SHOCK AND RANDOM VIBRATION EQUIVALENCE

By Tom Irvine
October 8, 2010

Email: tomirvine@aol.com

Introduction

Certain components must be designed and tested to verify that they can withstand a mechanical shock environment.

The shock specification may be given as a half-sine pulse, or some other classical pulse. Or the specification may be a shock response spectrum (SRS). Note that an SRS may be calculated for any time history, including classical pulses.

In some cases, a desired goal may be to show that a random vibration test covers the required shock specification for a given component. The random vibration test is conventionally specified as a power spectral density (PSD).

The purpose of this paper is to show how a random vibration test specification can be used to cover a shock specification in terms of peak response.

Model

The first step is to determine the acceleration response of the component to both the shock and vibration environments.

The component is modeled as an SDOF system as shown in Figure 1.

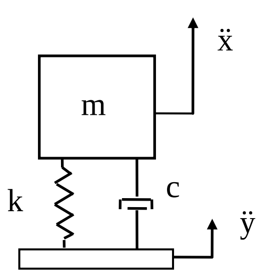

Figure 1.

where

m	is the mass
c	is the viscous damping coefficient
k	is the stiffness
x	is the absolute displacement of the mass
y	is the base input displacement

The natural frequency of the system fn is

$$\mathrm{fn} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{1}$$

The amplification factor Q is related to the viscous damping ratio ξ by

$$Q = \frac{1}{2\xi} \tag{2}$$

A shock pulse could then be applied as a base excitation to the SDOF system in Figure 1. The response of the mass could then be calculated via the method in Reference 1. The resulting response could then be presented as an SRS which gives the peak acceleration response as a function of natural frequency with a fixed damping ratio.

Furthermore, a random vibration PSD could be applied to the base of the SDOF system in Figure 1. The response of the mass could then be calculated via the method in Reference 2. The resulting response could then be presented as a vibration response spectrum (VRS) which gives the acceleration response as a function of natural frequency with a fixed damping ratio.

The VRS amplitude is typically given in terms of RMS or 3σ.

The standard deviation is 1σ.

Note the following relationships assuming a zero mean:

1σ = RMS

3σ = 3 x RMS

The 3σ value is often considered to be the peak value. But a better estimate of the peak λ is

$$\lambda = \sqrt{2 \ln (\mathrm{fn}\, T)} \tag{3}$$

where

λ	is a scale factor for the σ value
fn	is the natural frequency
T	is the duration

Equation (3) is taken from References 3 and 4. It models the response peaks via a Rayleigh distribution. The instantaneous base input values are assumed to have a normal distribution.

For example, a 4.33σ peak response is expected for a system with a natural frequency of 200 Hz exposed to random vibration over a 60 second duration.

$$\lambda = \sqrt{2 \ln (\mathrm{fn}\, T)} = \sqrt{2 \ln (200\,\mathrm{Hz})(60\,\mathrm{sec})} = 4.33 \tag{4}$$

Note that the VRS peak amplitudes can be expressed by applying the λ scale factor to the 1σ response, or equivalently to the RMS value assuming zero mean.

<u>Example</u>

A component must be subjected to a qualification shock test which is a 20 G, 11 msec half-sine pulse, with a single pulse. The component has already passed the random vibration test in Table 1. Determine whether the random vibration test covers the shock requirement.

Table 1. Qualification PSD, 180 seconds, 12.3 GRMS Overall	
Frequency (Hz)	Accel (G^2/Hz)
20	0.0212
150	0.16
600	0.16
2000	0.0144

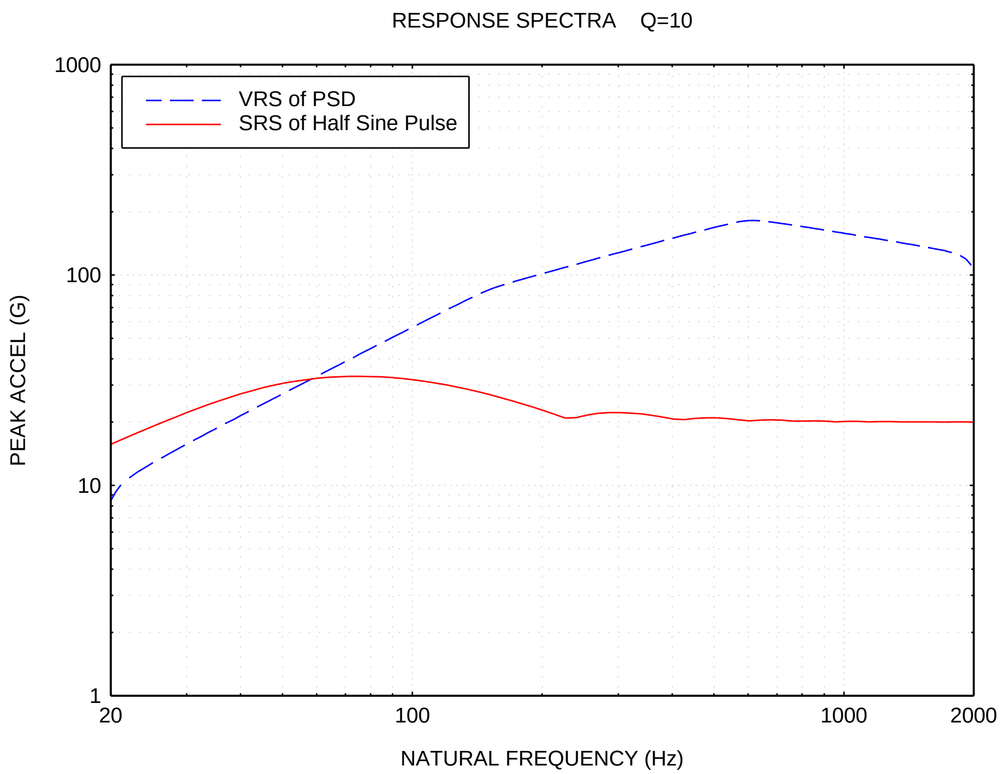

Figure 2.

The VRS curve is calculated from the random vibration PSD in Table 1, with the λ scale factor applied. Note that λ depends on both duration and natural frequency. The duration is 180 seconds. The natural frequency is an independent variable.

Again, the half-sine pulse was 20 G, 11 msec.

The resulting comparison shows that the random vibration test covers the half-sine pulse specification as long as the natural frequency of the component is 60 Hz or higher.

References

1. T. Irvine, An Introduction to the Shock Response Spectrum, Revision R, Vibrationdata, 2010.

2. T. Irvine, An Introduction to the Vibration Response Spectrum, Revison D, Vibrationdata, 2009.

3. DiMaggio, S. J., Sako, B. H., and Rubin, S., Analysis of Nonstationary Vibroacoustic Flight Data Using a Damage-Potential Basis, AIAA Dynamic Specialists Conference, 2003 (also, Aerospace Report No. TOR-2002(1413)-1838, 1 August 2002).

4. T. Irvine, Equivalent Statics Load for Random Vibration, Rev K, Vibrationdata, 2010.

5. H. Caruso and E. Szymkowiak, A Clarification of the Shock/Vibration Equivalence in MIL-STD-810D/E, The Journal of Environmental Sciences, Volume 32, Number 5 / September-October 1989.