

POWER SPECTRAL DENSITY UNITS: [ G<sup>2</sup> / Hz ]  
Revision B

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Introduction

Random vibration can be represented in the frequency domain by a power spectral density function. The typical units are acceleration [G<sup>2</sup>/Hz] versus frequency [Hz]. The acceleration can also be represented by metric units, such as [ (m/sec<sup>2</sup>)<sup>2</sup> / Hz ].

Note that the amplitude is actually [ GRMS<sup>2</sup> / Hz ], where RMS is root-mean-square. The RMS notation is typically omitted for brevity.

The Hz value in [G<sup>2</sup>/Hz] refers to a bandwidth rather than to the frequency in Hz along the X-axis.

The RMS value of a signal is equal to the standard deviation, assuming a zero mean. The standard deviation is usually represented by sigma  $\sigma$ .

A pure sinusoidal function has the following relationship:

$$\text{peak} = \sqrt{2} \text{ RMS} \quad (1)$$

Random vibration, however, is very complicated. Random vibration has no simple relationship between its peak and RMS values. The peak value of a stationary random time history is typically 3 or 4 times the RMS value.

A power spectral density can be calculated for any type of vibration signal, but it is particularly appropriate for random vibration.<sup>1</sup>

There are several equivalent methods for calculating a power spectral density function, as explained in Reference 1. This tutorial will focus on the bandpass filtering method.

Simple Example

A sample acceleration time history is shown in Figure 1. Its overall value is 1.49 GRMS. The signal was generated analytically. The entire energy content is between 10 Hz and 40 Hz.

The signal is shown bandpass filtered in Figures 2 through 4. The filtering was performed using the method in Reference 2.

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<sup>1</sup> A Fourier transform magnitude is more appropriate than a power spectral density for sine vibration. The converse is true for random vibration.

In some sense, the signal filtering process is analogous to passing sunlight through a prism. The prism separates the white light into its spectral color components. Likewise, the signal filtering process separates the time history into its spectral components.

The filtering parameters for the sample time history are shown in Table 1.

Table 1. Filtering Parameters		
Figure	Bandpass Filtering	Overall Level (GRMS)
1	Unfiltered	1.49
2	10 Hz to 20 Hz	0.68
3	20 Hz to 30 Hz	1.08
4	30 Hz to 40 Hz	0.73

Note that the overall level is equal to the "square root of the sum of the squares."

$$\sqrt{(0.68)^2 + (1.08)^2 + (0.73)^2} = 1.47 \text{ GRMS} \quad (2)$$

$$\sqrt{(0.68)^2 + (1.08)^2 + (0.73)^2} \approx 1.49 \text{ GRMS} \quad (3)$$

A slight error occurs due to imperfections in the filtering process. Nevertheless, the agreement is very good.

A power spectral density function can be calculated as shown in Table 2.

SAMPLE ACCELERATION TIME HISTORY, UNFILTERED  
Overall Level = 1.49 GRMS

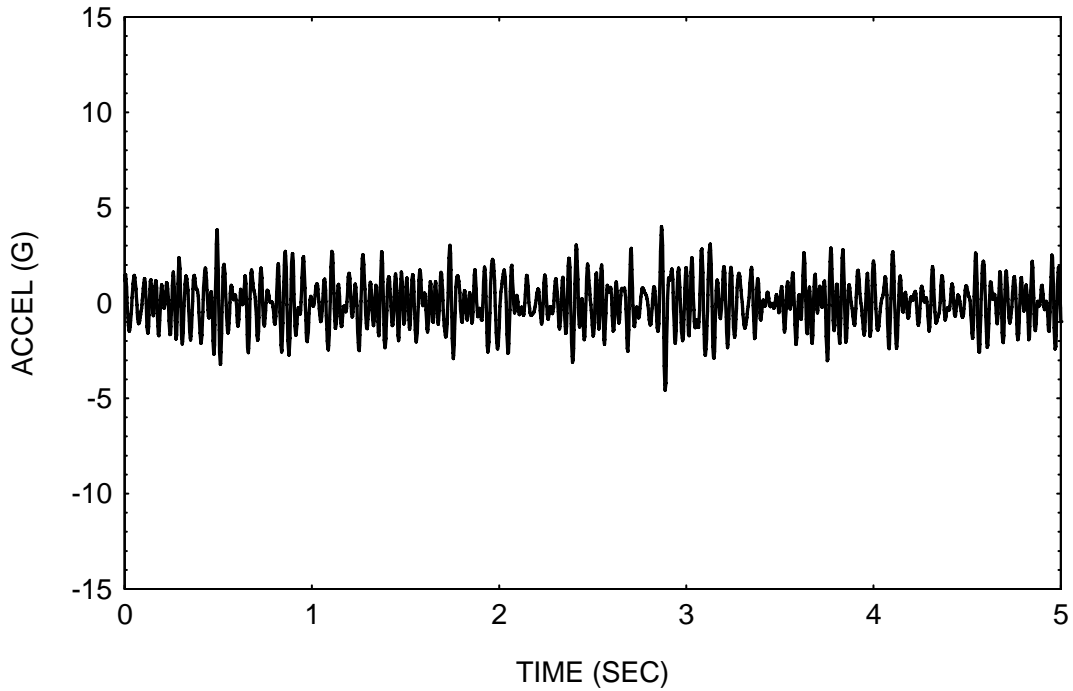


Figure 1.

SAMPLE ACCELERATION TIME HISTORY, 10 Hz to 20 Hz BP FILTERED  
Overall Level = 0.68 GRMS

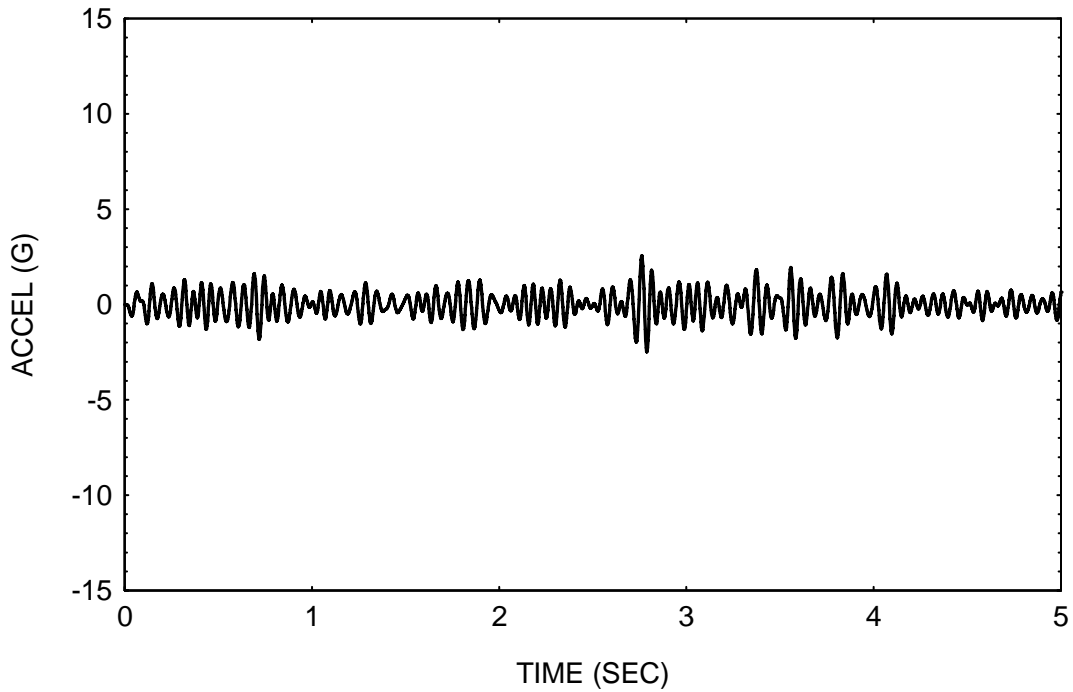


Figure 2.

SAMPLE ACCELERATION TIME HISTORY, 20 Hz to 30 Hz BP FILTERED  
Overall Level = 1.08 GRMS

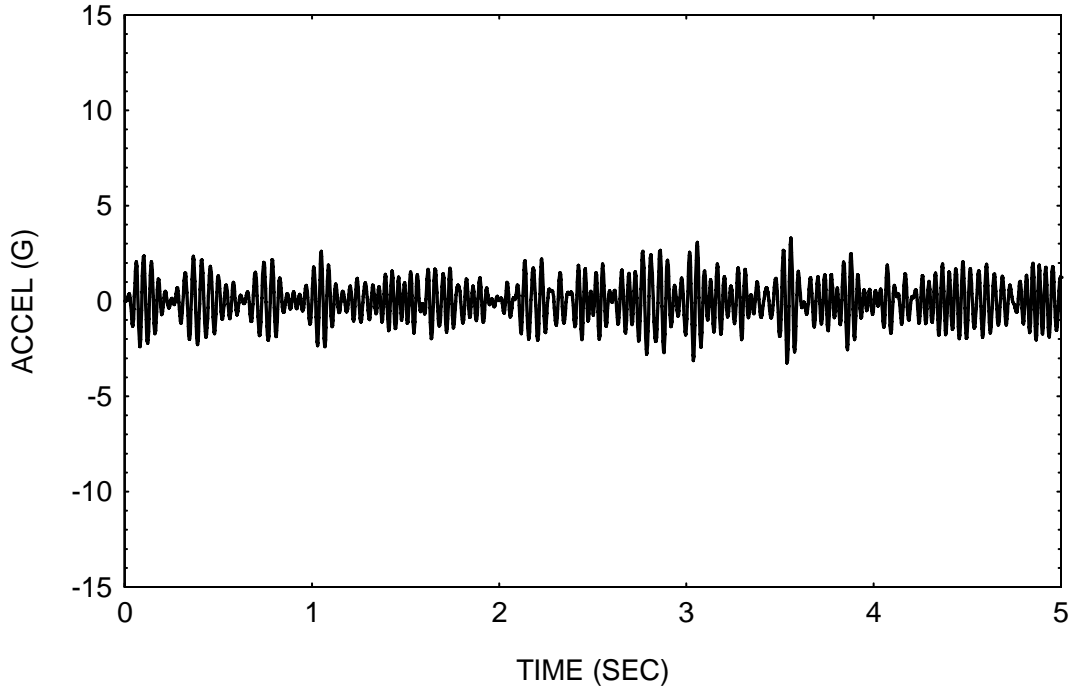


Figure 3.

SAMPLE ACCELERATION TIME HISTORY, 30 Hz to 40 Hz BP FILTERED  
Overall Level = 0.73 GRMS

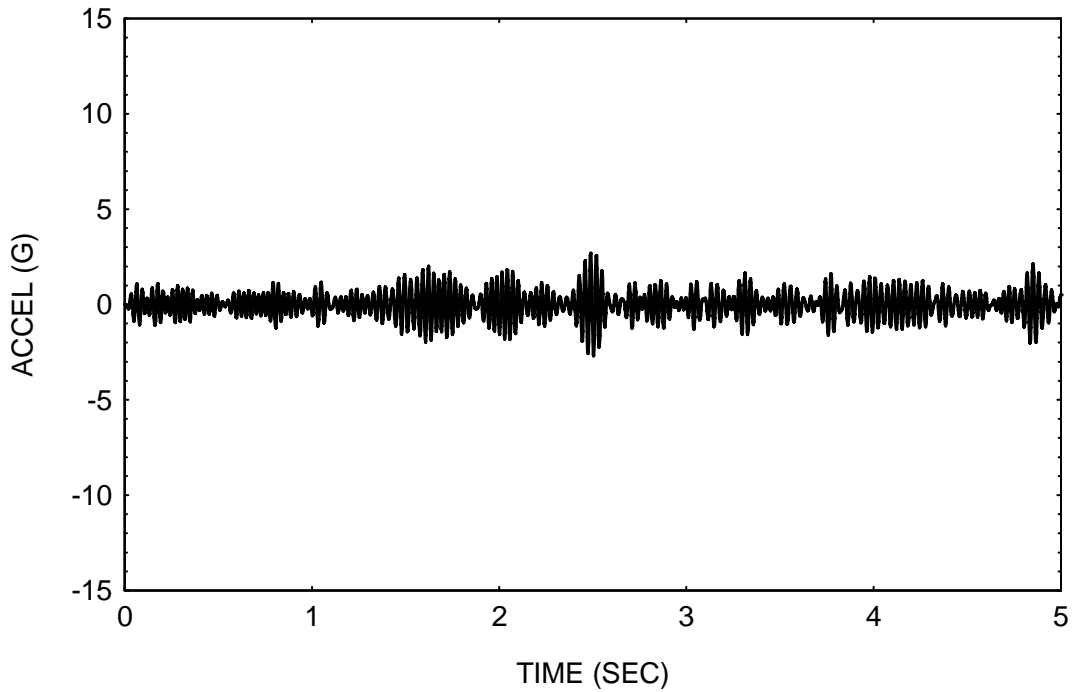


Figure 4.

Table 2. Power Spectral Density Calculation					
Bandpass Filter	Band Center Frequency (Hz)	Overall Level (GRMS)	Overall Level ^2 (GRMS^2)	Bandwidth (Hz)	PSD (GRMS^2/Hz)
10 Hz to 20 Hz	15	0.68	0.46	10	0.046
20 Hz to 30 Hz	25	1.08	1.17	10	0.117
30 Hz to 40 Hz	35	0.73	0.53	10	0.053

The bandwidth is the upper filtering frequency minus the lower filtering frequency. The bandwidth was constant in this example, but this was not a requirement.

The power spectral density (PSD) is simply the (overall level)^2 divided by the bandwidth.

Again, the unit [ GRMS^2 / Hz ] is typically abbreviated as [ G^2 / Hz ].

A plot of the power spectral density function is shown in Figure 5, represented as a bar graph.

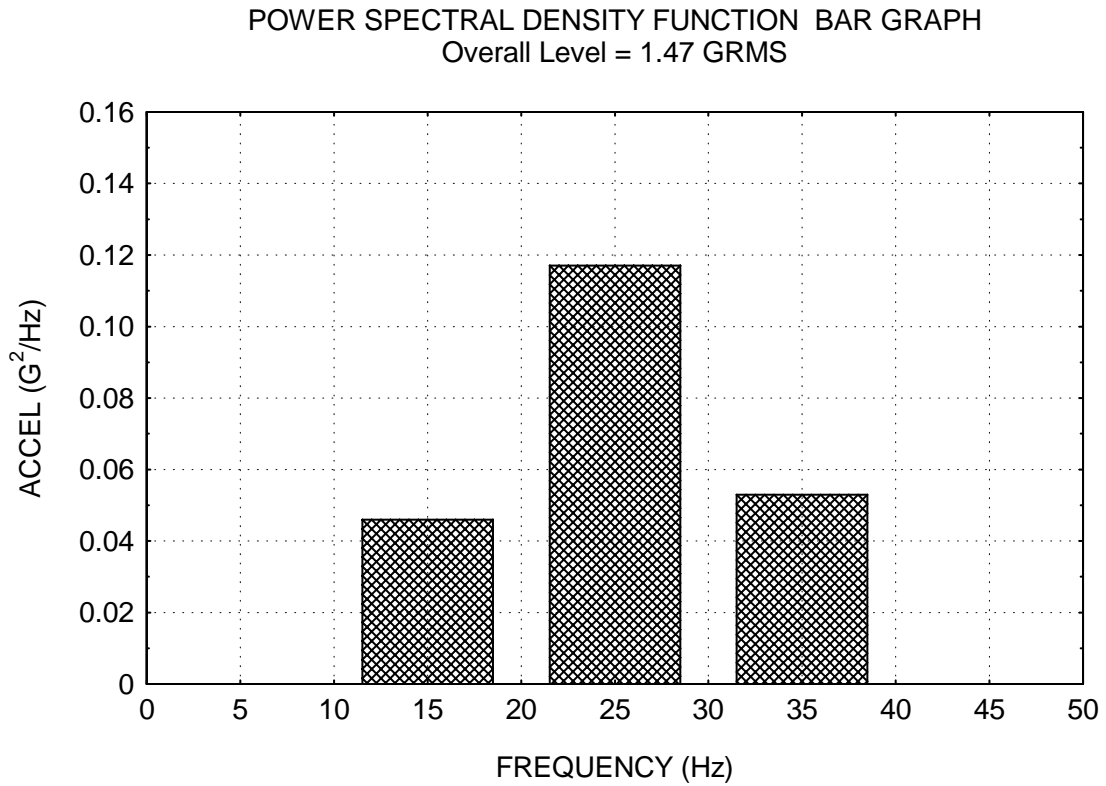


Figure 5.

The bar graph is a suitable format for a power spectral density function.

Convention, however, dictates a line-graph format as shown in Figure 6.

POWER SPECTRAL DENSITY FUNCTION - LINE GRAPH  
Overall Level = 1.47 GRMS

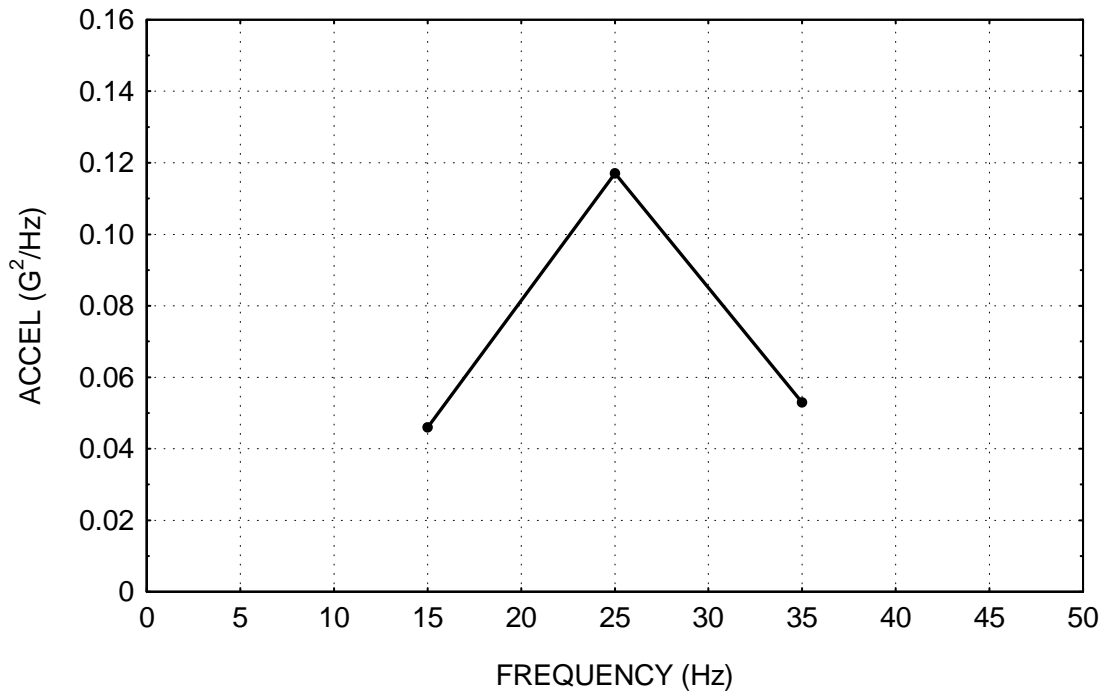


Figure 6.

The overall GRMS value can be obtained by integrating the area under the power spectral density curve. The GRMS value is then equal to the square root of the area.

This is equivalent to calculating the "square root of the sum of the squares," as performed in equation (2).

An alternate integration method is given in Reference 3.

### Conclusion

The bandpass filtering method was used to demonstrate a power spectral density calculation in terms of [ G<sup>2</sup> / Hz ].

The bandpass filter method is one of three equivalent methods for calculating power spectral density functions. The bandpass filtering method is inefficient, however. Thus, computer software is usually based on the Fourier transform method, as described in Reference 1.

## References

1. T. Irvine, An Introduction to Spectral Functions, Vibrationdata, 1998.
2. T. Irvine, An Introduction to the Filtering of Digital Signals, Vibrationdata, 2000.
3. T. Irvine, Integration of the Power Spectral Density Function, Vibrationdata, 2000.