i = A_i \cos \frac{i\pi x}{\ell} \cos \frac{i\pi a t}{\ell}. \tag{12}$$

As above, the stress in the ith mode is given by

$$\begin{aligned} \sigma &= E\epsilon_i = E\left(\frac{\partial u}{\partial x}\right)_i \\ &= -EA_i \frac{i\pi}{\ell} \sin \frac{i\pi x}{\ell} \cos \frac{i\pi a t}{\ell}, \end{aligned} \tag{13}$$

the maximum value of which is

$$\sigma_{i\,max} = \frac{EA_i i\pi}{\ell}. \tag{14}$$

From Eq. 12 note that the maximum velocity in the ith mode is

$$V_{i\,max} = \frac{A_i i\pi a}{\ell} \tag{15}$$

and now, as before, substituting Eq. 15 into 14 yields

$$\sigma_{i\,max} = \frac{EV_i}{a} \tag{16}$$

or the result identical to Eq. 7 and again using the definition of a, we find that the maximum stress in any mode is given by

$$\sigma_{max} = V_{max} \sqrt{E\rho}\,. \tag{8}$$

Finally, it is a simple matter to develop a generalized proof to show that the maximum stresses predicted by Eq. 8 apply to all cases of longitudinal vibrations in rods no matter what the end conditions, if the previous restrictions continue to apply. All possible vibrations of the rod are given by Eq. 1. As in Eq. 3, the stress is therefore given by

$$\sigma = E\frac{\partial u}{\partial x} = E\frac{p}{a}\left(-C\sin\frac{px}{a} + D\cos\frac{px}{a}\right)$$

$$(A\cos pt + B\sin pt).$$

In Ref. [13], the maximum values of the quantities in parentheses are shown to be $\sqrt{C^2 + D^2}$ and $\sqrt{A^2 + B^2}$ respectively, hence the maximum value of the stress is

$$\sigma_{max} = E\frac{p}{a}\sqrt{C^2 + D^2}\sqrt{A^2 + B^2}. \tag{17}$$

The particle velocity is found from Eq. 1 to be

$$\dot{u} = \frac{\partial u}{\partial t} = p\left(C\cos\frac{px}{a} + D\sin\frac{px}{a}\right)$$

$$\left(-A\sin pt + B\cos pt\right).$$

Again using the proof of Ref. [13], the maximum value of this velocity is

$$\dot{u}_{max} = p\sqrt{C^2 + D^2}\sqrt{A^2 + B^2}. \tag{18}$$

The substitution of Eq. 18 into Eq. 17 yields the desired result, namely

$$\sigma_{max} = \frac{E}{a}\dot{u}_{max}, \tag{19}$$

which is identical to Eqs. 7 and 16.

Thus it has been proved and illustrated that the maximum stress due to long-wave longitudinal vibrations in rods is completely specified by the material properties and the maximum modal velocity.

TRANSVERSE BEAM VIBRATIONS

Transverse uniform beam vibrations can also be classified according to maximum modal velocities. We shall again consider only the simplest type of vibrations in which the wavelength is long compared to the beam depth. (This neglects the so called rotary inertia and shear effects.) For these cases, simple bending theory suffices; Timoshenko's presentation (See Ref. 11, pp. 324-335) will be used as a foundation. He proves that the free vibrations of the neutral surface of such beams are expressed by the following solution

$$y = (C_1\sin kx + C_2\cos kx + C_3\sinh kx$$
$$+ C_4\cosh kx)\cdot(A\cos pt + B\sin pt), \tag{20}$$

where k, the wave number, and p, the circular frequency, are related by

$$k^4 = \frac{p^2}{\eta^2}\frac{\rho}{E}. \tag{20a}$$

In these equations the following definitions are used:

y = deflection of neutral surface,

x = distance down the beam,

C, A, B = arbitrary constants,

ρ = density,

E = Young's modulus,

η = radius of gyration $\equiv \frac{I}{A}$,

I = cross-sectional area moment of inertia about neutral axis,

A = cross-sectional area

Let us specialize Eq. 20 to consider a semi-infinite beam, which starts at $x = 0$ and continues on out to infinity. Again, since no reflections occur, the semi-infinite beam can accept sinusoidal vibration at all frequencies with amplitudes that do not exceed the elastic limit. The simplest case is that with zero shear and slope at its end, as shown in Fig. 6.

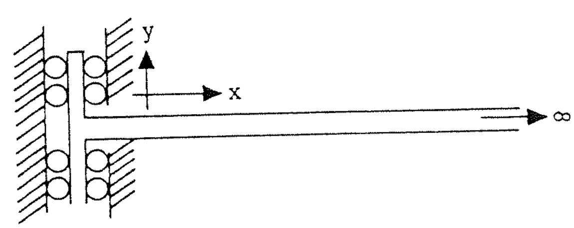


Fig. 6. Semi-infinite beam with zero slope and shear at end.

The boundary conditions to be imposed on Eq. 20 are:

1. The solution remains finite for very large x,
2. Zero slope at $x = 0$; $y' = 0$ at $x = 0$,
3. Zero shear at $x = 0$; $y''' = 0$ at $x = 0$.

The only way in which condition 1 can be satisfied is

$$C_3 = -C_4.$$

Thus the hyperbolic sine and cosine terms from Eq. 20 may be written

$$C_3 \sinh kx + C_4 \cosh kx = C_3 (\sinh kx - \cosh kx). \qquad (21)$$

The substitution of the definitions of the hyperbolic functions yields

$$C_3 (\sinh kx - \cosh kx) = C_3 e^{-kx}. \qquad (22)$$

Thus the shape portion of Eq. 20 may be written as

$$Y = C_1 \sin kx + C_2 \cos kx + C_3 e^{-kx}. \qquad (23)$$

By applying boundary condition 2, we obtain

$$0 = C_1 - C_3,$$

or

$$C_1 = C_3,$$

hence (23) becomes

$$Y = C_1 \sin kx + C_2 \cos kx + C_1 e^{-kx}.$$

Finally, application of boundary condition 3 proves that C_1 must be zero and so for this semi-infinite beam, the shape function is simply

$$Y = C \cos kx. \qquad (24)$$

Again without loss of generality, time can be started when the deflection is a maximum, making B zero, in Eq. 20. Thus the deflection of this beam may be expressed simply as

$$y = C \cos pt \cos kx. \qquad (25)$$

The maximum stress in any initially straight beam bent to a curvature given approximately by $\frac{\partial^2 y}{\partial x^2}$ is shown in beam theory[1] to be given by

$$\sigma = Eh \frac{\partial^2 y}{\partial x^2}. \qquad (26)$$

where

E = Young's Modulus;

h = maximum cross-sectional distance from neutral axis.

Substituting the maximum value of the second derivative of Eq. 25 into Eq. 26 gives the maximum stress to be

$$\sigma_{max} = C E h k^2. \qquad (27)$$

Substitution of the value of k^2 from Eq. 20a gives the expression for maximum stress as a function of the maximum deflection, C, to be

$$\sigma_{max} = C h \frac{p}{\eta} \sqrt{E\rho}. \qquad (28)$$

Note that in this case the stress is a function of both the deflection, C, and the frequency, p. Now the maximum value of the first time derivative of Eq. 25 shows the maximum velocity, V, to be

$$V = Cp. \qquad (29)$$

The substitution of this value into Eq. 28 yields our result:

$$\sigma_{max} = V \frac{h}{\eta} \sqrt{E\rho}. \qquad (30)$$

Again the maximum stress for any possible free vibration shape when specified by the maximum velocity does not depend upon frequency, but only on material properties and a beam cross-sectional shape factor, $\frac{h}{\eta}$.

Finally, and as a last example, we shall consider a finite beam to illustrate the previous result is not altered by finiteness *per se*. The bar of length ℓ with hinged ends is chosen, again proceeding from Timoshenko's lead in Ref. [11] on page 331. The general solution for all beams is Eq. 20; the following boundary conditions must be satisfied:

$$\text{At } x = 0,\ y = y'' = 0, \text{ and at } x = \ell,\ y = y'' = 0. \qquad (31)$$

These specify zero deflection and moment at the beam ends. Timoshenki[11] shows that these conditions require the shape function of Eq. 20 to reduce to

$$Y = C_1 \sin kx,$$

and the modes are such that

$$k\ell = i\pi$$

where $i = 1, 2, 3,$ etc.

Again we may select the starting time in Eq. 20 so that B is zero and hence each mode of the hinged beam is described by

$$y_i = C_i \sin k_i x \cos p_i t \qquad (32)$$

where k may only take on the values

$$k_i = \frac{i\pi}{\ell} \qquad (32a)$$

where $i = 1, 2, 3,$ etc. and k_i and p_i are related by Eq. 20a.

Application of Eq. 26 shows the maximum stress in any mode to be given by

$$\sigma_{i\,max} = E h C k_i^{\,2}, \qquad (33)$$

and the incorporation of Eq. 20a yields

$$\sigma_{i\,max} = C_i \frac{h}{\eta} p_i \sqrt{E\rho}. \qquad (33a)$$

Noting that C is the maximum modal displacement, again the stress in terms of displacement depends upon the frequency, p_i.

From Eq. 32 the maximum velocity of the beam in each mode is

$$V_i = \dot{y}_{i\,max} = p_i\, C_i,$$

and the substitution of this value of C_i into 33a yields Eq. 30, once again.

$$\sigma_{i\,max} = V_i \frac{h}{\eta} \sqrt{E\rho}. \qquad (30)$$

It might be commented that a generalized proof for beams is more complicated than the above two examples might lead one to expect. Most boundary conditions will require the presence of hyperbolic sines and cosines in the shape function, e.g., the cantilever beam. This analysis has been done, but it is too lengthy to report here. The above results hold away from the beam ends for modes greater than the second. Simple constants less than 2.0 come into Eq. 30 when root stresses or tip velocities are included.

THE PRACTICAL USE OF THE STRESS-VELOCITY EQUATION

In all the above cases, maximum modal stress is predicted by a single dynamic property, maximum modal velocity. Thus, in order to monitor shock modal response levels that may lead to failures as a result of high stress, modal velocity, at least for these simple cases, is the single, most significant parameter.

Table 1 lists values of the beam shape factor, $\frac{h}{\eta}$, of Eq. 30 for common cross sections. It is interesting to note that the hollow cross sections in bending are only slightly more sensitive than uniform stress.

TABLE 1
Shape Factors, $\frac{h}{\eta}$, for Dynamic Bending Stress

Solid rectangle	=	$\sqrt{3} \approx 1.73$
Solid round bar	=	2
Solid triangle	=	$2\sqrt{2} \approx 2.83$
Thin hollow tube	=	$\sqrt{2} \approx 1.41$
Thin hollow square	=	$\sqrt{6}/2 \approx 1.22$

Eq. 8 and 30 may be used in interpreting the comparative severity of shock spectra. Velocity spectra can be used directly. Acceleration spectra may be used by drawing the constant velocity lines, as mentioned previously. One must compare modal velocities with known damaging values. Severe velocity values may be computed for various metals and beam cross sections. A summary of such properties has been prepared and is included as Table 2. It will be noted that structural steel has the lowest velocity value. This does not indicate steel to be definitely the poorest choice as a shock resistant material. Steel is ductile and local yielding may be an entirely satisfactory behavior. The ideas presented here necessarily depend on a linear stress-strain relation. No similar theory has been developed for the yielding case. When a theory to include the mitigating effects of yielding is developed, shock severity will be much more amenable to evaluation.

TABLE 2
Severe Velocities*

Material	E (psi)	σ (psi)	ρg	V_{max} (ips) $\sigma/\sqrt{E\rho}$	Rectangular Beam V_{max} (ips)
Douglas fir	1.92×10^6	6,450	36 lb/ft^3	633	366
Aluminum 6061-T6	10.0×10^6	35,000	.098 lb/in^3	695	402
Magnesium AZ80A-T5	6.5×10^6	38,000	.065 lb/in^3	1015	586
Structural steel	29×10^6	33,000	.283 lb/in^3	226	130

*(properties taken from Ref. 15)

What has become apparent to us, with respect to complex actual shock motions, is that a broad band of frequencies is invariably present and that a great many of the structural modes are excited. The relative severity of the various frequency components can be assessed via the velocity spectrum of the transient motion. It is to be hoped that further study along these lines may lead us to improved procedures for estimating and testing for shock hardness without actually knowing detailed information about the multitude of possible modes in any real complex structure.

Available Transducers and Methods

Most common of commercially available velocity transducers is the seismic type. They are categorized by employment of a seismically suspended element which remains essentially motionless in space for motions of interest, while a second element of the device is forced to take on the motion of the surface to be measured. If one element has a magnetic field, and the other is a coil of wire, a voltage will be developed in the coil proportional to the relative velocity of the two elements.

Seismic velocity transducers function well, but to insure seismic behavior of the suspended element, internal clearances must exceed the peak displacements of the surface to be measured. If this requirement is not met, the seismic element will "bottom" as peak displacements are reached, and relative velocity between elements will suddenly drop to zero, as will the output voltage.[3, 16] A velocity-time history with such "bottoming discontinuities" is exceedingly difficult to decipher.

Seismic velocity transducers are available in a variety of displacement ranges and natural frequencies, but even for the smallest range and highest frequency, the weight of this type of transducer is too high for many applications. Unfortunately, as displacement range increases, or as natural frequency is lowered, weight goes even higher, and area of applicability of the seismic velocity transducer is further limited.[3, 16]

Since the arrival of the age of integrated circuits, there is now commercially available a "piezoelectric velocity transducer." Basically an accelerometer, this device contains an integrated circuit within the transducer housing which electronically integrates the acceleration signal to velocity. In size and weight, it is slightly larger than the average accelerometer, and therefore has a distinct advantage over the moving coil type of transducer. A major disadvantage in this approach is the wide dynamic signal range produced by the accelerometer when measuring shock. The electronic integrator is required not only to mechanically tolerate the shock, but at the same time provide satisfactory operation with a 70 dB dynamic range input signal. The "piezoelectric velocity transducers" now available are suitable for vibration measurements, but are too frail and lacking in dynamic range to be useful in any but the lightest shock measurements. Many advances have been made in the field of integrated circuits since these transducers were introduced, and greatly enhanced "piezoelectric velocity transducers" are possible, and may be forthcoming.

M. W. Oleson of the Naval Research Laboratory has reported success in on-line integration of accelerometer signals[7, 17] (see Fig. 5a). His method differs from the "piezoelectric velocity transducer" approach in that his electronic integrators are located some distance from the accelerometers, and are not restricted in size and weight. The freedom from restriction allows increased linearity and lower frequency response for his system, and also permits double integration of the acceleration signal so the shock can be described in displacement if desired.

Mr. Oleson is quite aware that his method has the disadvantage of having to deal with the wide dynamic range of shock acceleration, and has constructed accelerometer mounts which act as low-pass filters and isolate the accelerometers from high-frequency, high-level accelerations.

P. S. Hughes has reported on the use of a digital computer program titled "MR. WISARD" to integrate and double-integrate acceleration signals[18] (see Figs. 7, 8, and 9). In addition, the program computes shock spectra (see Figs. 1, 2, and 3). Assuming that complexity of the program necessitates off-line operation, shock acceleration signals must be recorded for later processing when using this approach. As explained earlier, no present analog-recording medium offers a dynamic range large enough to satisfactorily record shock acceleration. This is a severe limitation on "MR. WISARD."

Another use of digital computers is worthy of mention. G. O'Hara and P. Cuniff have reported on a method of correcting for bottoming discontinuities of velocity transducers.[19] It has been stated that velocity records corrected in this fashion provide information as accurate as the on-line integrated accelerometer approach.

SUMMARY

The basic facts presented in this paper are not new or unique. What is novel is that in this case the facts have been considered collectively rather than singly, and increased understanding of the damage mechanism of shock is the result.

Because of the direct relationship between stress and modal velocity, and the small dynamic range of vibrational velocity in shock, it is apparent that, of the three related parameters, velocity provides maximum measurement efficiency and accuracy.

Since modal velocity, not total vibrational velocity or translational velocity, bears the direct relation to stress, it is also apparent that to obtain shock-severity measurements, a velocity-measuring system need have a lower frequency response a little below the lowest modal frequency of the structure in question. For many usual structures, an instrumentation system with a lower frequency response of 5 Hz would be more than adequate.

Again, because of the direct relationship between modal velocity and stress, the wisdom of using normal mode theory[10] in shock-resistant design is indicated. The Dynamic Design-Analysis Method (DDAM),[20, 22] which is a normal-mode analysis, provides most of what is needed for vastly improved shock-resistant design.

Reviewing current techniques of measuring shock in terms of velocity, it is apparent that a velocity transducer different from ones presently available is needed. While computer correction of bottoming discontinuities appears feasible, it is certainly less than aesthetically satisfying. And use of integrated signals from accelerometers leaves much to be desired, particularly when analog recording must be interposed between accelerometer and integrator.

CONCLUSIONS

Conclusions are:

1. Modal velocity is the best criterion of shock severity.
2. Velocity should be the predominant parameter for shock measurement.
3. Development of an adequate velocity transducer is needed.

FROM NOLTR 69-64, FIG. 22

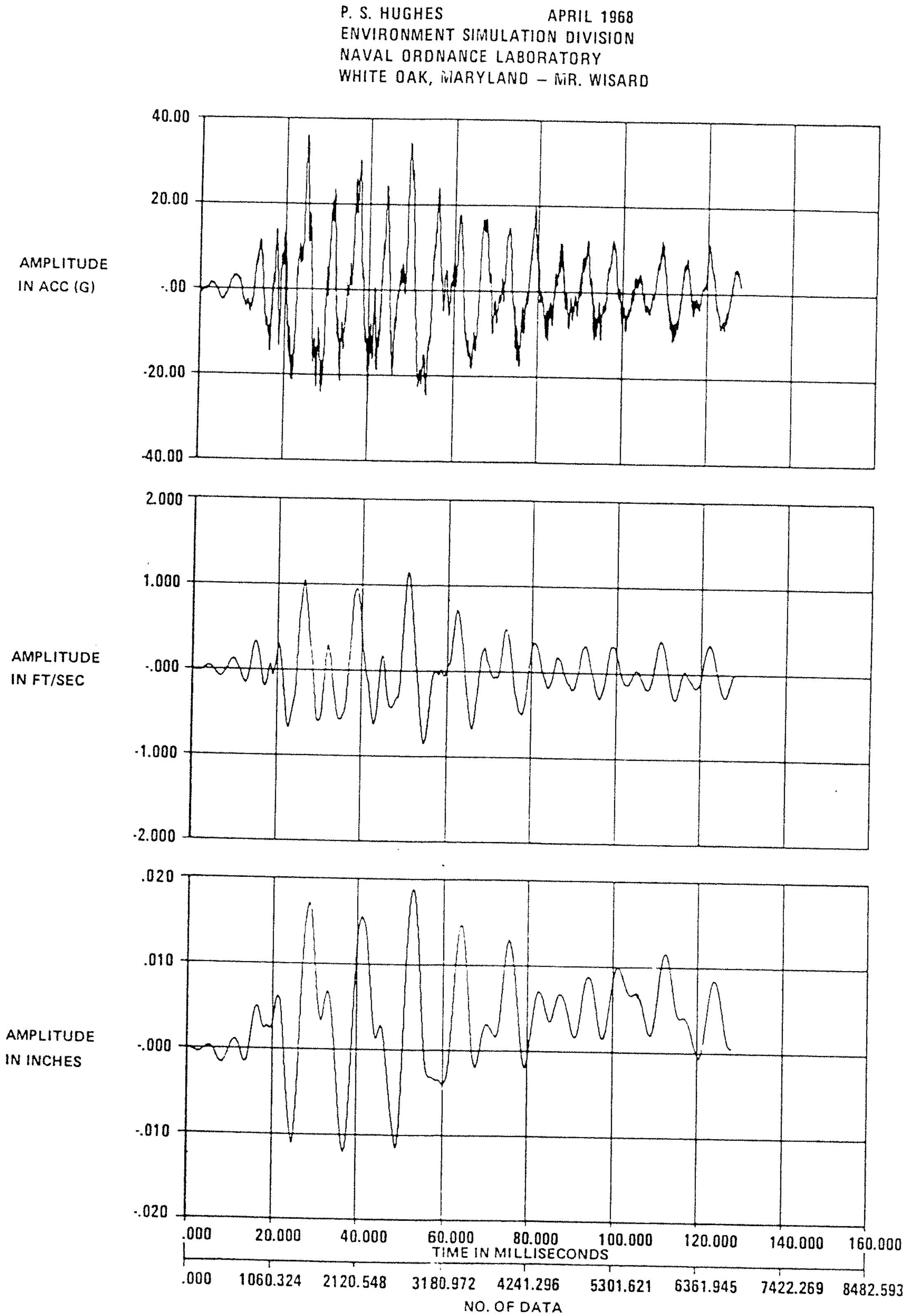


FIGURE 7. A-T, V-T, D-T SIGNATURES

FROM NOLTR 69-64, FIG. 12

P. S. HUGHES APRIL 1968
ENVIRONMENT SIMULATION DIVISION
NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND – MR. WISARD

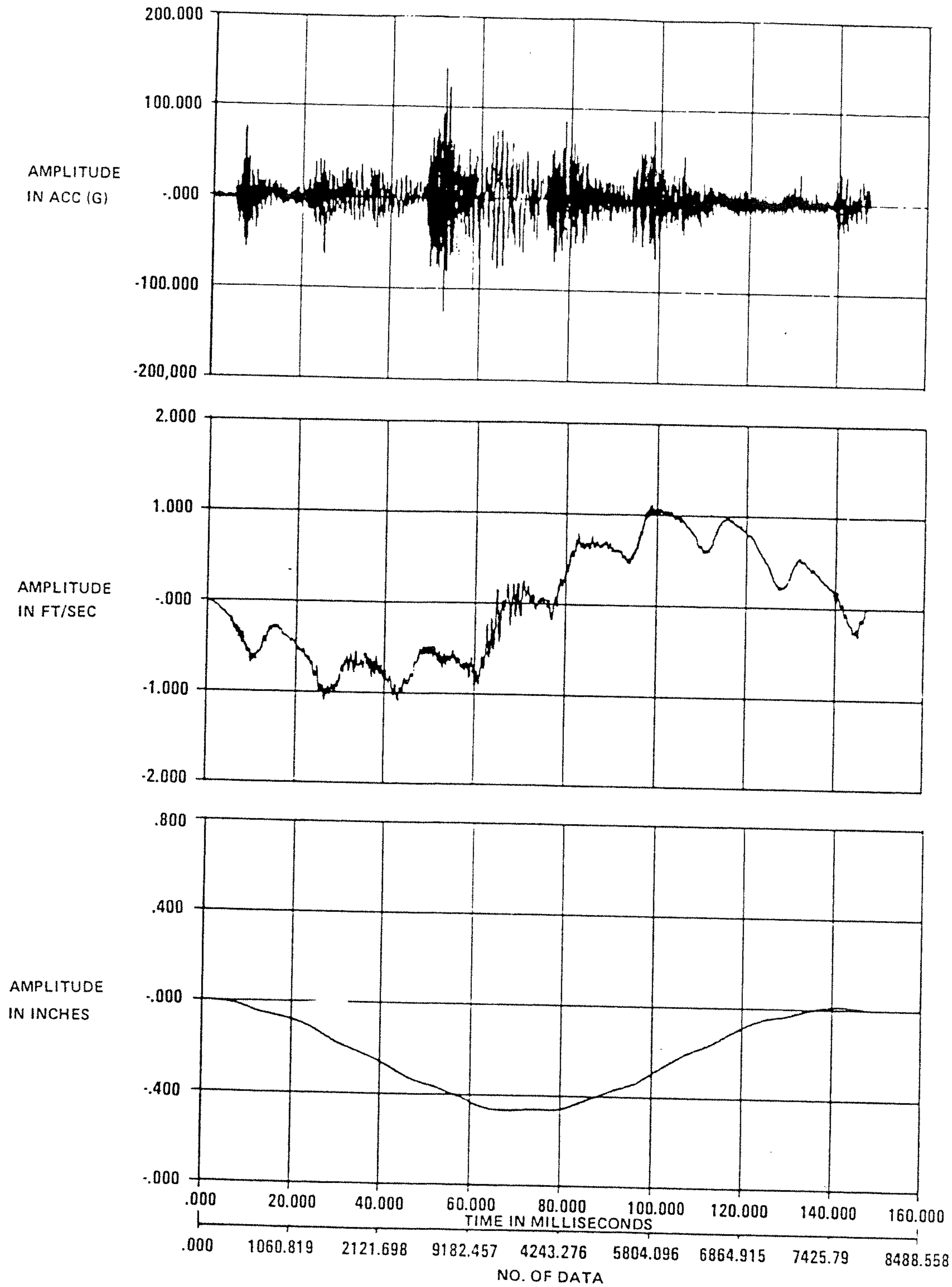


FIGURE 8. A-T, V-T, D-T SIGNATURES

FROM NOLTR 69-64, FIG. 13

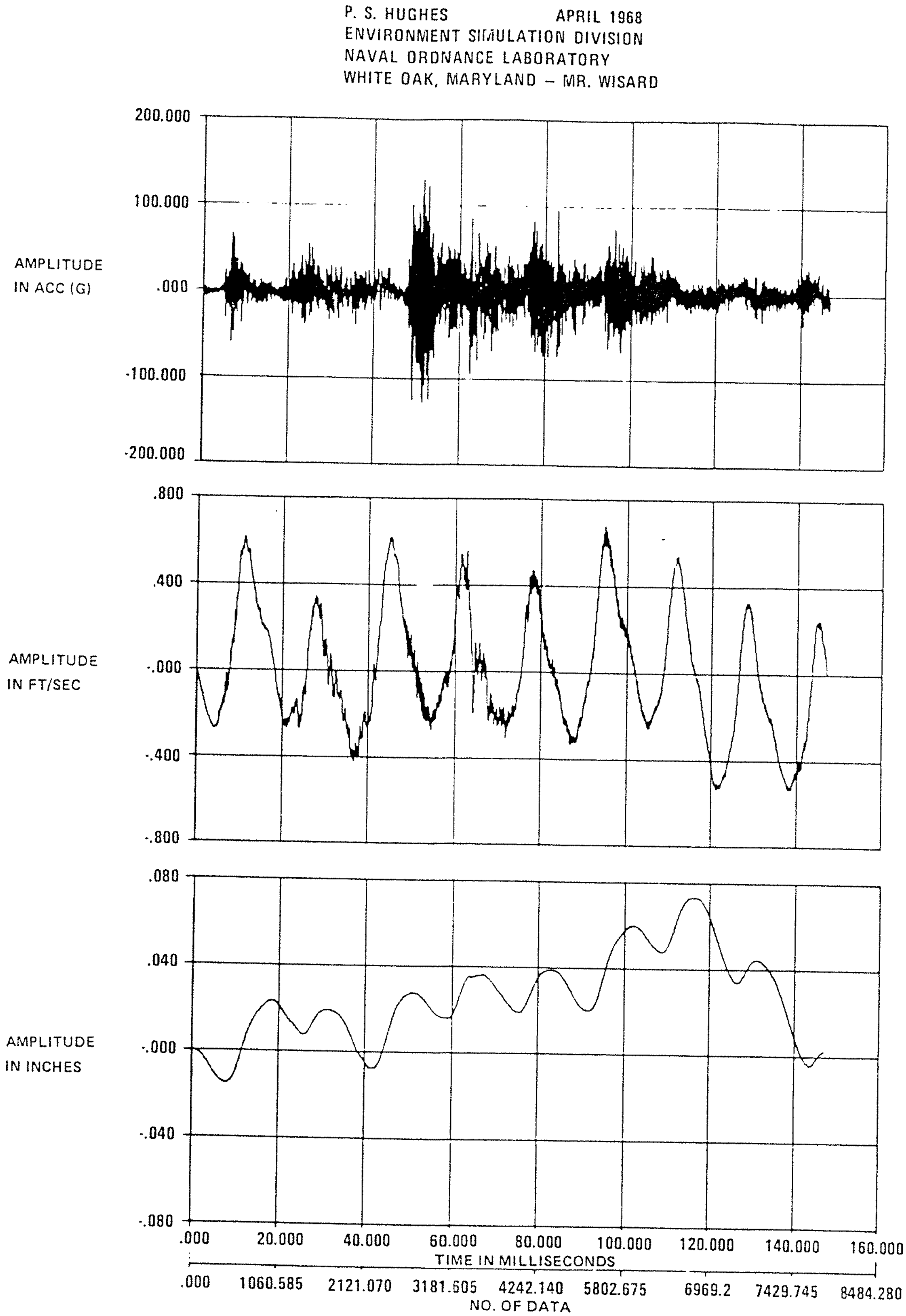


FIGURE 9 . A-T, V-T, D-T SIGNATURES

REFERENCES

1. Blake, R. E., "Problems of Simulating High Frequency Mechanical Shock," Proceedings of the Institute of Environmental Sciences, 1964, p. 145

2. U. S. Army Electronics Command Technical Report ECOM-2647A (Revised), A Mechanical Shock Pulse Survey, by F. Palmisano and L. L. Kaplan, March 1966

3. Vigness, I., "Introductory Remarks on Shock Instrumentation," Shock and Vibration Bulletin, No. 6, Naval Research Laboratory Report No. S3200, p. 92, November 1947

4. Naval Research Laboratory Report No. 0-1965, High Impact Shock Instrumentation and Measurements - First Partial Report, by I. Vigness, p. 20, November 1942

5. Forkois, H. M., "Problems in Getting Consistent Results in Shock Testing," Proceedings of the Institute of Environmental Sciences, 1965, p. 455

6. Walsh, J. P., "A Review of the Report on the Cameron Trials," Shock and Vibration Bulletin, No. 4, Naval Research Laboratory Report No. S-3151, p. 62, July 1947

7. Oleson, M. W., "Components of a New Shock Measurement System," Report of NRL Progress, Naval Research Laboratory, October 1967, p. 16-27

8. Shaw, J. E., "Brief Resume of Recent British Shock Trials in Ships," Shock and Vibration Bulletin, No. 9, Naval Research Laboratory Report No. S-3290, p. 36, April 1948

9. Hudson, D. E., Shock and Vibration Handbook, First Edition, ed. by Harris and Crede, volume 3, p. 50-36 to 50-37, McGraw-Hill Book Co., Inc., New York, 1961

10. Naval Research Laboratory Report No. 6002, Elements of Normal Mode Theory, by G. J. O'Hara and P. E. Cuniff, November 1963

11. Timoshenko, S., Vibration Problems in Engineering, Third edition, p. 297-299, p. 324-335, D. Van Nostrand, Inc., Princeton, New Jersey, January 1955

12. Den Hartog, J. P., Advanced Strength of Materials, p. 172, McGraw-Hill Book Co., Inc., New York, 1952

13. Den Hartog, J. P., Mechanical Vibrations, Fourth Edition, p. 5, McGraw-Hill Book Co., Inc., New York, 1956

14. Higdon, A., Ohlsen, E. H., and Stiles, W. B., Mechanics of Materials, p. 222-225, John Wiley and Sons, Inc., New York, 1960

15. Roark, R. J., Formulas for Stress and Strain, Fourth Edition, McGraw-Hill Book Co., Inc., New York, 1965

16. Oleson, M. W., "Shock Instrumentation Development and Random Vibration Research," Report of NRL Progress, Naval Research Laboratory, July 1963, p. 32

17. Oleson, M. W., "Integration and Double Integration - A Practical Technique," Shock and Vibration Bulletin, No. 35, Part 4, p. 1-10, February 1966

18. Naval Ordnance Laboratory Report NOLTR 69-64, 40 mm Gun Shock Environment Aboard Hydrofoil PCH-1, by P. S. Hughes, 18 March 1969

19. Naval Research Memorandum Report 1489, Velocity Meter Corrections, by G. J. O'Hara and P. E. Cuniff, December 1963

20. Bureau of Ships NAVSHIPS 250-423-30, Shock Design of Shipboard Equipment — Dynamic Design-Analysis Method, May 1961

21. Bureau of Ships NAVSHIPS 250-423-31, Shock Design of Shipboard Equipment — Interim Inputs for Submarine and Surface Ship Equipment, October 1961

22. Applied Technology Associates Report 124, Guide for Users of Dynamic Design-Analysis Method for Shock Resistance Evaluation of Shipboard Equipment, January 1969

DISCUSSION

Mr. Holland (Kinetic Systems): I have done some work for Frankford Arsenal on high frequency cutoff criteria for computing shock spectra. My study looked at simple cantilever beams and simply supported beams, both uniform. I added point masses at the end of the cantilevers and point masses in the middle of the simply supported beams. Using the modal participation factor of easily described structures like these and the fact that my velocity was assumed constant in a small area out to some frequency, I was able to find that my acceleration was constant. It is similar to the Navy shock spectrum where you have three backbones of constant displacement, constant velocity, and constant acceleration. Most of the stresses were contained in the first couple of modes, and by taking these modes I got about 90 percent of the stress in these simple beam structures. The problem that I was trying to solve was where to stop the computation. How high should the frequency be? We came up with the factor of 2000 Hz for some of the stiffnesses that we were looking at in the structure. But when you get into a complex system and have a resonant frequency in the range of 2000 Hz with your shock on the backbone of acceleration, you can easily excite these frequencies. This will create a stress at a higher mode in excess of what you are getting at your lower frequencies, so that you can get out of your velocity range. You had constant velocity, let us say, from 2 Hz to 100 Hz and, if you go down your constant acceleration line, you can easily find higher resonant frequencies where your acceleration is going to produce the failure.

Mr. Chalmers: On the part shown, the constant velocity centered out to 2000 Hz.

Mr. Holland: Your shock spectrum will vary for all different shocks.

Mr. Pakstys (General Dynamics Corp.): I certainly agree with your conclusions about the modal velocity being an important parameter. There is another way to look at this other than just looking at these simple cases. If we look on a mode-to-mode basis, we can look at the modal kinetic energy which then can be related to the velocity squared. Kinetic energy can then be related to strain energy and strain energy can be related to stresses. From that point of view you can rationalize the importance of velocity in a multi-degree-of-freedom system. The velocity does not have to be in the constant velocity range of the spectrum; it can be in the constant acceleration. The importance of the velocity criterion is not diminished.

Mr. Chalmers: That is correct. We are trying to do two things. First, if you are measuring in terms of velocity, insofar as shock is concerned, you have a very much reduced dynamic range required for your measurements. Secondly, Dr. Gaberson has shown that velocity is probably the best descriptor of stress in a simple beam.

Mr. Scharton (Bolt Beranek & Newman): I think the conclusion that much of the data which you have looked at is described by a constant velocity spectrum is very interesting. Two possible explanations of this occurred to me. One would seem to be that, at least in plates in bending and also in a beam in compression or torsion, the modal density is constant with frequency. This means that the frequency separation between modes is constant, so the observation that one has a constant velocity spectrum would be equivalent to saying that each mode of the structure has the same amount of energy. We can then ask, when you put a complex transient into the system does the structure somehow take this energy and distribute it equally among its modes? Also, since yield criteria are commonly related to strain energy, if the shock were so severe that in fact you got local yielding, it might be that the yield phenomenon would damp each mode and automatically bring the level of each mode down to some fixed energy level. Could you comment on that?

Mr. Gaberson: It is very interesting that you bring that up. You have had for a long time a theorem about equal modal density and that just had not occurred to me. I did some studies on what kind of modal distribution you get if you put an impulse on a beam. That gives equal velocity. If you twang it, that gives equal acceleration.