RIGID-BODY ROTATIONAL ACCELERATION OF A BEAM MOUNTED VIA SPRINGS TO A FRAME

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The following derivation is made to determine the spring forces in a rigid beam subjected to a quasi-steady rotational acceleration. The beam is mounted via springs to a rigid frame which is rotating about point A.

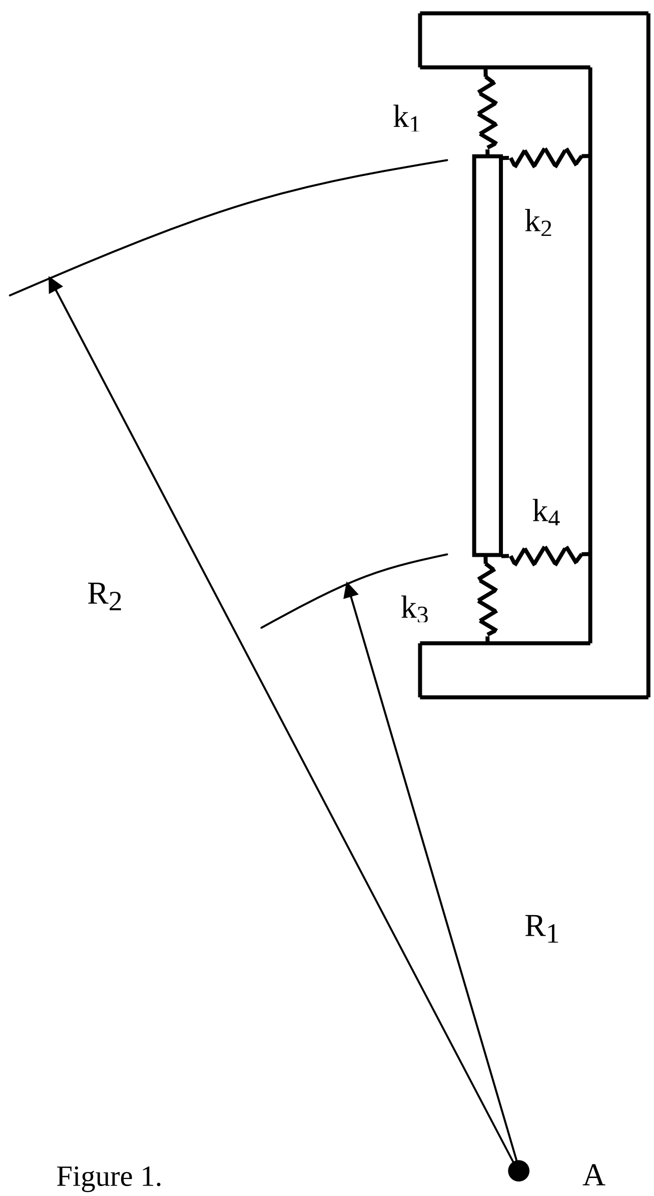

Figure 1.

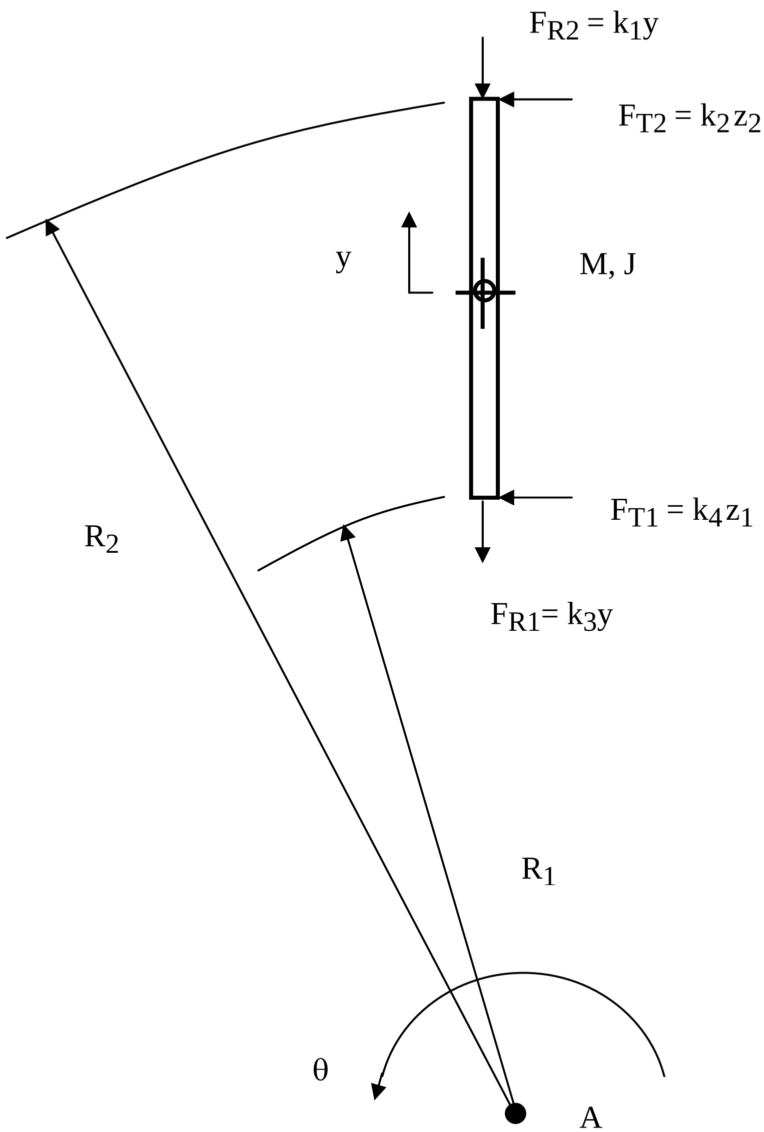

Figure 2.

Each of the terms y, z_1, z_2 represents a relative spring deflection.

Summation of the forces in the radial axis.

$$M\dot{\theta}^2 R_{cg} = (k_1 + k_3)y \tag{1}$$

$$y = \frac{M\dot{\theta}^2 R_{cg}}{(k_1 + k_3)} \tag{2}$$

$$F_{R2} = k_1 y \tag{3}$$

$$F_{R1} = k_3 y \tag{4}$$

Summation of the moments about point A.

$$J_A \ddot{\theta} = F_{T1} R_1 + F_{T2} R_2 \tag{5}$$

Summation of the tangential force.

$$M\ddot{\theta} R_{cg} = F_{T1} + F_{T2} \tag{6}$$

Assemble the equations in matrix form.

$$\begin{bmatrix} 1 & 1 \\ R_1 & R_2 \end{bmatrix} \begin{bmatrix} F_{T1} \\ F_{T2} \end{bmatrix} = \begin{bmatrix} M\ddot{\theta} R_{cg} \\ J_A \ddot{\theta} \end{bmatrix} \tag{7}$$

$$\det \begin{bmatrix} 1 & 1 \\ R_1 & R_2 \end{bmatrix} = R_2 - R_1 \tag{8}$$

$$F_{T1} = \det \begin{bmatrix} M\ddot{\theta} R_{cg} & 1 \\ J_A \ddot{\theta} & R_2 \end{bmatrix} / [R_2 - R_1] \tag{9}$$

$$F_{T2} = \det\begin{bmatrix} 1 & M\ddot{\theta}R_{cg} \\ R_1 & J_A\ddot{\theta} \end{bmatrix} / [R_2 - R_1] \tag{10}$$

$$F_{T1} = \ddot{\theta}\left[\frac{M\,R_{cg}R_2 - J_A}{R_2 - R_1}\right] = k_4\,z_1 \tag{11}$$

$$F_{T2} = \ddot{\theta}\left[\frac{J_A - MR_{cg}R_1}{R_2 - R_1}\right] = k_2\,z_2 \tag{12}$$

Note that

$$J_A = J + M\,R_{cg}{}^2 \tag{13}$$