ESTIMATING THE TRANSMISSIBLITY Q FOR RANDOM VIBRATION

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Introduction

Steinberg gives a method for estimating the transmissibility Q for a sinusoidal input in Reference 1.

The two main conclusions from empirical data are:

1. Increasing the natural frequency will increase the transmissibility.
2. Increasing the input acceleration G level will decrease the transmissibility.

Steinberg's equation for approximating Q for a system is:

$$Q = A\left(\frac{f_n}{(G_{in})^{0.6}}\right)^{0.76} \tag{14.21}$$

where

A	=	1.0 for beam-type structures
A	=	0.5 for plug-in PCBs or perimeter supported PCBs
A	=	0.25 for small electronic chassis or electronic boxes
f_n	=	natural frequency (Hz)
G_{in}	=	sinusoidal vibration input acceleration G level

Definitions of structures:

Beam structures = several electronic components with some interconnecting wires or cables

PCB = well populated with an assortment of electronic components

Small electronic chassis = 8-30 inches in its longest dimension, with a bolted cover to provide access to various types of electronic components such as PCBs, harnesses, cables, and connectors.

Random Vibration Derivation

Steinberg's methods can be adapted for a random vibration input.

The system is assumed to be a single-degree-of-freedom system.

Again, Steinberg's equation for sine vibration is

$$Q = A\left(\frac{f_n}{(G_{in})^{0.6}}\right)^{0.76} \tag{1}$$

The sine and random equivalence formula is taken from Reference 2.

$$(G_{in}) = \frac{1.95\ (G_{1\sigma})}{Q} \tag{2}$$

where $G_{1\sigma}$ is the 1-sigma response to random vibration.

Substitute equation (2) into (1).

$$Q = A\left(\frac{f_n\ Q^{0.6}}{\left(1.95\ (G_{1\sigma})\right)^{0.6}}\right)^{0.76} \tag{3}$$

$$Q = 0.6\ A\ f_n^{0.76}\ Q^{0.46}\ G_{1\sigma}^{-0.46} \tag{4}$$

$$\frac{Q}{Q^{0.46}} = 0.6\ A\ f_n^{0.76}\ G_{1\sigma}^{-0.46} \tag{5}$$

$$Q^{0.64} = 0.6\,A\,f_n^{\,0.76}\;G_{1\sigma}^{\,-0.46} \tag{6}$$

$$Q = \left[0.6\,A\,f_n^{\,0.76}\;G_{1\sigma}^{\,-0.46}\right]^{1/0.64} \tag{7}$$

$$Q = \left[0.6\,A\,f_n^{\,0.76}\;G_{1\sigma}^{\,-0.46}\right]^{1.56} \tag{8}$$

$$Q = 0.45\,A^{1.56}\,f_n^{\,1.2}\,G_{1\sigma}^{\,-0.72} \tag{9}$$

Miles equation for the 1-sigma response $G_{1\sigma}$ to random vibration is

$$G_{1\sigma} = \left[\left(\frac{\pi}{2}\right) f_n\;Q\;\hat{Y}_{APSD}(f_n)\right]^{0.5} \tag{10}$$

where

$\hat{Y}_{APSD}(f_n)$ is the power spectral density (G^2/Hz) at the natural frequency fn.

Substitute Miles equation (10) into equation (9).

$$Q = 0.45\,A^{1.56}\,f_n^{\,1.2}\left\{\left[\left(\frac{\pi}{2}\right) f_n\;Q\;\hat{Y}_{APSD}(f_n)\right]^{0.5}\right\}^{-0.72} \tag{11}$$

$$Q = 0.45\,A^{1.56}\,f_n^{\,1.2}\,f_n^{\,-0.36}Q^{-0.36}\left(\frac{\pi}{2}\right)^{-0.36}\left[\hat{Y}_{APSD}(f_n)\right]^{-0.36} \tag{12}$$

$$Q\ \ Q^{0.36} = \quad 0.45\, A^{1.56}\ f_n^{\,0.84} \left(\frac{\pi}{2}\right)^{-0.36} \left[\hat{Y}_{APSD}(f_n)\right]^{-0.36} \tag{13}$$

$$Q^{1.36} = \quad 0.38\, A^{1.56}\ f_n^{\,0.84} \left[\hat{Y}_{APSD}(f_n)\right]^{-0.36} \tag{14}$$

$$Q = \quad \left\{0.38\, A^{1.56}\ f_n^{\,0.84} \left[\hat{Y}_{APSD}(f_n)\right]^{-0.36}\right\}^{1/1.36} \tag{15}$$

$$Q = \quad \left\{0.38\, A^{1.56}\ f_n^{\,0.84} \left[\hat{Y}_{APSD}(f_n)\right]^{-0.36}\right\}^{0.74} \tag{16}$$

$$Q = \quad 0.49\ A^{1.15}\ f_n^{\,0.62} \left[\hat{Y}_{APSD}(f_n)\right]^{-0.27} \tag{17}$$

$$Q = \quad \frac{0.49\ A^{1.15}\ f_n^{\,0.62}}{\left[\hat{Y}_{APSD}(f_n)\right]^{0.27}} \tag{18}$$

Note that the transmissibility Q is equivalent to (G out / G in) at the system’s natural frequency.

The power transmissibility is Q^2.

References

1. Steinberg, Vibration Analysis for Electronic Equipment, Third Edition. See Section 14.16, “*More Accurate Method for Estimating the Transmissibility Q in Structures*.”

2. T. Irvine, Sine and Random Damage Equivalence, Revision A, Vibrationdata, 2004.