SINGLE-DEGREE-OF-FREEDOM SYSTEM RESPONSE TO SINE-ON-RANDOM VIBRATION

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Introduction

Consider a single degree-of-freedom system.

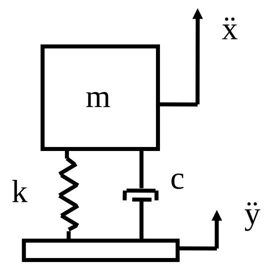

Figure 1.

where

m	=	mass
c	=	viscous damping coefficient
k	=	stiffness
x	=	absolute displacement of the mass
y	=	base input displacement

Assume that the amplification factor is Q=10. Allow the natural frequency to be an independent variable. Three natural frequencies are considered: 100, 200, 400 Hz.

The system is subjected to simultaneous sine and random base excitation per the specification in Figure 2.

Calculate the SDOF response in both the time and frequency domains.

Specification

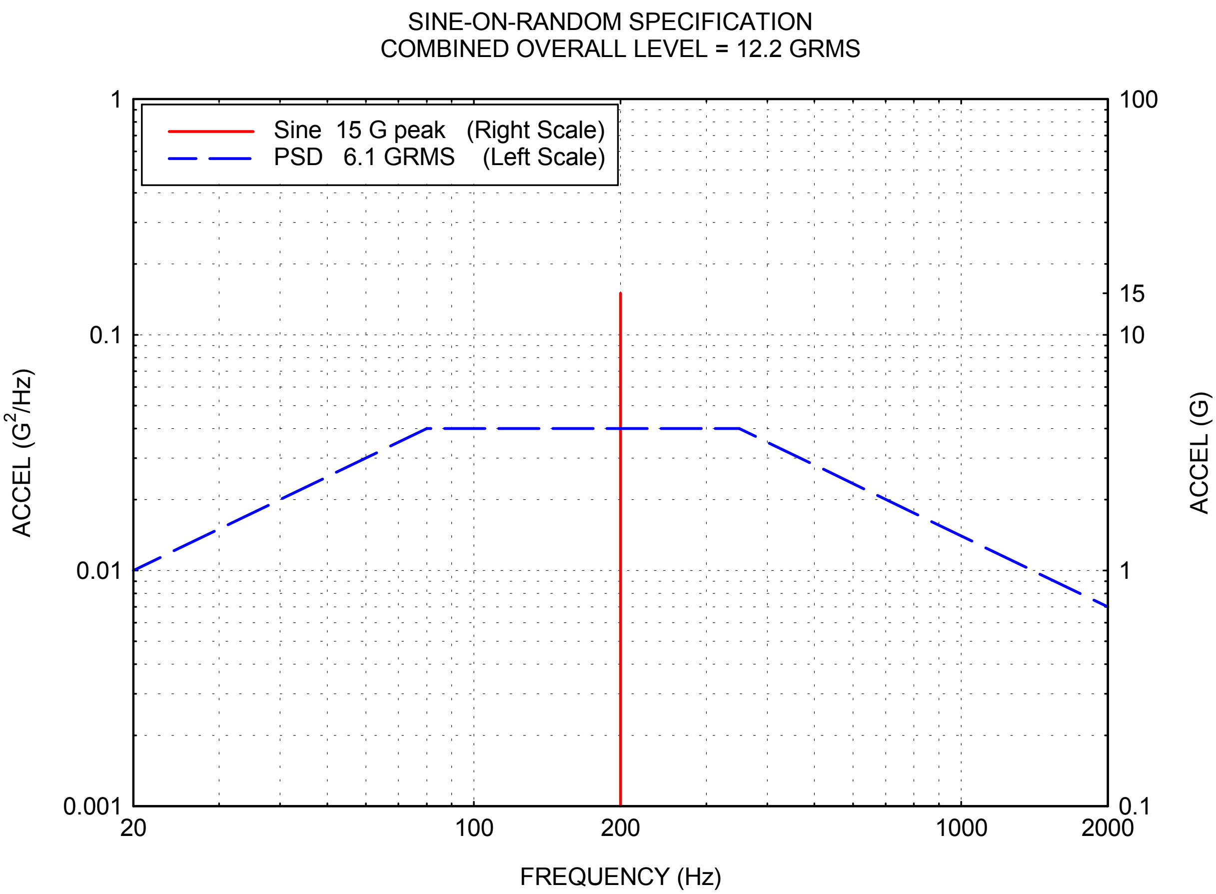

Figure 2.

Table 1. PSD, 6.1 GRMS, 60 seconds	
Frequency (Hz)	Accel (G^2/Hz)
20	0.01
80	0.04
350	0.04
2000	0.007

Time Domain Synthesis

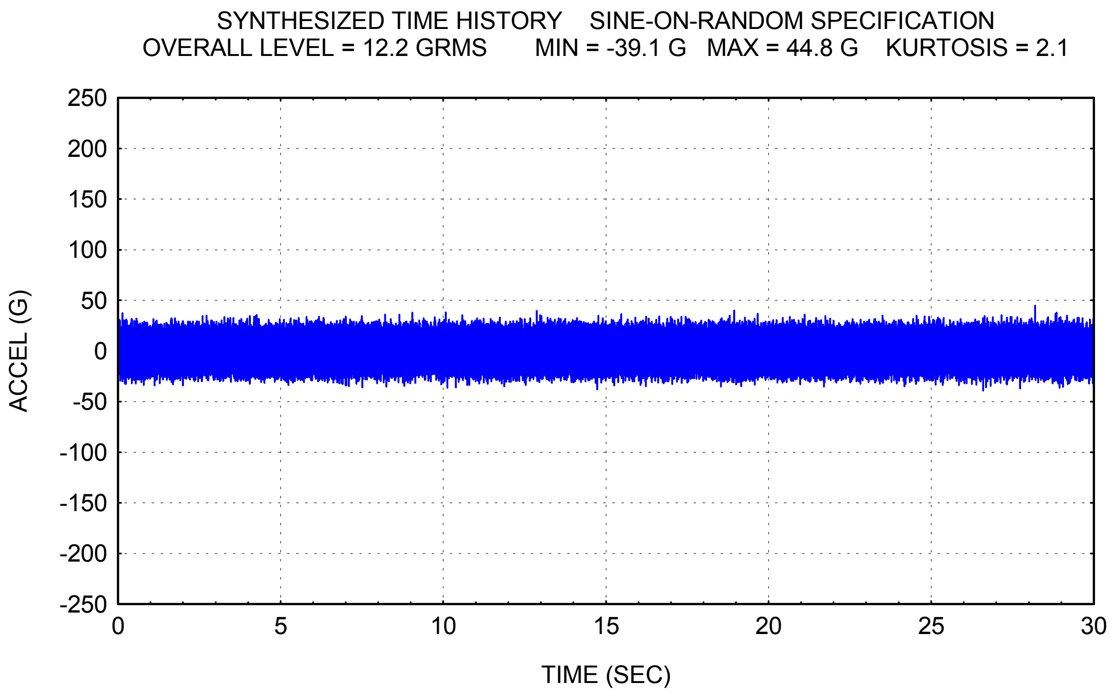

Figure 3.

A time history was synthesized in Figure 3 to meet the specification in Figure 2. The time history was shortened to 30 seconds for brevity.

A close-up view is given in Figure 4.

The corresponding histogram is shown in Figure 5. The histogram has a somewhat symmetric, bimodal shape.

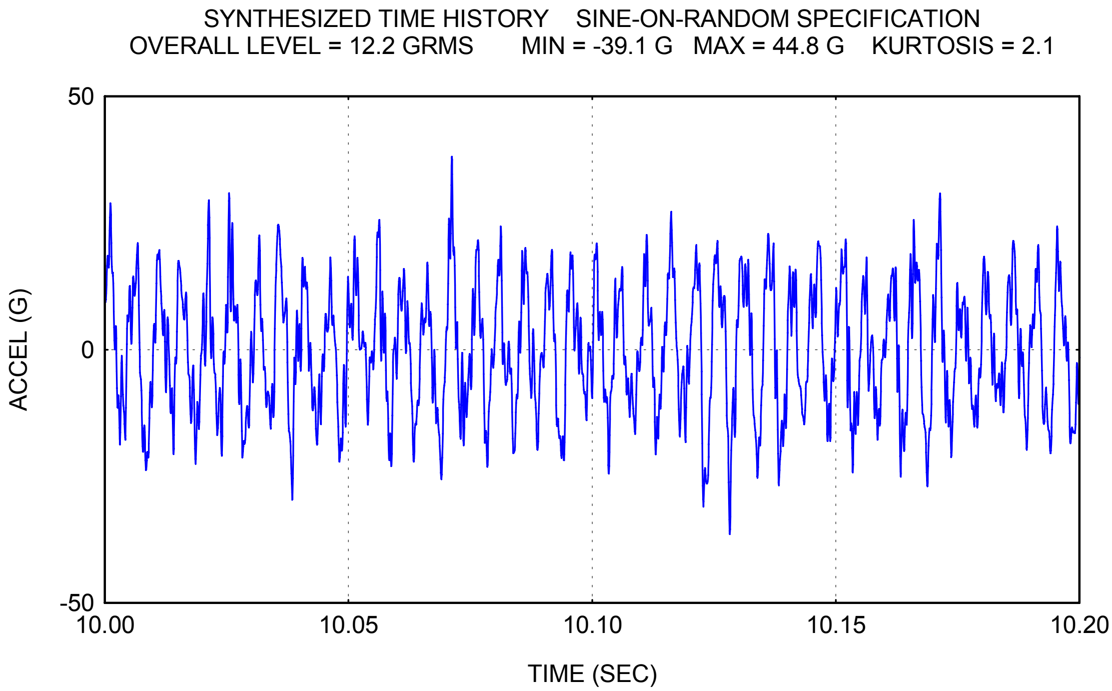

Figure 4.

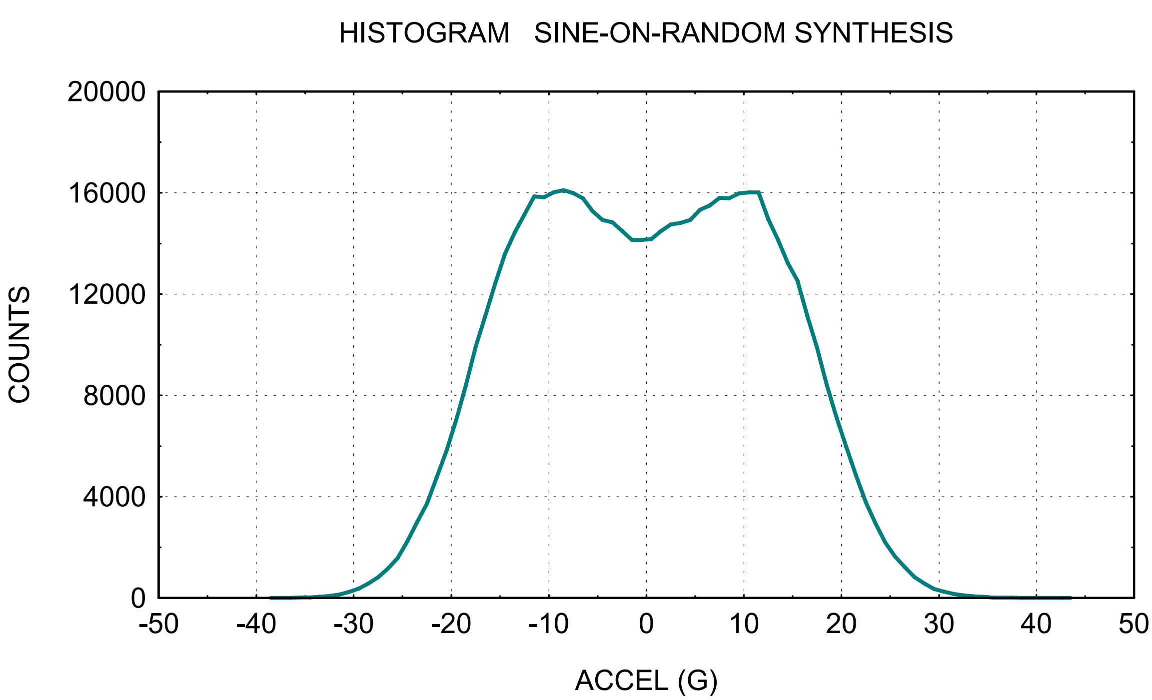

Figure 5.

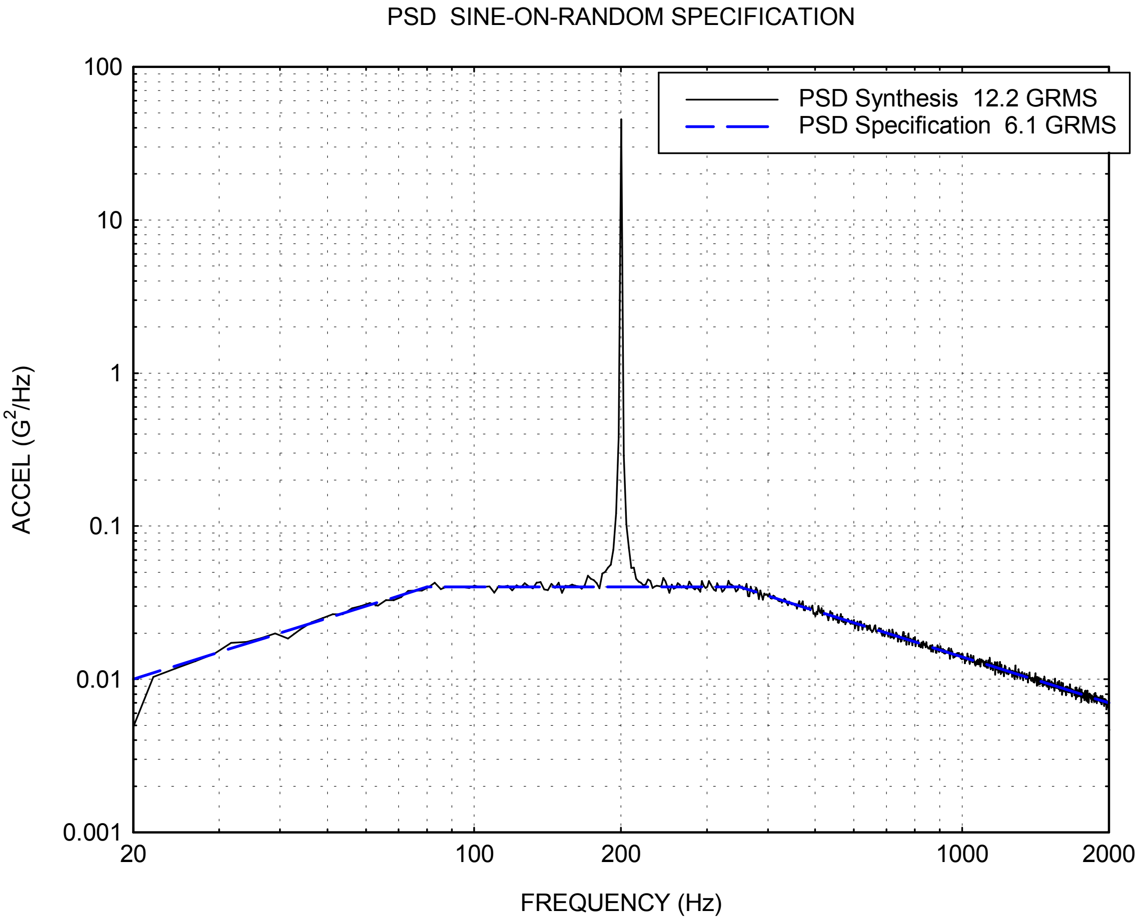

Figure 6.

The PSD of the synthesis is compared to that of the specification in Figure 6.

Note that the G^2/Hz amplitude at 200 Hz depends on the Δf bandwidth. There is no exact way to convert pure sine G levels into G^2/Hz levels.

Time Domain Response, fn =100 Hz

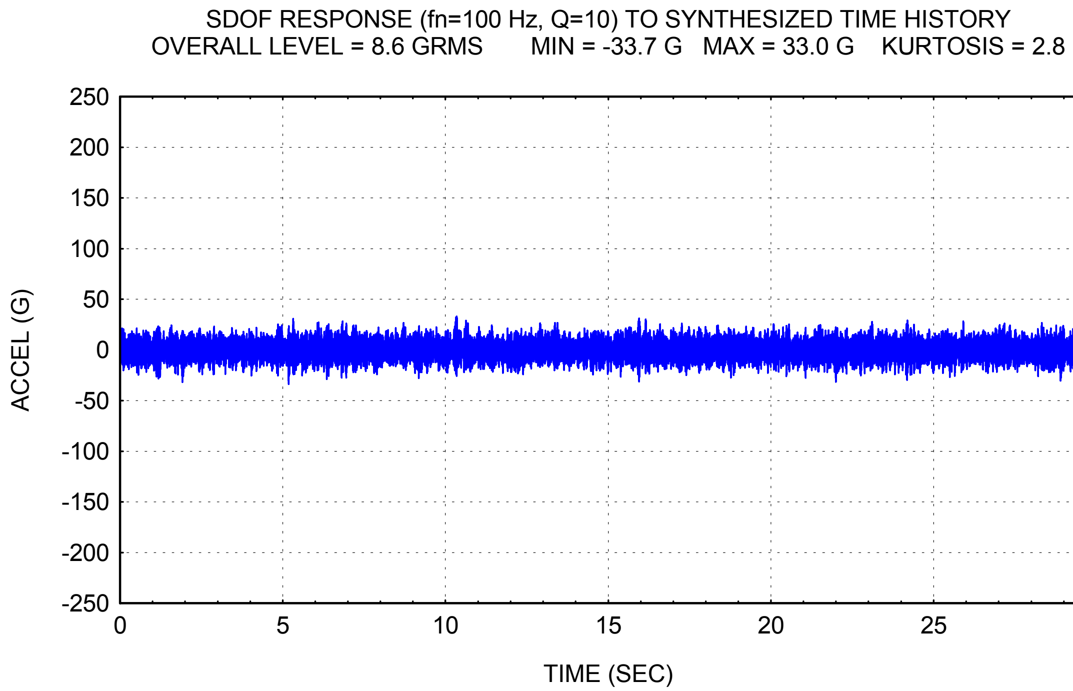

Figure 7.

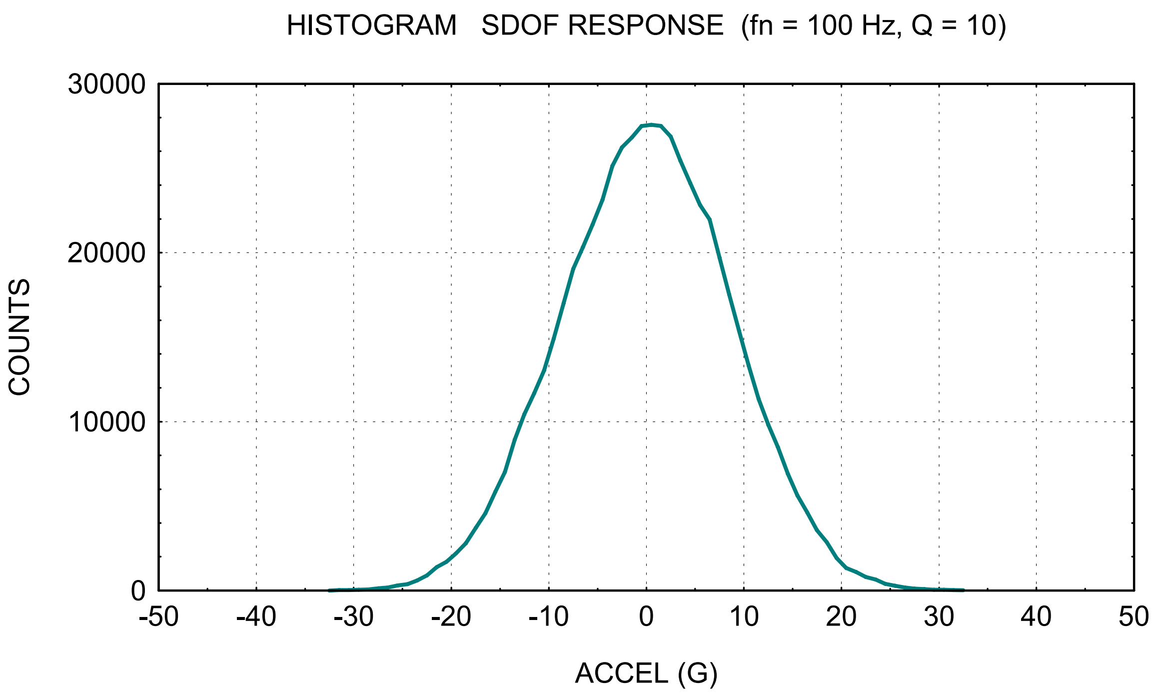

Figure 8.

Time Domain Response, fn =200 Hz

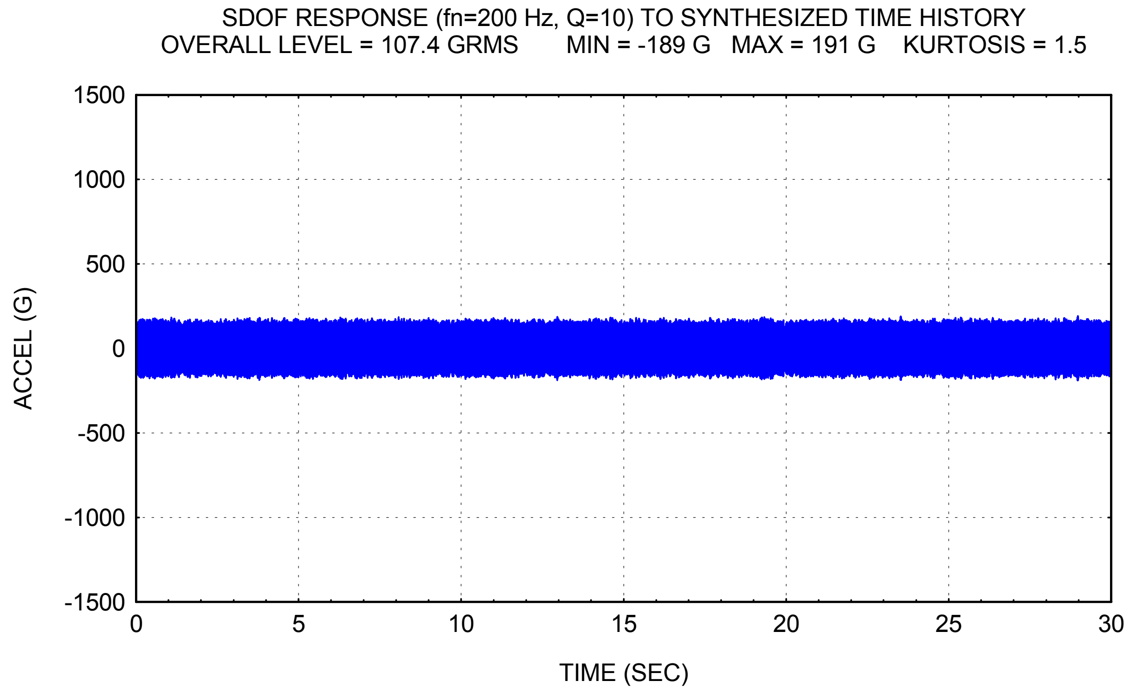

Figure 9.

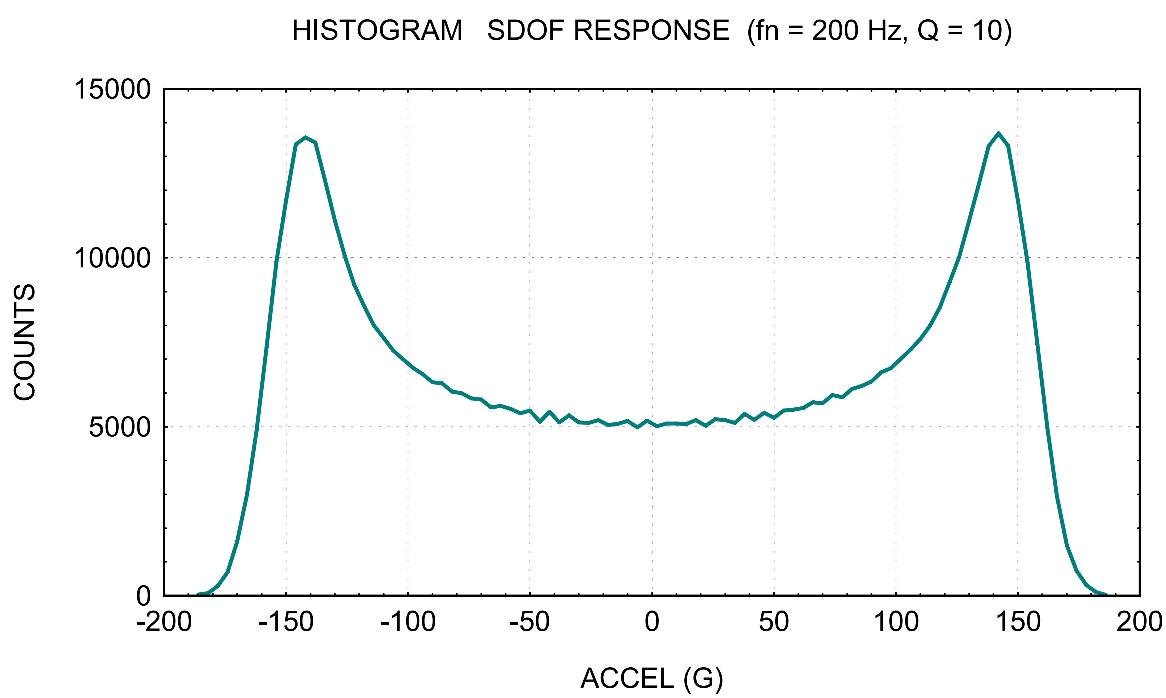

Figure 10.

Time Domain Response, fn =400 Hz

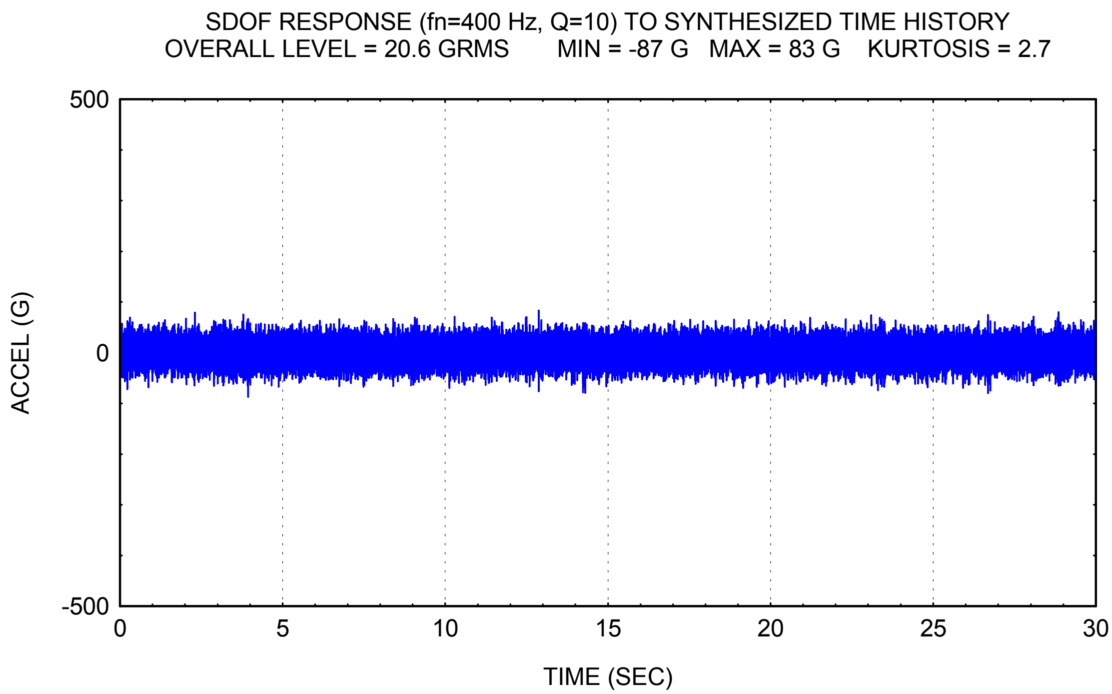

Figure 11.

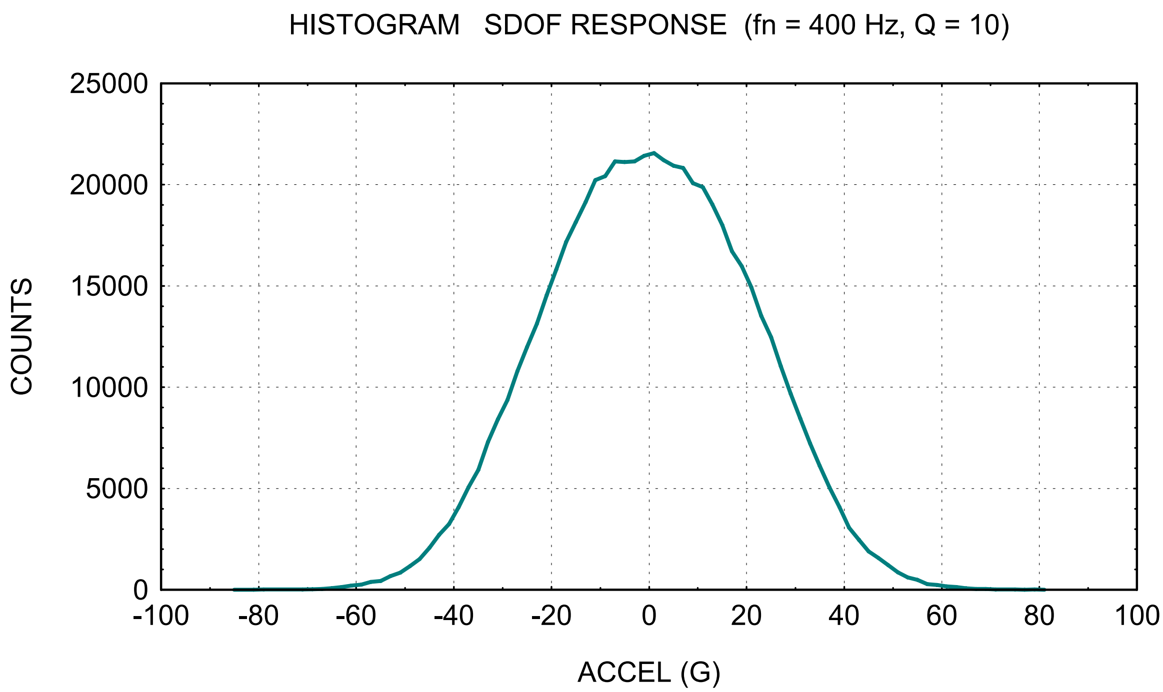

Figure 12.

Frequency Domain Response

The frequency domain calculations are performed using the equations in Appendix A. Each Combined Response value is the "square root of the sum of the squares."

The Combined Response values agree very closely with the corresponding levels calculated in time domain.

Table 2. Response Levels from Frequency Domain Analysis

fn (Hz)	Response to Sine (GRMS)	Response to Random (GRMS)	Combined Response (GRMS)
100	3.6	7.82	8.6
200	106.8	11.18	107.3
400	14.1	14.9	20.5

References

1. T. Irvine, A Method for Power Spectral Density Synthesis, Rev B, Vibrationdata, 2000.
2. T. Irvine, An Introduction to the Vibration Response Spectrum Rev C, Vibrationdata, 2000.
3. T. Irvine, The Steady-state Response of Single-degree-of-freedom System to a Harmonic Base Excitation, Vibrationdata, 2004.

APPENDIX A

Variables

ξ	=	fraction of critical damping
f_n	=	natural frequency
f_i	=	base excitation frequency
ρ_i	=	non-dimensional frequency parameter
A	=	sine acceleration amplitude
$\hat{Y}_{APSD}$	=	PSD base input acceleration
$\ddot{x}_{GRMS}$	=	overall acceleration GRMS response

Response to Sine Vibration

The peak response is

$$\ddot{x} = A\sqrt{\frac{1+\left(2\xi\rho_i\right)^2}{\left(1-\rho_i^2\right)^2+\left(2\xi\rho_i\right)^2}}\ ,\ \ \rho_i = f_i / f_n \qquad \text{(A-1)}$$

The corresponding RMS value is calculated by multiply the peak response by 0.7071.

Equation (A-1) is taken from Reference 3.

Response to Random Vibration

The overall acceleration response is

$$\ddot{x}_{GRMS}\left(f_n,\xi\right)=\sqrt{\sum_{i=1}^{N}\left\{\frac{1+\left(2\xi\rho_i\right)^2}{\left[1-\rho_i^2\right]^2+\left[2\xi\rho_i\right]^2}\right\}\hat{Y}_{APSD}(f_i)\Delta f_i}\ ,\quad \rho_i=f_i/f_n \qquad \text{(A-2)}$$

Equation (A-2) is taken from Reference 2.

APPENDIX B

Kurtosis is a parameter that describes the shape of a random variable's histogram or its equivalent probability density function.

The kurtosis for a time series Y_i is

$$\text{Kurtosis} = \frac{\sum_{i=1}^{n} \left[Y_i - \mu \right]^4}{n\sigma^4}$$

where

μ	=	mean
σ	=	standard deviation
n	=	number of samples

The term in the numerator is the "fourth moment about the mean."

A pure sine time history has a kurtosis of 1.5.

A time history with a normal distribution has a kurtosis of 3.

Some alternate definitions of kurtosis subtract a value of 3 so that a normal distribution will have a kurtosis of zero.

A kurtosis larger than 3 indicates that the distribution is more peaked and has heavier tails than a normal distribution with the same standard deviation.