ON THE USE OF A NARROWBAND RANDOM POWER SPECTRAL DENSITY TO COVER A PURE SINE VIBRATION ENVIRONMENT VIA RAINFLOW FATIGUE

Revision A

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The reader should review the references prior to reading this paper.

Introduction

Consider that a component must be tested on shaker table to the following pure sine, base input.[1]

Amplitude	1 G
Frequency	100 Hz
Duration	30 seconds

Assume that the component can be represented as a single-degree-of-freedom (SDOF) system with a natural frequency of 100 Hz and an amplification factor of Q=10.

The 100 Hz natural frequency would be a conservative assumption if the component's natural frequency was unknown.[2]

The steady-state response of the SDOF system to the sinusoidal input is 10 G, with 3000 cycles.

Now consider that an "equivalent" narrowband random power spectral density (PSD) must be derived as a substitute for the sinusoidal input.

The PSD frequency domain will be from 90 and 110 Hz, for the examples in this paper. The resulting bandwidth is approximately one-sixth octave. The bandwidth choice is somewhat arbitrary.

The input PSD level must be selected so that the SDOF system would be subjected to greater "fatigue damage" during the narrowband random test than during the sinusoidal test.

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[1] In reality, the 1 G amplitude would be rather benign, but it is used for a convenient comparison baseline.

[2] The component's natural frequency should be separated by one octave from the input sine's frequency as a good engineering design practice.

The fatigue assessment is performed with respect to the SDOF system's *response* for each input type.

Two assessment methods are considered in this paper. Both use the rainflow cycle counting method from References 1 and 2.

Notes

1. All of the acceleration ranges in the following sections are in terms of G peak-to-peak.

2. The SDOF system has a natural frequency of 100 Hz with Q=10.

3. The sine input parameters are those previously given in the Introduction section.

4. The SDOF system response to the sine input has 3000 reversals with a range of 20 G.

Note that the range is double-sided.

First Method

The first method is conservative. It can be used when the slope of the S-N curve is unknown. Note that selecting a fatigue slope for an aerospace component is difficult because the component is usually composed of numerous materials. Furthermore, stress concentration factors may have an unknown effect on the S-N curve.

The first method is demonstrated by the following example.

The goal is to derive a narrowband PSD input such that the response of the SDOF system has at least 3000 cycles greater than or equal to 20 G.

Note that this method is conservative because it does not account for the true magnitude effect of reversals above 20 G. i.e. A reversal of 40 G is counted the same as a reversal of 20 G.

Furthermore, the duration for the PSD must be longer than that for the sine input. The PSD duration will be set to 60 seconds in this example, which is twice that of the sine duration.

The derivation method is performed in an iterative, trial-and-error method using the following steps.

1. Select a PSD amplitude is that is flat between 90 and 110 Hz.
2. Synthesize a random time history for the PSD.
3. Perform a modal transient analysis to determine the SDOF response to the synthesized time history.
4. Perform a rainflow cycle analysis on the response time history, and count the number of cycles with a range $\geq$ 20 G.

This method showed that an input PSD with an amplitude of 0.07 G^2/Hz over 90 to 110 Hz would yield 3103 response cycles with a range ≥ 20 G. Again, this is for a duration of 60 seconds.

The input PSD's overall level is 1.2 GRMS. This is 4.5 dB greater than the input sine's overall level.

Again this method is conservative. Its advantages are that it is rigorous and defensible. It also makes no assumptions about fatigue slope and stress concentration factors.

Second Method

The second method is somewhat similar to that in Reference 3. It accounts for the amplitude range in terms of fatigue damage from an S-N curve approach.

Assume a fatigue slope of 4. Note that a lower slope is more conservative for using random vibration to cover sine

The sinusoidal response fatigue damage D_{sine} is

$$D_{sine} = (3000 \text{ cycles})(20 \text{ G})^4 = 4.80\text{E}+08 \quad (1)$$

The goal is to derive a PSD such that the response fatigue damage is $\geq D_{sine}$

The PSD duration will be the same as that of the sine duration or this case, which is 30 seconds.

The steps are

1. Select a PSD amplitude is that is flat between 90 and 110 Hz.
2. Synthesize a random time history for the PSD.
3. Perform a modal transient analysis[3] to determine the SDOF response to the synthesized time history, using an amplification factor of Q=10.
4. Perform a rainflow cycle analysis on the response time history, and calculate the fatigue damage by applying an exponent of 4 to the amplitude ranges.

This method showed that an input PSD with an amplitude of 0.03 G^2/Hz over 90 to 110 Hz would yield a damage value of 4.85E+08, which is slightly greater than that for the sine input.

The input PSD's overall level is 0.77 GRMS. This is 0.74 dB greater than the input sine's overall level.

As an aside, the PSD response had 664.5 cycles ≥ 20 G. Again, the sine response had 3000 cycles at 20 G.

[3] The numerical engine for the modal transient analysis can be the same as that for the shock response spectrum calculation.

But the PSD response had numerous cycles above 20 G. For example, it had 4 cycles $\geq$ 44.8 G.

So the second method relies on the fatigue damage from the cycles at higher ranges to show that the PSD input covers that of the sine input.

Summary

The first method yielded an input PSD with and overall level of 1.2 GRMS.

The second method gave an input PSD with an overall level of 0.77 GRMS.

The first method is thus 3.9 dB higher in terms of overall level.

Furthermore, the duration for the first method was twice that of the second method.

The first method is useful when conservatism is allowable. It makes no assumption about the fatigue slope.

The second method is useful when "sharpening the pencil" is needed to "justify" a lower input PSD level. It requires a fatigue slope assumption.

Alteration Effects

The following applies to both methods.

The PSD amplitude could be reduced by extending the duration. The overall level would also decrease.

The PSD amplitude could be decreased for either method by widening the frequency bandwidth, although the overall level would increase.

Conclusion

The rainflow fatigue method is useful for deriving "equivalent" levels, but further work is needed.

Furthermore, the rainflow fatigue method is rather concrete because it relies on direct counting of actual cycles.

Other equivalency methods use the area under the normal distribution curve to evaluate the dwell time within certain amplitude ranges, such as the method in Reference 3. These methods are somewhat abstract and rely on probabilities.

References

1. T. Irvine, Rainflow Cycle Counting in Fatigue Analysis, Rev A, Vibrationdata, 2010.

2. ASTM E 1049-85 (2005) Rainflow Counting Method, 1987.

3. T. Irvine, Sine and Random Vibration Equivalent Damage, Rev A, Vibrationdata, 2004.