STRUCTURAL RESPONSE TO AN APPLIED POINT FORCE IN TERMS OF MECHANICAL IMPEDANCE

By Tom Irvine
Email: tomirvine@aol.com

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Variables

Z_F	=	driving point impedance
C_L	=	longitudinal wave speed
ρ	=	mass/volume
h	=	thickness
w	=	width
S	=	cross-section area
F_o	=	applied force
V_o	=	velocity at force input location

Structural Waveforms

The purpose of this tutorial is to consider the propagation of forced-induced waveforms in mechanical structures. The excitation source is assumed to be an applied harmonic force at a discrete point. Furthermore, the source may be a pyrotechnic shock pulse.

The resulting structural velocity is assumed to be proportional to the mechanical impedance for the given structure and its waveform. Note that the mechanical impedance is a measure of how a structure resists external forces or moments, per Reference 1.

Longitudinal and bending waves[1] are considered in this report. Each of these waveforms may be regarded as structural-borne sound.[2]

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[1] Bending waves are also referred to as flexural waves.

[2] Airborne sound waves are longitudinal.

The following waveforms are neglected for brevity but may be considered in a future revision:

1. Shear
2. Torsion
3. Love
4. Rayleigh
5. Lamb

Driving Point Impedance

The driving point impedance $\tilde{Z}_F$ for an applied harmonic force is

$$\tilde{Z}_F = \frac{\tilde{F}_o}{\tilde{V}_o} \qquad (1)$$

Each of the variables in equation (1) consists of both amplitude and phase.[3]

As a simplification, the impedance phase angle will be omitted in this report.

As a further simplification, the impedance will be reduced to a scalar quantity in the remainder of this report. This is achieved by considering the structure to be "infinite," or at least "semi-infinite," so that the modal responses, or equivalent standing waves, are negligible.

Beam, Longitudinal Waves

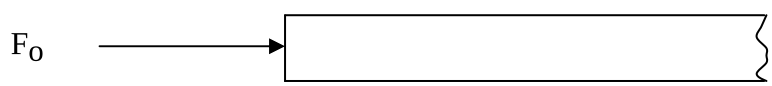

Figure 1. Semi-infinite Beam

The following driving impedance formula is taken from Reference 2, p 317. It applies to beams or rods where the excitation axis is longitudinal.

$$Z_F = C_L \rho S \qquad (2)$$

[3] Or equivalent magnitude and phase.

Note that $S = hw$ for a rectangular cross-section.

As an example, assume that a pyrotechnic device applies a point force excitation to a beam with a rectangular cross-section only exciting longitudinal waves.

The beam is aluminum, 0.125 in thick. The beam is "sufficiently long" that the response at the excitation point is dominated by traveling waves rather than by modes.

The resulting velocity in the longitudinal axis is measured at the force application point. Perform a trade study to determine how an increase in thickness would reduce the velocity response. Note that the width is unspecified but fixed.

Table 1. Beam Example, Longitudinal Waves, Velocity Reduction at Force Application Point

Thickness (in)	Reduction (dB)
0.125	0
0.25	6
0.5	12
1	18
2	24
4	30
6	34

Thin Plate, Bending Waves

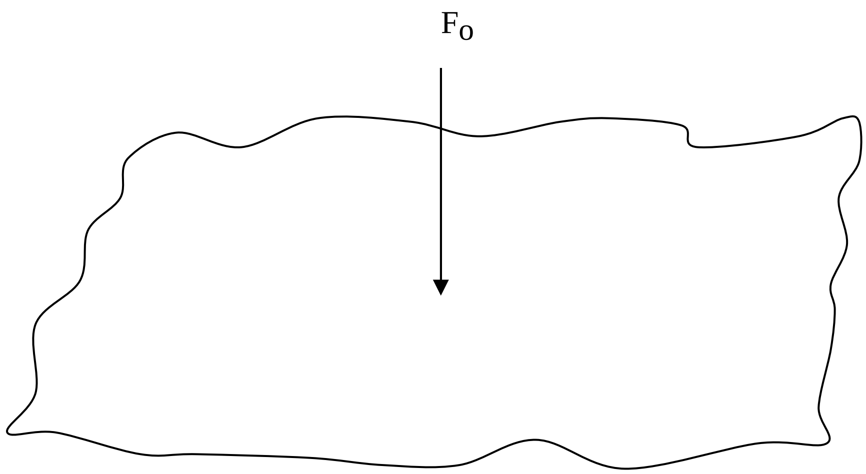

Figure 2. Infinite Plate

The following driving impedance formula is taken from Reference 2, p 317.

$$Z_F \approx 2.3 C_L \rho h^2 \quad (3)$$

As an example, assume that a pyrotechnic device applies a point force excitation to a plate exciting bending waves only.

The plate is aluminum, 0.125 in thick. The plate length and width are each “sufficiently large” that the response at the excitation point is dominated by traveling waves rather than by modes.

The resulting velocity in the perpendicular axis is measured at the force application point. Perform a trade study to determine how an increase in thickness would reduce the velocity response.

Table 2. Plate Example, Bending Waves, Velocity Reduction at Force Application Point

Thickness (in)	Reduction (dB)
0.125	0
0.25	12
0.5	24
1	36
2	48
4	60
6	67

Note that a separate impedance formula must be used for “thick plates,” but the transition thickness is not immediately clear.

References

1. L. Beranek & I. Ver, Noise and Vibration Control Engineering, Principles and Applications, Wiley, New York, 1992.

2. L. Cremer and M. Heckl, Structure-Borne Sound, Springer-Verlag, New York, 1988.

3. T. Irvine, Radiation & Driving Point Impedance of a Thin, Isotropic Plate, Rev A, Vibrationdata, 2006.