A METHOD FOR USING AN EQUIVALENT SHOCK SPECIFICATION TO COVER A PURE SINE VIBRATION REQUIREMENT

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Introduction

This tutorial considers two potential scenarios:

1. A component has been previously tested to a shock level. The goal is to show that the shock test covers a sine requirement.
2. A component has a sine requirement. The amplitude is too high to perform on a shaker table. A shock specification must be derived to cover the sine requirement.

In either case, the shock specification must envelop the sine level in terms of both peak response level and fatigue. The fatigue requirement tends to be the more rigorous consideration.

Furthermore, the shock level is assumed to be in the form of a shock response spectrum.

Model

Consider a single degree-of-freedom system.

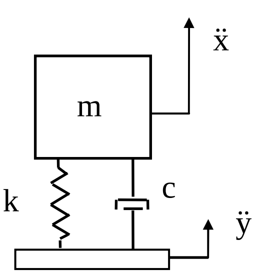

Figure 1.

where

m	is the mass
c	is the viscous damping coefficient
k	is the stiffness
x	is the absolute displacement of the mass
y	is the base input displacement

The double-dot denotes acceleration.

By convention,

$$\omega_n = \sqrt{\frac{k}{m}} \tag{1}$$

$$\xi = \frac{c}{2\omega_n m} = \frac{c}{2\sqrt{km}} \tag{2}$$

Note that

$$Q = \frac{1}{2\xi} \tag{3}$$

The sine transmissibility magnitude (G out / G in) is

$$\left| H(\rho) \right| = \sqrt{\frac{1 + (2\xi\rho)^2}{(1-\rho^2)^2 + (2\xi\rho)^2}} \tag{4}$$

where

$$\rho = \frac{\omega}{\omega_n} = \frac{f}{f_n}$$

Equation (4) is taken from Reference 1.

Assume as a sine acceleration function $Y(\hat{f})$ at a discrete frequency $\hat{f}$.

The sine response spectrum $X(f_n)$ is:

$$X(f_n) = \left|H(\rho)\right|\left|Y(\hat{f})\right| \tag{5}$$

where

$$\rho = \frac{\hat{f}}{f_n}$$

Amplification Factor

The Q value should be taken from test data.

Otherwise, a Q value must be assumed. A higher Q value will yield a more conservative equivalent shock test level.

A general guideline is: $10 \leq Q \leq 50$.

Fatigue Exponent

The fatigue exponent is denoted by b. It is taken from the S-N curve.

Typically, $4 \leq b \leq 6.4$

Note that b=6.4 is taken from Reference 2, equation (8.3).

Peak Level

The SRS must satisfy

$$SRS(f_n) \geq X(f_n) \tag{6}$$

where $X(f_n)$ is taken from equation (5).

Fatigue

Shock Fatigue

The formula for the shock fatigue damage is

$$\text{Shock Fatigue Damage}(f_n) = N\left[\,\mathrm{SRS}(f_n)\,\right]^b \sum_{i=1}^{\infty} D^i$$

$$= \frac{N}{1-D}\left[\,\mathrm{SRS}(f_n)\,\right]^b \tag{7}$$

where

N = number of hits per axis

$$D = \exp\left[\frac{2\pi\xi}{\sqrt{1-\xi^2}}\right]$$

In addition, the index “i” represents the cycle number, in equation (7).

The convergence of the infinite sum assumes $-1 < D < 1$.

Typically,

$$0 < \xi < 1.$$

Thus,

$$0 < D < 1.$$

And convergence is guaranteed.

Furthermore, the D term accounts for the reverberation of the natural response of the SDOF oscillator assuming exponential decay.

Sine Fatigue

The formula for the sine fatigue damage is

$$\text{Sine Fatigue Damage}\,(f_n) \;=\; f_n T \left| X(f_n) \right|^b \tag{8}$$

where T is the sine duration

Comparison of Sine and Shock with respect to Fatigue

The following requirement must be met:

$$\text{Shock Fatigue Damage}\,(f_n) \geq \text{Sine Fatigue Damage}\,(f_n) \tag{9}$$

$$\frac{N}{1-D}\left[\,\mathrm{SRS}(f_n)\,\right]^b \;\geq\; f_n T \left| X(f_n) \right|^b \tag{10}$$

$$\mathrm{SRS}(f_n) \;\geq\; \left[\frac{(1-D)\, f_n T}{N}\right]^{1/b} \left| X(f_n) \right| \tag{11}$$

<u>Matlab Script</u>

The equations in this tutorial are embedded in the Matlab script: shock_sine.m

Example

Assume that a component must be tested to a sine test with the following specifications

Parameter	Value
Frequency	500 Hz
Amplitude	100 G
Duration	5 seconds

Assume that the shock test will be 3 hits per axis. Also, assume that the component's properties are

Property	Value
Natural Frequency	$100 < fn < 10{,}000$ Hz
Amplification Factor	$Q = 10$
Fatigue Exponent	$b = 6.4$

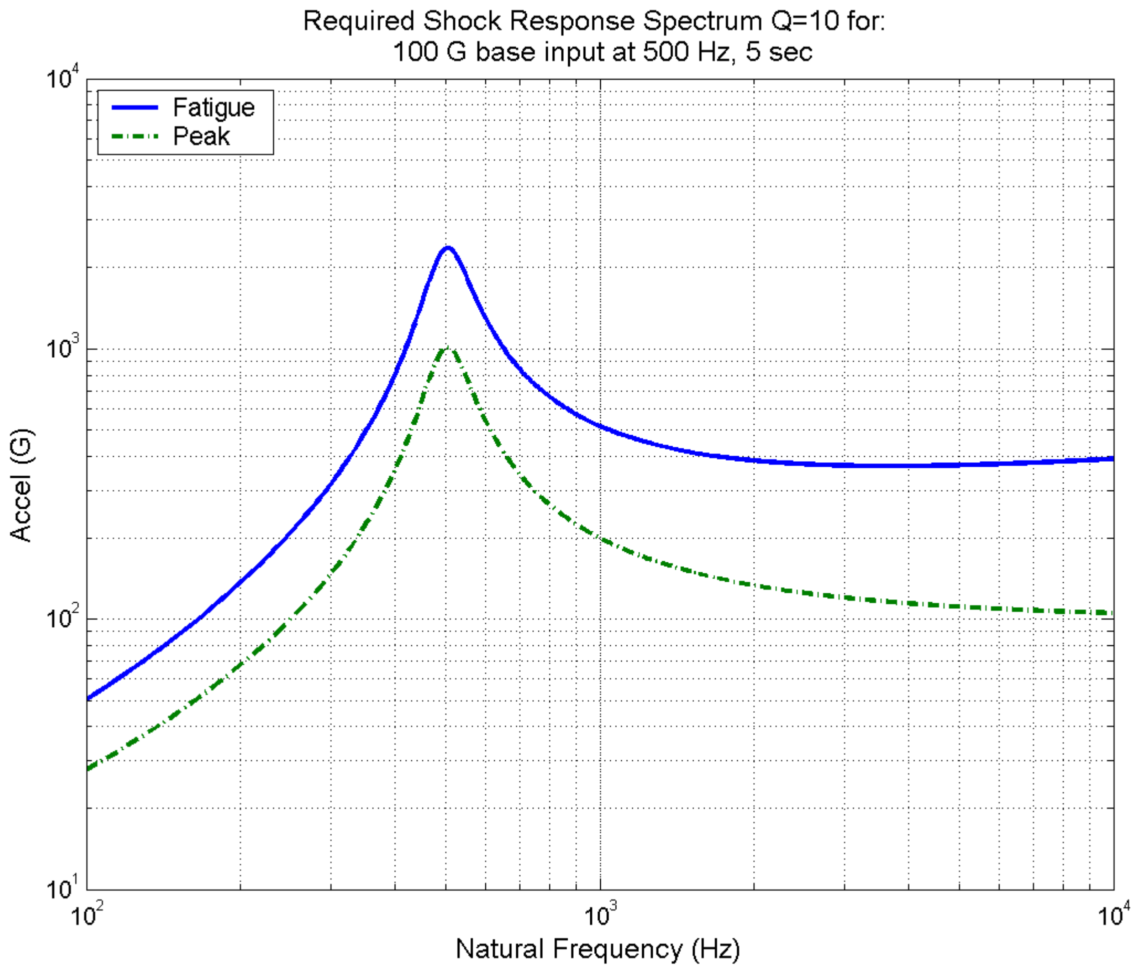

Figure 2.

The resulting curves for the Peak and Fatigue are given in Figure 2.

The Fatigue curve thus becomes the minimum SRS requirement.

Note that a typical SRS specification would have an initial ramp, followed by a plateau.

<u>References</u>

1. T. Irvine, The Steady-State Response of a Single-Degree-of-Freedom System Subjected to a Harmonic Base Excitation, Revision A, Vibrationdata, 2007.

2. D. Steinberg, Vibration Analysis for Electronic Equipment, Third Edition, Wiley-Interscience, New York, 2000.